Sample Maple Program

The Numerical Evaluation of the Motion of a Mass Attached to an Elastic Spring

In this sample problem, Newton's second law is integrated numerically to produce the motion of a mass m on a spring with constant k. Newton's second law is

- k x = m dv/dt.

To intergrate this numerically, we take discrete values for time (t), postion (x) and velocity (v). Thus, the times are t[0], t[1], t[2],...,t[n], the positions are x[0], x[1], x[2],...,x[n], and the velocities are v[0], v[1], v[2],...,v[n]. The second law then becomes

-k(x[i]+x[i-1])/2 = m (v[i] - v[i-1])/delt,   (1)

where x is replaced by its average value, and dx -> x[i]-x[i-1], dt ->delt = t[i] - t[i-1].   Note that x[i] is the position at time t[i] and v[i] is the velocity at time t[i].   The velocity is related to position as

v = dx/dt,

and this translates into discrete language as

(v[i] +v[i-1])/2 = (x[i]-x[i-1])/delt.   (2)

Putting equations (1) and (2) together, we get

x[i] = x[i-1] (1 - k delt^2/(4m))/ (1 + k delt^2/(4m)) + v[i-1] delt/ (1 + k delt^2/(4m)).   (3)

The values of positon at time t[i] are found form equation (3) from the values of positon and velocity at the preceding time t[i-1]. Starting from the intial values for time, position and velocity, one can then numerically evaluate the position at future times.

The Maple program below does just that, and produces a plot of x versus t, which you might recognize as the oscillating motion of an object on a spring:

```
STUDENT > k:=12;m:=0.1;delt:=0.01;x[0]:=2;v[0]:=0;t[0]:=0;fac1:=1-k*
          delt^2/(4*m);fac2:=1+k*delt^2/(4*m);
```

$$k := 12$$

$$m := .1$$

$$delt := .01$$

$$x_0 := 2$$

$$v_0 := 0$$

$$t_0 := 0$$

$$fac1 := .9970000000$$

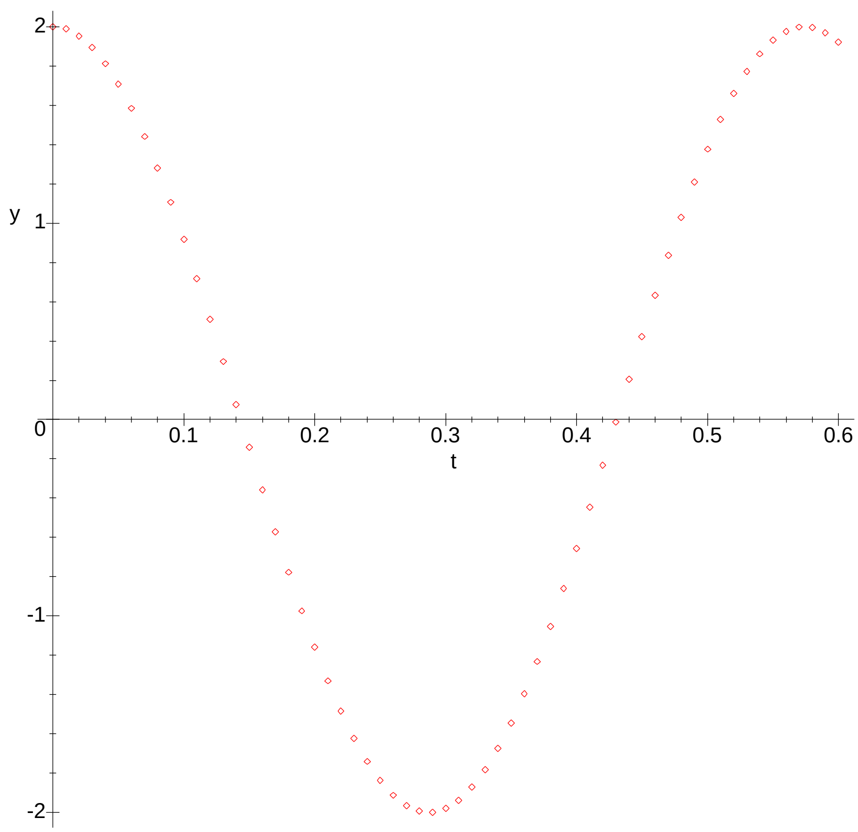
$$fac2 := 1.003000000$$

```
STUDENT > for i from 1 to 60 do
          x[i]:=x[i-1]*fac1/fac2+v[i-1]*delt/fac2;
          v[i]:=2*(x[i]-x[i-1])/delt-v[i-1];
          t[i]:=t[i-1]+delt;   od:
STUDENT >
```

Plotting points in Maple follows through a plot statement containing a loop:

```
STUDENT > plot([[t[n],x[n]]$n=0..60],labels=[t,y],title=`position
          versus time`,style=point);
```

position versus time



The potential, kinetic and total energies are now evaluated and printed in four columns t, u(potential), ek(kinetic energy) and e(total energy):

```
STUDENT > for j from 0 to 60 do
          u[j]:=k*x[j]^2/2;ek[j]:=m*v[j]^2/2;e[j]:=u[j]+ek[j]; od:
```

Looking at these results, is energy conserved? To plot the energies, we use the following plot statement:

```
STUDENT > plot({[[t[n],u[n]]$n=0..60],[[t[n],ek[n]]$n=0..60],[[t[n],
          e[n]]$n=0..60]},labels=[t,energy],title=`potential,kinetic
          and total energy`,style=point);
```