Sample Maple Assignment
Toy rocket Problem : Winter 2000

A toy rocket is launched from **ro** = xo **i** + yo **j** with an inital velocity of **vo** =vox **i** +voy **j**. It burns f

We now write Newton's second law for the motion of the rocket in the form

Fext = d**p**/dt = [m(**v**+**dv**) + dm **ufuel**) – (m+dm) **v**]/dt, (1)

where ufuel is the velocity of the fuel relative to the earth, dm is the mass of the fuel ejected in the time dt, and the external force is gravitational force

Fext = – mg **j** (2)

This equation will be solved numerically following the motion of the rocket in time intervals of delt. The mass of the rocket decreases because fuel is ejected, and if we use the index i to count the steps, with i = 0,1,2,...., N, the mass is:

m[i] = mo – dm/dt *delt * i. (3)

Re-arranging equation (1) allows us to find the velocity at a time t = i delt:

– g **j** = d**v**/dt + [dm/dt (**ufuel – v**)]/m, (4)

where we take as our numerical approximation

d**v** = **v**[i] – **v**[i–1] and (**ufuel – v**) = velocity of fuel relative to the rocket = – ufr **v**/v,

v[i] = sqrt(vx[i–1]^2 + vy[i–1]^2).

Incorporating these assumptions into equation (4) allows us to step from the initial velocity **vo** to future velocities with an approximate solution that improves as delt gets sufficiently small. Once the velocities are known in time steps, the x and y positions can be obtained as d**r** = **v** dt, starting from the initial position **ro** .

The program for this is given below. Let us assume the following: Initial mass of the rocket is 5.0 kg, Burn rate = 0.33 kg/s, velocity of fuel relative to the rocket= 100 m/s, initial velocity of projection is 14 m/s and the angle of projection is 45°.

```
> N:=110:delt:=0.025:mo:=5.0:dmdt:=0.33:g:=9.8:ufr:=100.0:vo:=14.0:a
  ngle:=45.0*3.14159265/180.0:vy[0]:=vo*sin(angle):
  vx[0]:=vo*cos(angle):x[0]:=0.0:y[0]:=0.0: ti[0]:=0.0:
  vel[0]:=sqrt(vx[0]^2+vy[0]^2):
```

above:

```
> 
  for i from 0 to N do
  v[i]:=sqrt(vx[i]^2+vy[i]^2);
  m[i]:=mo-dmdt*i*delt;
  ti[i]:=delt*i;
  vy[i+1]:=vy[i]-g*delt+dmdt/m[i]*ufr*delt*vy[i]/v[i];
  vx[i+1]:=vx[i]+dmdt/m[i]*ufr*delt*vx[i]/v[i];
  x[i+1]:=x[i]+vx[i]*delt;
  y[i+1]:=y[i]+vy[i]*delt; if (y[i] < 0.5 )and (y[i] > -1) then
  print(x[i],y[i]) fi;
  od:
```

0, 0

.2474873736, .2474873733

.4978915626, .4917665621

45.52521773, .2041067972

46.19918365, -.1398240731

46.87756732, -.4921343571

47.56036128, -.8528600714

Note that we have printed some data points when the toy-rocket passes through the same level of projection.
Finally, we plot these calculate points x versus y in a point graph. This is the trajectory of the rocket.

```
> 
  plot([[x[n],y[n]]$n=0..N],style=point);
```