PREPARATION FOR ENGINEERING STUDIES

(a WebCT based course)

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PREPARATION FOR ENGINEERING STUDIES

Introduction

The course is divided into five **UNITS**:

UNIT 01 – Simultaneous Equations

UNIT 02 – Fundamentals of Plane Geometry

UNIT 03 – Use of Trigonometric Functions

UNIT 04 – Fundamentals of Solid Geometry

UNIT 05 – Vectors

Our experience over the past many years strongly indicates that a mastery of these topics is essential for fully benefiting from the first sequence of Engineering Physics courses offered at Drexel and elsewhere.

Each UNIT is organized in four sections:
[i] Introduction; **[ii]** Interactive Problems; **[iii]** Sample Problems; and **[iv]** Tests.

Although all UNITS are self-contained, you may find it useful to consult other texts. Familiarize yourself with the contents of each UNIT by reading the Introduction to each UNIT.

The **introductory section** to each UNIT briefly describes the main concepts contained and the skills you will learn by studying that UNIT. This section also contains solved examples at the level you will need to successfully take the Test at the end of the UNIT. Study this section thoroughly and review all the solved examples. Close all notes and see if you can solve most of the Examples yourself without referring to the solutions provided. You may study these solutions until you feel comfortable with the concepts involved and the techniques employed to solve the problems.

The next section provides three **Interactive Problems**, and leads you step by step through the solution of typical problems in that UNIT. The "Feedback" part at the end of your response will describe the rationale behind the correct response. This helps you develop a thinking process to solve problems and take tests from the unit.

The third section of every UNIT is **Sample Problems**. This is what you would ordinarily call "Homework". You should be able to solve these problems without much difficulty. We have provided answers to all such problems for you to ascertain if you have solved the problems correctly. If you find it difficult to do any of these problems, you should go back to the beginning of the UNIT and study those concepts that you find difficult or you feel uneasy with. If you feel comfortable with your mastery of a particular UNIT to your satisfaction, you are ready to take **TEST-01** of a given UNIT.

When taking the test, read the instructions carefully. You can use paper, pencil and any calculator to work out the detailed solutions to the TEST problems. You are not required to present the detailed, step-by-step solution to a Test problem pass the test. You will be asked to enter your answer in the Answer Box. Follow the instructions provided carefully about the form, significant figures, etc. that you must use to input your response. If you pass the test (do all three problems correctly) you can proceed to the next UNIT. If you do not pass the test in the first try, don't get unduly concerned. You will be given one more chance to take another test. You will go to **TEST-02** of that UNIT. After you pass the test, you are ready to go to the next UNIT. If you do not pass the test after the second trial, you can still go to the next UNIT. You must show mastery (pass the tests successfully) of at least four of the five UNITs to qualify out of taking the TDEC-199 (Preparation for Engineering Studies) during the Fall term at Drexel.

Other Useful Information

You may already be familiar with some of the topics discussed below. Some of the information deals with how you would input your answers when you take a test at the end of each UNIT. If you don't input your answer in the form required, it may be marked wrong by the computer. Since you will not get a chance to explain your answer to your computer, make sure you read the information discussed here carefully and follow the instructions when you take the tests.

Units of Measurement

The measurements of mass(M), length(L), and time(t) are, respectively, kilogram (kg), meter (m), and second (s or sec) in what is known as the International System of Units (also called the Metric or the MKS system of units) In the British (also called the FPS) system of units, the corresponding units for the same three quantities, M, L, and t, are, respectively, pound (lb), foot (ft), and second (s or sec).

Conversion of Units

Often the units of basic quantities (M, L, and s) or derived quantities (some combination of two or more of the basic quantities) may not be given in standard units. For example, the length may be given in inches or centimeters instead of feet or meters. Or the speed may be given in miles per hour and not in feet per sec. In such cases you will need to know how to convert one set of units into another. In other cases you may need to convert FPS units into MKS units. In the following we describe some examples.

Simple Conversion Factors

You should thoroughly familiarize yourself with the following simple conversions.

$1\text{ cm} = 10^{-2}\text{ m}$

$1\text{ cm}^2 = (1\text{ cm})(1\text{ cm}) = (10^{-2}\text{ m})(10^{-2}\text{ m}) = 10^{-4}\text{ m}^2$

$1\text{ cm}^3 = (1\text{ cm})(1\text{ cm})(1\text{ cm}) = (10^{-2}\text{ m})(10^{-2}\text{ m})(10^{-2}\text{ m}) = 10^{-6}\text{ m}^3$

Some simple useful prefixes are, centi- (one hundredth), milli- (one thousandth), and micro- (one millionth). For example, one millimeter is (1/1000)th of a meter or 10^{-3} m. A kilo- means a thousand times, thus a kilometer is 1000 m or 10^3 m. Similarly 1kg = 1000 grams = 10^3gm.

1 inch (1”) = 2.54 cm
1 foot = 12”
1 yard (yd) = 3 ft.
1 mile = 1760 yds.

Compound Conversions

Example 1.
Express 60miles per hr in units of ft per sec

1 mile = 1760 yd = 1760*3 ft = 5280 ft.
1 hr = 60 min = 60*60 sec = 3600 sec.
Therefore, 60 miles/hr = 60*5280 ft/ 3600 sec = 88 ft/sec.

Example-2

Density (mass per unit volume) of water is 1gm/ cm^3. Express the density of water in units of kg/ m^3.

1gm/ cm^3 = 10^{-3} kg/10^{-6} m^3 = 10^3 kg/m^3.

Significant Figures and Decimal Places

Consider a simple problem: Suppose you have a disc of aluminum and a scale that has centimeter markings with each centimeter further subdivided in ten divisions. Thus the precision of your scale is 1mm. You can not claim to measure any length with this scale to an accuracy of better than plus or minus 1 mm. Suppose the radius of the disc you measured with your scale is 7.7 cm. If you were now to calculate the area, *A*, of one of the two faces of the disc, the answer would be $A = \pi r^2 = 3.141 * (7.7)^2 = 186.22989\ cm^2$**.** Mathematically your answer is correct, but are all the digits in your answer physically meaningful? Does it make sense to claim a precision to the fifth decimal place in the measurement of area when that number is generated from measurements with two significant figures? The answer is no. One would be completely justified in quoting the answer as 190 cm^2. It is a sort of convention that you can quote your calculated answer to one more significant figure than the least precise quantity in the problem. Thus $A = 186\ cm^2$ would also be an acceptable answer. In this course you would be given explicit instructions about how to enter your answers in the ANSWER Box when you take the Test at the end of each UNIT. Some ways in which you may be asked to enter the above answer are given below.

Area to three significant figures, $A = 186\ cm^2$
Area to four significant figures, $A = 186.2\ cm^2$
Area rounded to the first decimal place, $A = 186.2\ cm^2$
Area rounded to the second decimal place, $A = 186.23\ cm^2$

For rounding your answer to the nth significant figure, discard all digits to the right of the n^{th} significant figure if the $(n+1)^{th}$ digit is less than 5 otherwise raise the n^{th} digit by one and reject all digits to the right of the n^{th} place digit. Follow a similar procedure to round your answer to the n^{th} decimal place. This statement should not be taken to mean that the decimal places and significant figures mean the same. For example, consider the lengths 12.3 cm, 1.23 m, and 0.00123 m. The three lengths are all quoted with three significant figures, but they have, respectively, one, two, and five decimal places. In this course we will usually write 1000 m instead of 1000.0m (and similarly for other quantities). All four digits in 1000 m are thus significant.

Figures

If you were to build a home or manufacture a machine part, you will need very precise drawings of the various sections and they will have to b e drawn to scale. However, the role of diagrams in physics and engineering problems is often different. The aim of a diagram in such cases is to abstract the essential details of the problem in a concise visual, geometrical form. Such figures, unlike schematics for machine parts or building plans, need not be – and they rarely are- to scale. Furthermore, it will be very difficult to show an atom and a crystal, a satellite and a planet to scale on the same sheet of paper of manageable dimensions. So, read the labels on figures carefully, and never make assumptions about the dimensions from the appearance of a diagram. Similarly, it is a good habit to draw a well-labeled diagram when you are trying to solve a problem.

How to solve Problems?

One of the most important components of the courses (physics, chemistry, engineering, etc.) that you take is the problem solving skills. There are some general strategies to solve such problems. There is a definite underlying structure to such strategy. The problem-solving process is complex, although not necessarily a difficult one once you know what is involved. There are certain essential steps that are taken as you proceed towards the solution. We are going to illustrate the strategy by discussing one of your homework problems.

> ***For example consider Problem 11 from Unit 02:*** *A series of six poles are to be erected along a city block 50 m long. These poles will be used to light up the neighborhood at night. Each pole is 10m high. The specs require each pole to be supported by three guywires that are connected to the pole at a height of 5m. The other end of the guywire is connected to a point (a hook attached to a concrete block) 4m from the base of the pole. If each pole costs $ 185, each concrete block $15, and the guywire costs $ 1.50 per meter, determine the total material cost of erecting the poles.*

1. Recast the problem in your own words.

The person who composed the problem had a certain understanding of the physical situation on which the problem is based. Recasting it in your own words will help you gain some of that familiarity. For example, as I understand it , the problem is about finding *the cost of putting up six poles. Each pole is held in place by three wires. Each wire is connected to the pole on one and the other end is connected to a concrete block.* Therefore the total cost = the cost of six poles + the cost of 18 concrete blocks + the cost of 18 pieces of guywire. Just the essence — don't get bogged down in details at this stage. Notice that the length of the city block is not required to solve the problem. Not every piece of information in the problem may be relevant.

(***NOTE***: This step may appear to you to be tedious. However, we would recommend that you actually write down every problem in your own words. When you have gained some experience, you can recast the problem but you don't have to really write it. This recasting is unavoidable, all of us do it. The difference between good and poor performers(problem solvers) is largely dependent on the efficiency with which proper recasting is done. If you can not express the problem in your own words, you have not understood the problem.)

2. Create an image.

Try to imagine how these poles would look, if they were actually erected. This mental picture would actually be evolving simultaneously with and as a byproduct of the first stage described above.

3. Details

From step 1 above, the total cost = the cost of six poles + the cost of 18 concrete blocks + the cost of 18 guywires.

What's known :
[a] you know the cost of one pole, therefore you can calculate the cost of six poles.

(This step would involve simple multiplication. The problem is not a test of whether you can multiply. We are assuming that you can. As you progress in your coursework, this circle of assumption would keep on widening. For example, very soon we will assume that you know how to work with sine and cosine functions, etc. Make sure you keep up with the developments in the course. If some concept or mathematical topic is confusing you, get help immediately. Otherwise the gap between what you know and what we assume you know will get deeper and you will feel lost. Don't let that happen. We are here to help.)

[b] you know the cost of one concrete block, therefore you can calculate the cost of eighteen blocks.

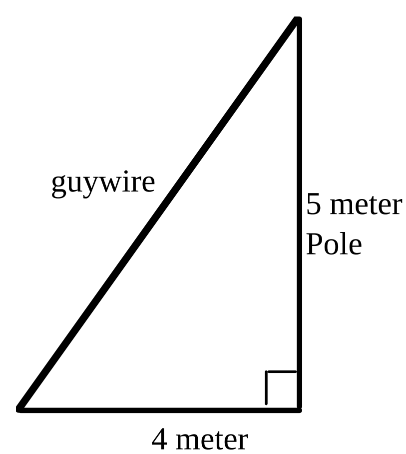
[c] there are eighteen pieces of guywire.

What is yet to be determined to get the final answer:

The cost per unit length of the guywire is given. But this information cannot be used directly since you don't know the total length of the guywire. Therefore the main part of the problem is to find out the length of one piece of guywire.

4. Put your mental image on paper.

Draw a clearly labeled diagram.
The adage: a picture is worth a thousand words aptly describes the importance of this step. In our example there are six identical poles, each pole has three identical wires, therefore the diagram should look something like shown to the right



5. Casting in mathematical form and solving.

This step should be easy if you have gone through steps 1-4 carefully. Some students would find this part the most difficult one only if some confusion has crept in one or more of the first four steps.
***Note*:** The mathematical equations that you need here may be as simple as multiplying two numbers or an equation such as Newton's second law, F = ma. This will be dictated by the nature of the problem. Which equation to use when will often can be guessed from the context, but there is no substitute for practice. The three most important factors in doing well in your technical courses are practice, practice and more practice.

The diagram in step 4 suggests that to get the length, W, of one of the pieces of the guywire you will have to use the Pythagorus' theorem

Length of each guywire is = $(5^2 + 4^2)^{½} = (41)^{½}$ = 6.4 meter.

Knowing the length of one piece of the guywire, we are now ready to write down the total cost as:

Total cost (in $) =*185x6+18x15+18x6.4x1.50* = ***1552.80***

or $ 1553 to four significant figures.

At this stage, relax, check your solution to make sure you have not made any obvious mistakes, see if your answer looks reasonable : if the answer above were 15.52 million dollars, you know there is something wrong. Tell yourself you have done a good job and go to the next problem.

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Simultaneous Equations

Introduction:

Many problems in engineering, physics, chemistry, economics and other disciplines can be expressed as systems of equations or inequalities. Actually the occurrence of such systems of equations in engineering is so pervasive that you should regard solving a set of equations as one of the more important core competencies in any engineering discipline. Today, with the easy availability of inexpensive but powerful computers it is easy to solve systems of equations in such a short time that was unthinkable even a few decades ago.

In this Unit, we will study some of the most common systems of equations that you will encounter in the beginning of your engineering education at Drexel.

We are interested in solving two linear equations in two unknowns of the form:

$ax + by = m$
$cx + dy = n$

where a, b, c, d, m, and n are real numbers. To solve this set of equations, we have to determine all possible pairs (x, y). [**Note**: in most cases of our interest, there will be just a single pair (x,y) that satisfies the two equations].

There are several ways to solve such a system of equations. We will discuss two of them: [1] the Elimination Method, and [2] the Substitution Method.

Example 1: Solve the following set of equations:

$5x - 2y = 24$ ……..[1]
$2x + y = 6$ …….. [2]

Solution:

1. The Elimination Method

Step1. Multiply each equation by a constant so that in the resulting equations the coefficients of either x or y will differ only in sign.

For the present example multiply the second equation by 2 and the first equation by 1 to get two equations in which the coefficients of y are equal in magnitude but opposite in sign.

Step 2. Add the two equations to get rid of (eliminate) the y–term, to get,

$(5x - 2y) + 2*(2x + y) = 24 + 2*6$

5x –2y+ 4x +2y = 24+12
9x = 36
Therefore, x = 4.

Note: if the coefficients of y were of the same sign, we would subtract one equation from the other to get rid of the y–term.

Step 3. Substitute this value of x in the second equation to get,
2*4 + y = 6 or y = 6 – 8 = –2.

2. The Substitution Method

Step 1. From one of the equations, express one of the unknowns in terms of the other.

In our case , we get from eq.[2], y = 6 – 2x.

Step.2 Substitute this expression for y in eq. [1] to obtain,

5x – 2 (6–2x) = 24
5x – 12+ 4x = 24,
which yields 9x = 24+12 =36 or x = 4 as earlier.

Step 3. Proceed as above (substitute this value of x in eq.[2] or [1]) to solve for y. We get from eq.[2], y = 6 – 2x = 6 – 2*4 = – 2.

3. From Words to Symbols

In real world, problems don't present themselves as systems of equations for you to solve. First, you have to be able to: state the problem in words; change the statement into algebraic equation; and, finally, try to solve these equations. We start with a simple example.

Example 2.

A wine merchant buys some wine at $3/liter and a better quality wine at $5/liter. She mixes the two wines and sells the resulting mixture at $5/liter to make a 6% profit on her initial investment of one thousand dollars. How many liters of each type of wine did the wine merchant buy?

Solution:

Step 1. Give the unknown quantities names (algebraic symbols). Here, let us call the number of liters of the lower quality wine as L, and the higher quality wine as H.

Step 2. Translate the statement in words into symbols. For our example:

3L + 5 H = 1000 ……….[1]

(L+H)*5 =1000*(1.0 + 6/100)
=1000*(1.0+ 0.06) = 1060 …….[2]

Step 3. Solve the resulting equations ([1] and [2] above) using one of the two methods outlined above. For this example we will use the substitution method.

From eq.[2], L+ H = 1060/5 = 212,
L = 212– H
Substitute this value of L in eq.[1] to get,

3*(212 – H) + 5H = 1000, or 2H = 1000 – 636 = 364, or H = <u>182</u> liters,
therefore, L = 212 – 182 = <u>30</u> liters.

To Test Your Progress:

One of the things that you learn in studying this Unit is how to convert verbal statements into algebraic symbols. This ability to convert words into relations among symbols will be useful throughout your engineering education and your professional life. If you feel comfortable with reducing a statement to an equation then you have learnt a useful technique. However, if you still feel a bit uneasy about such conversion from words to symbols, try this: Take a set of equations from the Unit, and construct a story that you think converts the equations to verbal statements. Ask a classmate to convert your statements back to a set of simultaneous equations. If your friend can convert your statements to equations, then you are getting better at understanding the correspondence between the symbolic relations and verbal statements. This will teach you how to extract the relevant information from a problem statement. This is a crucial first step in solving any engineering problem.

Simultaneous Equations

Interactive Problem 1:

Another Problem on the Elimination Method

You have to solve the following equations by elimination method and find the values of x and y.

$5x - 3y = 11$ ……..[1]
$2x + 4y = 20$ …….. [2]

Step1. Multiply each equation by a constant so that in the resulting equations the coefficients of either *x* or *y* will differ only in sign.

Suppose we choose the equations and we want to eliminate "y"

Question 1: Where do we start?

Choose from the following.

a) multiply eqn [1] by 3 and multiply eqn [2] by 4
b) multiply eqn [1] by 2 and multiply eqn [2] by 5
c) multiply eqn [1] by 4 and multiply eqn [2] by 3

Answer 1: The correct choice is (c).

Feedback 1: Another possible choice is – 4 and –3. Note. Changing the signs of both multipliers lead to the correct result.

Now we have

$20x - 12y = 44$ ……..[3]
$6x + 12y = 60$ …….. [4]

Question 2: What is to be done now?

Choose from the following.

a) Subtract eqn [3] from eqn [4]
b) Subtract eqn [4] from eqn [3]
c) Add eqn [3] to eqn [4]

Answer 2: The correct choice is (c).

$26x = 104$ This gives $x = 4.0$.

Feedback 2: Knowing the value of "x", what do we have now?

Take the value of x and substitute in eqn [1] or eqn [2].

Let us choose eqn [1] and we get $5*4 - 3y = 11$. This gives $20 - 11 = 3y$ or $3y = 9$ and hence $y = 3$.

Try substituting for x in eqn [2] and see what we get.

Also, substitute values of x and y in eqn [1] and eqn [2] to check your answers. The values you obtained should satisfy both equations. Hence the name simultaneous equations.

Interactive Problem 1: Another Method

Let us do the same problem by substitution method.

$5x - 3y = 11$ ……..[1]
$2x + 4y = 20$ …….. [2]

Step 1. From one of the equations, express one of the unknowns in terms of the other.

In our case , we get from eqn[2], $2x = (20 - 4y)$. Thus, $x = 10 - 2y$

Step.2 Substitute this expression for x in eqn. [1] to obtain,

$5(10 - 2y) - 3y = 11$
$50 - 10y - 3y = 11$,
which yields $50 - 11 = 13y$. Thus $13y = 39$ or $y = 3$ same answer as by the elimination method.

Step 3. Proceed as above (substitute this value of y in eqn.[2] or [1]) to solve for x.

Interactive Problem 2:

You are given two equations

$3x + 4y = 23$ ……..[1]
$7x – 9y = 17$ …….. [2]

We want to solve this problem by the substitution method.

Question 1: Using eqn [1], express 4y in terms of x. What is your answer?

a) $4y = (23 + 3x)$
b) $4y = (23 – 3x)$
c) $4y = – 23 + 3x$

Answer 1: The correct choice is (b)

Question 2: What is y in terms of x? Choose from below.

a) $y = (27 + 7x)$
b) $y = (19 – 3x)$
c) $y = (23 – 3x)/4$

Answer 2: The correct choice is (c)

Question 3: Substitute this expression for y in eqn [2] and you get

a) $7x – 9*(23 – 3x)/4 = 17$
b) $7x + 9*(23 + 3x)/4 = 17$
c) $7x – 9*(23 + 3x)/4 = 17$

What is your choice?

Answer 3: The correct choice is (a)

Question 4: Simplify (a). What do we get?

a) $28x – 9*(23 – 3x) = 58$
b) $28x – 9*(23 – 3x) = 68$
c) $28x + 9*(23 – 3x) = 68$

Answer 4: Correct choice is (b)

From (b) we get $(28x – 207 + 27x) = 68$ gives $55x = 207 +68 = 275$. Thus, $x = 5$.

Solving for "y". Proceed as above (substitute this value of *x* in eqn.[2] or [1]) to solve for *y.* And use this value in eqn [1], we have 3*5 + 4y =23. This gives 4y = 23 – 15 = 8. Thus, y = 2.

Feedback:

Interactive problem (2) by the Elimination Method

3x + 4y =23 ……..[1]
7x – 9y = 17 …….. [2]

Step1. Multiply equation [1] by 7 and eqn [2] by 3 to obtain resulting equations the coefficients of either *x* or *y* will differ only in sign.

21x + 28y =161 ……..[3]
21x – 27y = 51 …….. [4]

Step 2. Subtract eqn [4] from eqn [3] and we get rid of (eliminate) the *x*–term, to get,

28y – (–27y) = 161 – 51 = 110

55y = 110 and thus, y = 110/55 = 2.

Note: if the coefficients of x were equal and of opposite signs, we would added the equations to get rid of the x–term.

Step 3. Substitute this value of *y* in the first equation to get,
3x + 4*2 = 23 or 3x = 23 – 8 = 15. Thus x, =5.

Interactive Problem 3:

Another word problem on the Elimination Method

Mary went to a candy store and bought some candies. She bought five Snickers bars and six Kitkat bars for three dollars and eighty cents. As she was walking out, she decided to return one Kitkat bars and buy five more Snickers bars. She had to pay an additional one dollar and seventy cents. You have to write simultaneous equations and solve the equations by elimination method to find the cost of each Snickers bar and Kitkat bar.

Let the cost of each Snickers bar be "x" cents and the cost of each Kitkat bar be "y" cents. Her first purchase of five Snickers bars and six Kitkat bars can be written as

$5x + 6y = 380$ [1]
$5x - y = 170$ [2]

Step1. Multiply each equation by a constant so that in the resulting equations the coefficients of either *x* or *y* will differ only in sign.

Suppose we choose the equations and we want to eliminate “y”

Question 1: **What do we do to eliminate y?**

Choose from the following.

a) multiply eqn [1] by 3 and multiply eqn [2] by 4
b) multiply eqn [1] by 6
c) multiply eqn [2] by 6

Answer 1: The correct choice is (c).

Feedback 1: Another possible choice is multiply eqn [1] by –1 and multiply eqn [2] by – 6. Note. Changing the signs of both multipliers lead to the correct result.

Now we have

$5x + 6y = 380$ ……..[3]
$30x - 6y = 1020$ …….. [4]

Question 2. What is to be done now to eliminate "y"?

Choose from the following.

a) Add eqn [3] to eqn [4]
b) Subtract eqn [4] from eqn [3]
c) Subtract eqn [3] from eqn [4]

Answer 2: The correct choice is (a).

35x = 1400 This gives x = 40. Thus, cost of a Snickers bar is 40 cents.

Feedback 3: After finding the cost of a Snickers bar, what do we have now?

Take the value of x and substitute in eqn [1] or eqn [2].

Let us choose eqn [1] and we get $5*40 + 6y = 380$. This gives $200+6y = 380$ and hence $6y = 380 – 200 = 180$ and hence $y = 30$. The cost of a Kitkat bar is 30 cents.

Try substituting for x in eqn [2] and see what we get.

Also, substitute values of x and y in eqn [1] and eqn [2] to check your answers. The values you obtained should satisfy both equations. Hence the name simultaneous equations.

Interactive problem 3 by the Substitution Method:

We have

$5x + 6y = 380$ [1]
$5x – y = 170$ [2]

From equation [2], we have $5x = 170 + y$.

Substitute this in the first equation.

We get $5x + 6y = 170 + y + 6y = 380$ [3]

Thus, $7y = 380 – 170 = 210$.

Solve this $y = 30$ i.e. the cost of a Kitkat bar is 30 cents.

To find the cost of Snickers bar, what do we do?

Substitute the value of y in either equation [1] or [2] and we can get the value of "x'.

Let us choose equation [1]. We get $5x + 6*30 = 380$.

This gives $5x = 380– 180 = 200$. Solve this and we get $x = 40$.

The cost of a Snickers bar is 40 cents.

Simultaneous Equations

Sample Problems:

Solve the following sets of two linear equations *(you may use the Substitution method, the Elimination method or both)*

1. $3x + 4y = 9$
$6x - 16y = 6$

Ans: $x = 2.3$, $y = 0.5$

2. $3x + y - 7 = 0$
$2x - 4y - 14 = 0$

Ans: $x = 3.0$, $y = -2.0$

3. $x + 2y = 5$
$3x - 5y = 81$

Ans: $x = 17.0$, $y = -6.0$

4. $2x + y = 7$
$3x + 2y = 12$

Ans: $x = 2.0$, $y = 3.0$

5. $2x + y = -4.5$
$-6x + 2y = 21$

Ans: $x = -3.0$, $y = 1.5$

6. A movie theater admission ticket costs \$6 per adult (A) and \$4 per child (C). If \$3700 worth of tickets are sold for a show attended by 700 people, how many tickets of each type were sold ?

Ans: Number of Adults A = 450, Children C = 250

7. A pile of 50 coins is made up of nickels(N) and dimes (D). If the total value of the coins is \$4.25, how many of each type of coin are in the pile ?

Ans: N = 15, D = 35

8. A manufacturer produces a breakfast cereal by mixing two types of grain, *X* and *Y*. Each unit of grain *X* contains 2 grams of fat and 80 calories, while each unit of grain *Y* contains 3 grams of fat and 60 calories. If a single serving of the cereal has to supply 18 grams of fat and 480 calories, how many units of each grain are used to produce a single serving of the breakfast cereal?

Ans: x = 3.0, y = 4.0

9. Suppose the daily cost of producing *X* kg of a brand of dog–food is given (in US dollars) by $C = 1500 + 0.5X$, while the revenue (in US dollars) generated by selling the daily output pf dog–food by $R = 0.6X$. Find:
[a] the break–even point (no profit or loss)
[b] the total revenue at the break–even point.

Ans: X = 15000 kg, R = $ 9000

10. A stationary wholesaler sells two types of notepads *N1* and *N2*, respectively, for 40 and 60 cents each. The wholesaler receives an order for 600 notepads from the Drexel bookstore, together with a check for $280. If the order fails to specify the number of each type of notepads, how many of *N1* and *N2* should be shipped to the Drexel bookstore?

Ans: N1 = 400, N2 = 200

11. The treatment of a certain viral disease requires a combination dose of drugs *D1* and *D2*. Each unit of *D1* contains 1 milligram of factor *X* and 2 milligrams of factor *Y*, and each unit of *D2* contains 2 milligrams of factor *X* and 3 milligrams of factor *Y*. If the most effective treatment requires 13 milligrams of factor *X* and 22 milligrams of factor *Y*, how many milligrams of *D1* and *D2* should be administered to the patient?

Ans: D1 = 5.0 mg, D2 = 4.0 mg

To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

Unit 01 – Simultaneous Equations – Test 01

1. Solve the following simultaneous equations:

$3x - 2y = 35/2$
$6x + 8y = 65$

Answer : $x = 7.5$, $y = 2.5$

2. A person invests a total of $5000.00 in two mutual funds in the amounts of, x and y respectively. The annual yields from these amounts x and y are, respectively, 5% and 10%. If the person earned a total of $300.00 in interest from the two accounts, how much did the person invest in each ?

Answer : x = $ 4000 , y = $ 1000

3. A car manufacturing company (the Company) can produce a car for $3000.00 and a truck for $5000.00. Suppose "C" cars and "T" trucks are sold to the dealer at mark-ups, respectively, of 20 and 30 percent. If the Company made a profit of $27 million on sales of $137 million in one particular year, how many cars and trucks were sold by the company in that year?

Answer : C = 20,000 cars , T = 10,000 trucks.

Unit 01 – Simultaneous Equations – Test 02

1. Solve the following simultaneous equations:

$5x - 2y = \frac{1}{2}$
$4x + 6y = 46$

Answer : x = 2.5 , y = 6.0

2. Suppose you have two alcohol + water solutions. One solution (volume x liter) is 45% alcohol, while the second solution (volume y liter) is 75% alcohol. What values of "x" and "y" (in liters) will fill a total volume of 5.0 liter which contains 2.88 liters of alcohol ?

Answer : x = 2.1 liter , y = 2.9 liter.

3. According to a rating agency, a car's MPG (miles per gallon) ratings are: 25 MPG for city and 30 MPG for highway driving. A driver spent $20.00 (the gasoline costs $1.25/gallon) on a combined city and highway trip of 450 miles. How many miles "C" in city and "H" on highway was the car driven ?

Answer : C = 150 miles, H = 300 miles.

Fundamentals of Plane Geometry

Introduction:

Skills you will learn

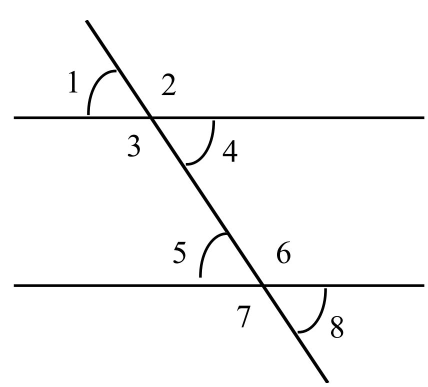
1. Identify relationships between angle measures.
2. Classify simple plane geometrical figures.
3. Calculate perimeter and other distances of basic geometrical figures (rectangles, circles, squares, etc.), and complex figures that can be obtained by combining such simple geometrical figures.
4. Calculate areas of simple geometrical figures and complex figures that can be obtained by combining such simple geometrical figures.
5. Use properties of similar triangles to calculate unknown parameters of plane geometrical figures.
6. Solve real world problems involving perimeters and areas.
7. Solve real world problems using properties of right-angle triangle (Pythagorus' Theorem)

Some Concepts in Plane Geometry.

Angles

A geometrical figure formed by two rays (straight lines) extending from a common point (the vertex) is called an *angle*. When these lines are perpendicular to each other the angle they subtend is called a *right angle* (90^o). The angle is called straight (180^0) when the two lines forming an angle lie along a straight line. An angle is called *acute* if it is less than the right angle, and *obtuse* if it exceeds 90^o. Two adjacent angles adding to 180^0 are called *supplementary*. When the two adjacent angles add up to a right angle the angles are termed *complementary*. *Vertical* angles are nonadjacent angles formed by two intersecting straight lines.

An angle is measured in degrees(^{0}), minutes (') and seconds("). 1^0 = 60'; 1' = 60"



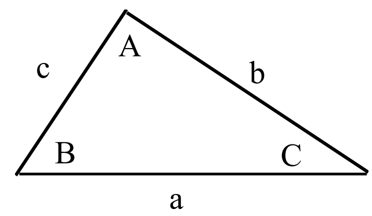
Angles obtained when a line intersects two parallel lines are shown in the diagram on the right.

Angles 3 and 6 are equal and angles 4 and 5 are alternate interior angles

1= 8 and 2 = 7 are alternate exterior angles

1= 5 are called corresponding angles, other pairs of corresponding angles are 2 and 6 ; 3 and 7; and 4 and 8

Triangles.



A triangle is a plane figure bounded by three line segments, called sides. A triangle has three vertices (a vertex is a point common to two sides).
An *altitude* is a perpendicular segment drawn from a vertex to the opposite side (known as the *base*).
The three interior angles of a triangle add up to 180^0.

Let a, b, and c represent three sides of a triangle whereas, A, B and C are the angles in a triangle.

1. when, $a \neq b \neq c$, the triangle is called scalene.
2. when, $a = b \neq c$, the triangle is called isosceles
3. when, $a = b = c$, the triangle is called equilateral
4. If none of the three angles exceeds a right angle the triangle is called *acute*, otherwise it is termed *obtuse*.
5. If one of the three angles $A = 90^0$, the triangle is called a right-angle triangle. The three sides of such triangles are related through the Pythagorean theorem $a^2 = b^2 + c^2$, where "b" and "c" are perpendicular to each other, and "a" is the hypotenuse - the side opposite to the right angle.

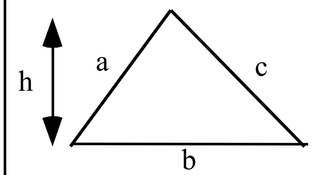
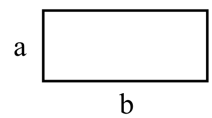
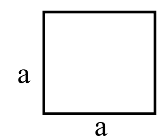
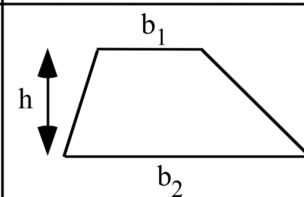
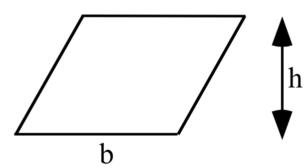
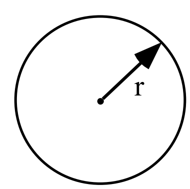
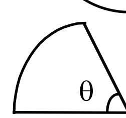
Similarity of Triangles

Two geometrical figures are called similar if the two figures can be exactly superimposed on each other by scaling the corresponding dimensions of one of the figures by a constant factor. Similarity of triangles can be expressed in several ways. We will express it as follows: Two triangles are similar if: [a] their corresponding sides are proportional or equivalently, [b] their corresponding internal angles are equal. The triangles are called *congruent* if the three sides of a triangle are equal to the three corresponding sides of another triangle.

1. Perimeter. Distance measured around the plane figure. The units of measurement are inches, feet, yards, mile (in the English system of units) and centimeter (cm), meter (m), and kilometer (km) in the Metric system of units.

2. Area is measured in dimensions of $(\text{length})^2$.

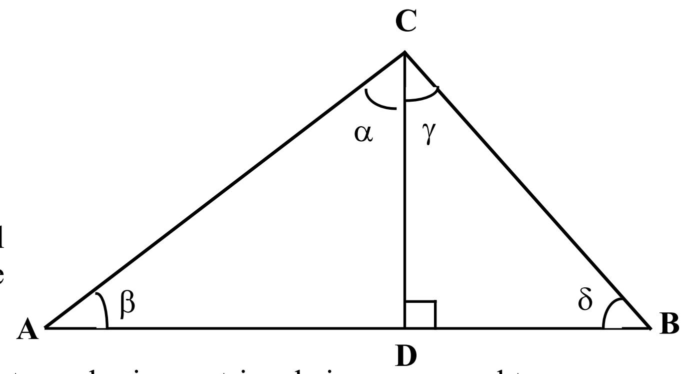
Formulas for perimeters and areas for some common geometrical shapes are given in the following table.

Object	Formulas P = perimeter A = area	Typical Figure
Triangle	$P = a + b + c$ $A = (bh)/2$	
Rectangle	$P = 2(a + b)$ $A = ab$	
Square	A square can be viewed as a special case of rectangle, a = b, therefore $P = 4a$ and $A = a^2$	
Trapezoid	$A = h\,(b_1 + b_2)/2$ P = sum of the four sides. No general formula can be provided	
Parallelogram	$A = bh$ P = 2 x sum of two adjacent sides.	
Circle	P(circumference) $= 2\pi r$ $A = \pi r^2$ For a sector of a circle of radius r $P = 2r\,(1 + \pi\,(\theta/360))$ $A = (\theta/360)\,\pi r^2$ where θ is measured in degrees	

Fundamentals of Plane Geometry:

Solved Examples:

Example 1. ΔABC is a right angle triangle, CD is perpendicular to AB. Show that ΔBCD, and ΔADC are similar.



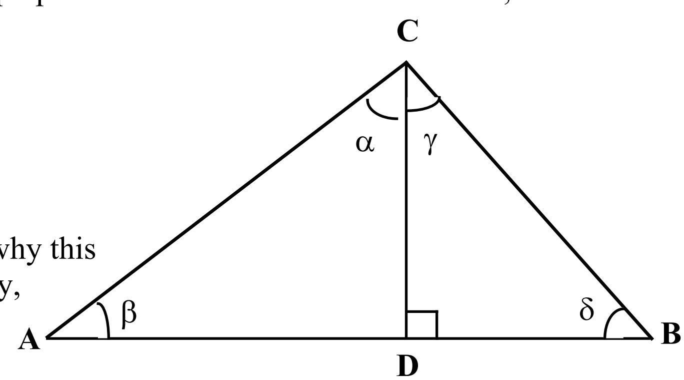
Solution:

To show similarity of two triangles, you only need to show that any two of the angles in one triangle are equal to the corresponding two angles in the other triangle. In case of a right-angle triangle this requirement is reduced to showing that one of the acute angles in one triangle is equal to one of the angles in the other triangle.

Thus, the similarity is easily shown by noting that; $\delta = 90^0 - \gamma$ and since the angle at C is 90^0, $\alpha = 90^0 - \gamma = \delta$.
Thus, the two triangles BCD and ADC are similar.

Example 2.

Δ ABC is a right angle triangle, CD is perpendicular to AB. Show that Δ BCD, and Δ ABC are similar.



Solution:

The similarity is easily shown by noting that angle δ is common to the two triangles BCD and ABC (For why this is justified as an argument for similarity, see Example 1 above).

Example 3.

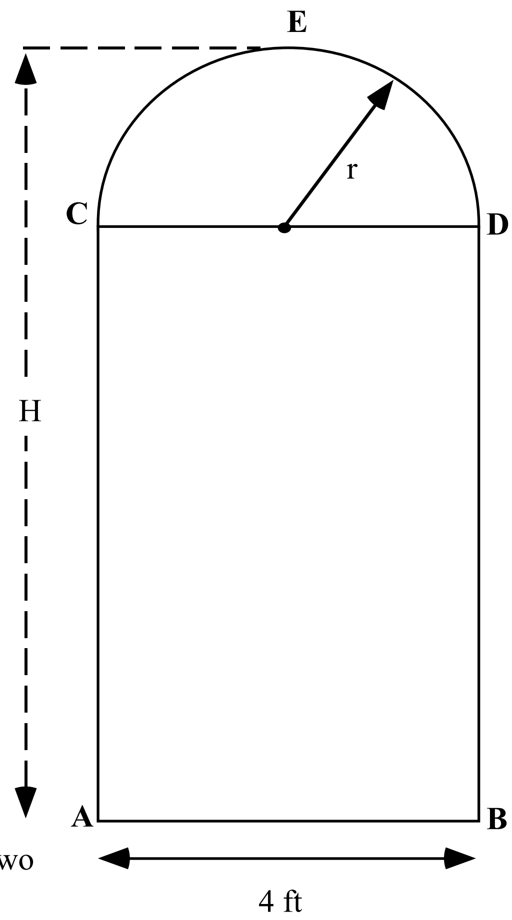
If in Example 1 above, AB = 50 cm, and AC = 40 cm., determine CD, using the similarity of triangles.

Solution.

Using Pythagorean theorem**,** $BC = [\ (50)^2 - (40)^2]^{1/2}$ cm = 30cm
Next we note that ΔBCD, and ΔABC are similar, therefore,
BC/AB = CD/AC
therefore, CD = (BC/AB)*AC =(30/50)*40 = 24 cm

Example 4.

A 4-ft wide door is to be constructed out of aluminum. The door is in the shape of a semi-circle, CDE sitting atop a rectangle ABCD. If the area of the door is 47 square feet, determine the height, H of the door.



Solution.

The total area of the door can be expressed as the sum of two parts,
i.e., a semicircle of radius, r = 2ft and
a rectangle 4-ft wide and of height, (H– r).

Thus,

$\pi r^2/2 + 4*(H-r) = 47$ ft^2

Substituting for r =2ft gives,

$[(3.14)(2^2)/2] + 4H - 4*2 = 47$, or

4H = 47 + 8 – 6.28 = 48.72

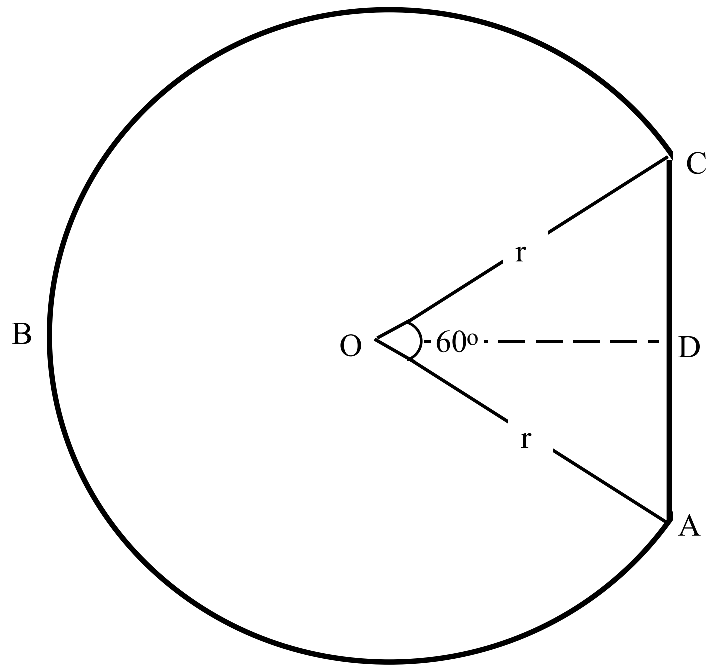
H = 48.72/4 = 12.18 ft.

= 12.2 ft (if rounded to the first decimal place).

=12 ft (if expressed as “whole” feet or to two significant figures.)

Example 5.

A machine part ABCDA is to be fabricated from a circular piece of aluminum of radius 6 cm. The angle subtended by the two radii, OA and OC is 60^o.Determine the area of the machine part.



Solution

We can view the area to be determined as composed of two parts: a circular sector COABC of radius r = 6 cm and central angle of $360^o - 60^o = 300^o$ and an equilateral triangle of 6cm on the side. Thus, the total area of the machine part ABCDA is:

$\pi r^2 (300/360) + (3/4)^{1/2} r^2/2 = 30\pi + 15.59 = 109.8\ cm^2 = 109.8 \times 10^{-4}\ m^2 = 1.1 \times 10^{-2}\ m^2$

(REMEMBER: $1cm = 10^{-2}$ m and $1cm^2 = 10^{-4}\ cm^2$)

Fundamentals of Plane Geometry:

Interactive Problem 1:

"Harry the painter" is chosen to paint the inside of an auditorium that is 8.5 meters high, 40.0 meters wide at the front and back and 80.0 meters long along the sides. Each side wall is 8.5 meter tall and 80 meter long and if we neglect the doors at the front or back side, the front and back walls are 40 meter in length and 8.5 meter tall.
Note: You will find it very helpful to draw a picture from the given information.

Question 1:

What is the area of each side wall?

Choose from the following.

a. 3200 $meter^2$
b. 340 $meter^2$.
c. 680 $meter^2$.

Answer 1: The correct choice is (c).

Feedback: Each wall is 8.5 meter tall and 80 meter long.
Hence the area is (8.5 meter x 80 meter) = 680 $meter^2$

Question 2:

What is the area of the front wall if the doorway is 4 m wide and 3 m tall?

Choose from the following.

a. 328 $meter^2$
b. 340 $meter^2$.
c. 368 $meter^2$.

Answer 2: The correct choice is (a).

Feedback: Front wall is 8.5 meter tall and 40 meter wide. Front wall area without any door will be (8.5 meter x 40 meter) = 340 $meter^2$. But the doors take up (4 meter x 3 meter) = 12 $meter^2$ area.

Thus, 340 $meter^2$ – 12 $meter^2$ = 328 $meter^2$ area.

Question 3:

What is the area of the ceiling?

Choose from the following.

a. 3200 $meter^2$

b. 340 $meter^2$.

c. 680 $meter^2$.

Answer 3: The correct choice is (a).

Feedback: Ceiling (or the floor of the auditorium) is 40 meter wide and 80 meter long. Thus, the area is 3200 $meter^2$.

Question 4:

If "Harry the painter" wants to paint the interior walls and the ceiling, what is the total area to be painted assuming that no painting is done for all the windows and the doors in the auditorium with total area 1600 $meter^2$?

Choose from the following.

a. 5240 $meter^2$

b. 3640 $meter^2$.

c. 6840 $meter^2$.

Answer 4: The correct choice is (b).

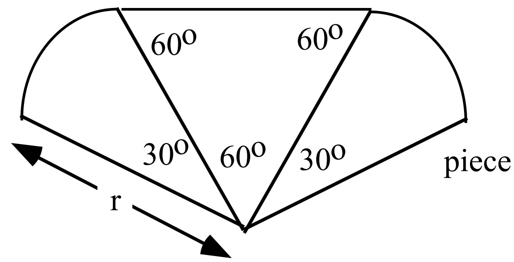
Feedback: Ceiling (or the floor of the auditorium) is 40 meter wide and 80 meter long. Thus, the area is 3200 $meter^2$. Side walls are each 680 $meter^2$ area and the front and back walls excluding any doors or windows will be each 340 $meter^2$ in area.

Thus, total area to be painted is

3200 $meter^2$ + 2*680 $meter^2$ + 2*340 $meter^2$ – 1600 $meter^2$ = 3640 $meter^2$

Interactive Problem 2:

Three pieces of colored glass are to be put together to form a stained glass piece in the shape shown. Two of them are identical sectors of a circle of radius r = 8 inches and the third is an equilateral triangle.



Question 1:

What is the side of the middle triangular piece?

Choose from the following.

a. 3 inches

b. 8 inches

c. 12 inches

Answer 1: The correct choice is (b).

Feedback: Each side of the triangle is the same as the radius of each circular segment. .

Question 2:

What is the area of each circular segment piece?

Choose from the following.

a. 61.2 inch^2

b. 33.6 inch^2

c. 16.76 inch^2

Answer 2: The correct choice is (c).

Feedback: Each circular segment subtends an angle of 30^0 at its center. The area

of each segment is $(\pi r^2)*(30/360) = [(\pi)(8 \text{ inch})^2]*(30/360) = 16.76 \text{ inch}^2$.

Question 3:

What is the total area of the stained glass piece?

Choose from the following.

a. 61.2 $inch^2$

b. 33.6 $inch^2$

c. 16.76 $inch^2$

Answer 2: The correct choice is (c).

Feedback: Each circular segment subtends an angle of 30^0 at its center.

The area of each segment is

$(\pi r^2)*(30/360) = [(\pi)(8 \text{ inch})^2]*(30/360) = 16.76 \text{ inch}^2$.

Triangular piece has an altitude $(8^2 - 4^2)^{1/2} = 6.93$ inch.

Its area is $(1/2)*(8 \text{ inch})*(6.93 \text{ inch}) = 27.71 \text{ inch}^2$.

Total area is $(27.71 \text{ inch}^2 + 2* 16.76 \text{ inch}^2) = .\ 61.2 \text{ inch}^2$

Fundamentals of Plane Geometry

Sample Problems

1. Write a concise statement that will define parallel lines. (do not copy statements *verbatim* from any textbooks.)

2. Write a concise statement that will define intersecting lines. (do not copy statements *verbatim* from any textbooks.)

3. Determine the value of β in terms of θ in Figure 3 .

(Answer: $\beta = \theta$.)

4. In terms of α and β, determine the value of the angle at which lines AB and CD intersect. See Figure 4.

(Answer: $\alpha - \beta$)

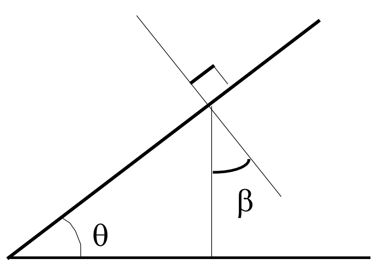


Figure 3

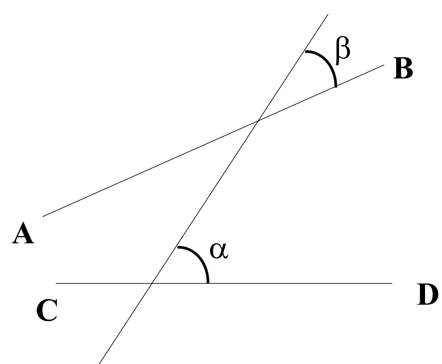


Figure 4

5. For the triangle below, show that $(AD)^2 + (BC)^2 = (AB)^2 + (DC)^2$

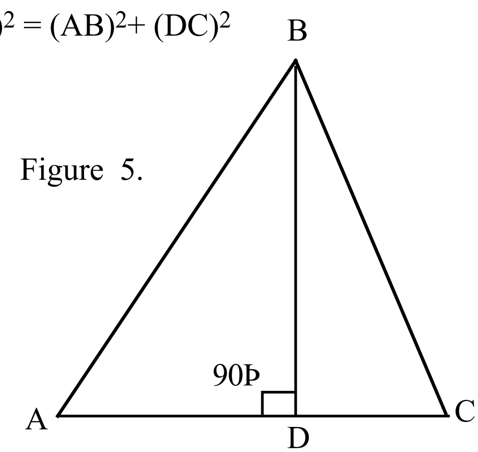
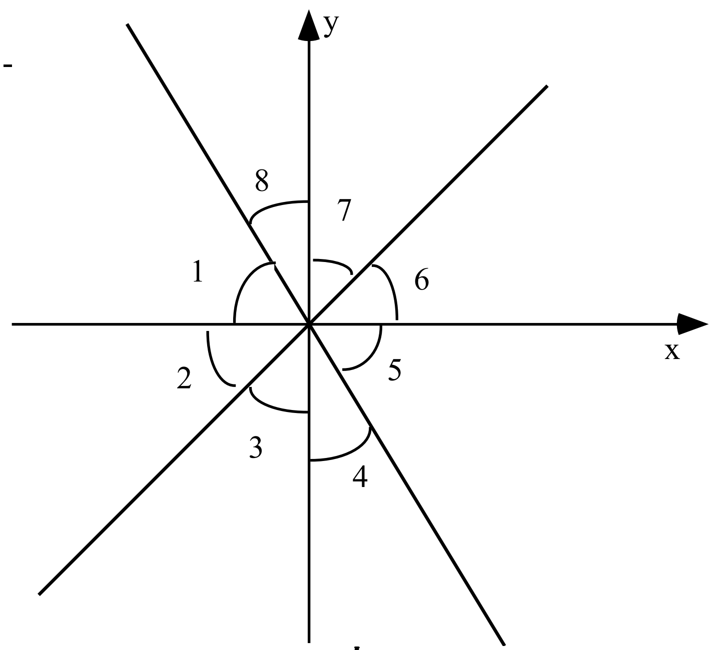


Figure 5.

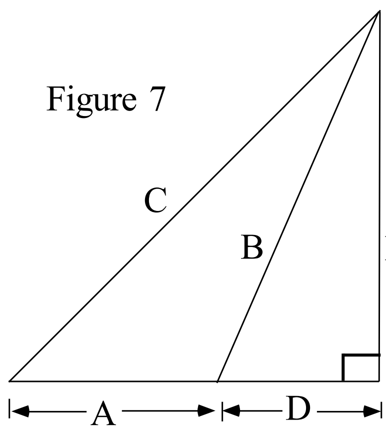
6. Identify all pairs of complementary angles in the accompanying diagram - Figure 6 on the right. .

(Answer: Angles 2 and 3; 4 and 5; 6 and 7; 8 and 1; 6 and 3; 2 and 7; 8 and 5 and, 4 and 1)



7. In Figure 7, show that
$C^2 = A^2 + B^2 + 2\,A\,D$

Figure 7



8. Show that angle ß > angle **Ω** in Figure 8.

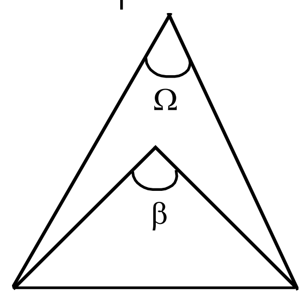
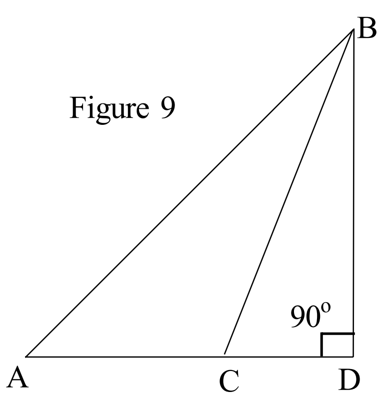


Figure 8

9. Consider Δ ABC and Δ CBD, with 2*CD = AC. Determine the ratio of the areas of the smaller to the larger triangle.

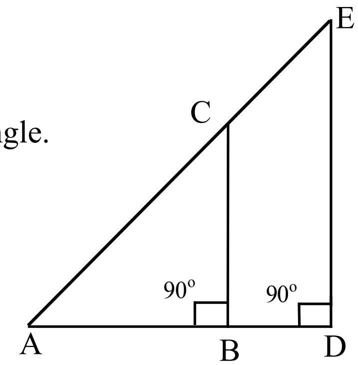
(Answer : The ratio of areas is ½).

Figure 9



10. Consider two similar triangles ABC and ADE. If AC = 8m and CE = 4m, determine the ratio of the areas of the larger to the smaller triangle.

(Answer : The ratio of areas is 2.25).



11. A series of six poles are to be erected along a city block 50 m long. These poles will be used to light up the neighborhood at night. Each pole is 10m high. The specs require each pole to be supported by three guywires that are connected to the pole at a height of 5m. The other end of the guywire is connected to a point (a hook attached to a concrete block) 4m from the base of the pole. If each pole costs $ 185, each concrete block $15, and the guywire costs $ 1.50 per meter, determine the total material cost of erecting the poles.

Answer: Cost of poles = $ 1110; Cost of 18 concrete blocks = $ 270 and cost of wire = $ 173. Total cost is $1553.

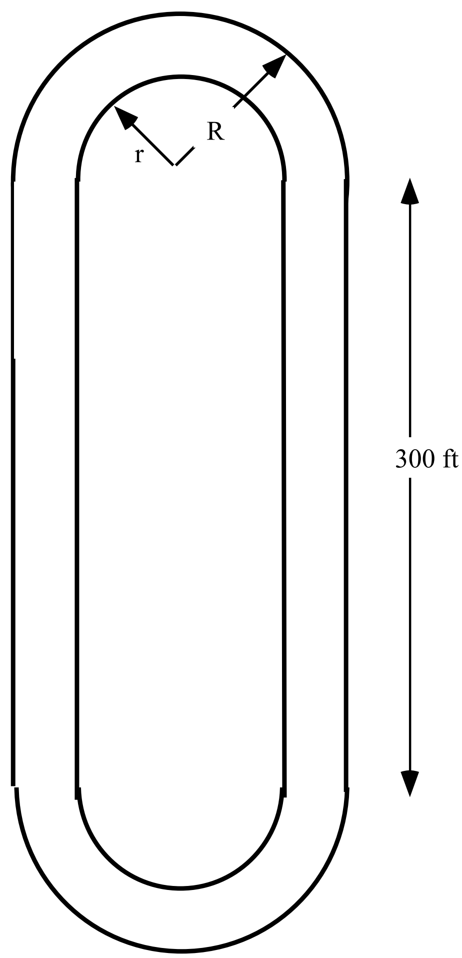
12. You will like to build a racecourse in the shape shown to the right. Given: R = 110 ft and r = 90 ft.

a. Calculate the cost of building the racecourse if it costs $200.00 per linear foot.

The length is to be measured along a line equidistant from the outer and the inner edges of the racecourse.

b. Using the cost from part [a], calculate the cost per square foot of the racecourse.

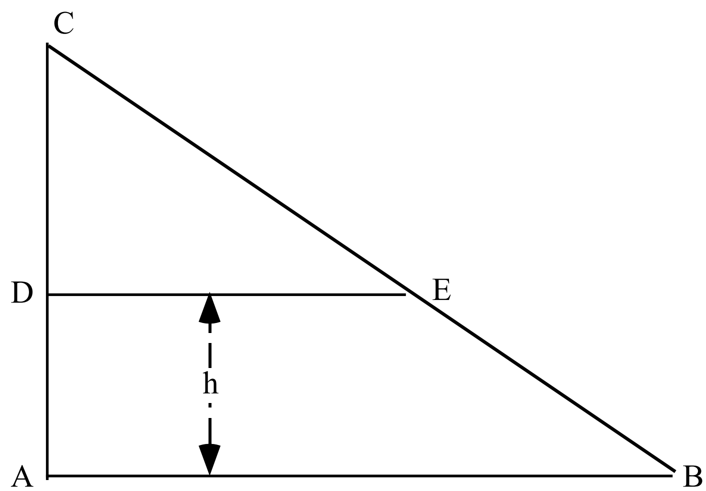
(Answer: a) Cost $ 245, 600 and
b) $ 10 per sq. ft.)



To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

Unit 02 Plane Geometry: TEST-01

Problem 1. Consider two right angle triangles CDE and ABC. If CA= 20 cm and the area of the smaller triangle is half that of the larger one, calculate the value of h in units of cm. (Answer = 5.9)

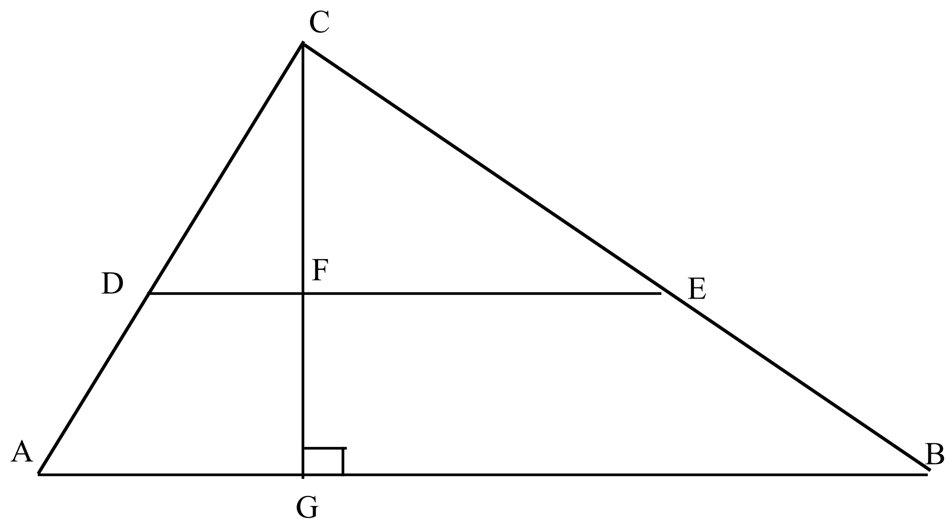


Problem 2. Determine the area of the largest rectangle whose length is twice its width that can be cut from a piece of circular cardboard of diameter 30 cm. Express your answer in units of cm^2. (Answer = 360)

Problem 3. Consider a brick of dimensions, length = 8”, width = 4”, and thickness = 3”. What’s the minimum number of bricks required to cover a floor of width = 8 ft and length = 16 ft. (Answer = 576)

Unit 02 Plane Geometry: TEST-02

Problem 1. Consider two similar triangles ABC and CDE. . If the altitude CG = 4.0 cm and the area of the smaller triangle is half that of the larger one, determine the height FG in cm units. (Answer = 1.17)



Problem 2. Consider an area of 19π cm^2 bounded by two concentric circles (the figure is called an annulus). Determine the radius of the inner circle (in units of cm) if the outer and the inner diameters of the of the annulus differ by 2 cm.(Answer = 9)

Problem 3. Consider a brick of dimensions, length = 8", width = 4", and thickness = 3". What's the maximum number of bricks that can be stored ,one layer thick, on a floor of width = 8 ft and length = 16 ft. (Answer = 1536)

Use of Trigonometric Functions

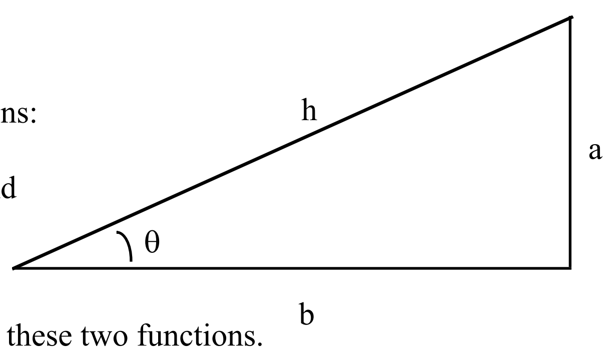
1. Introduction

The term *trigonometry* means measurement *(-metry*) involving triangles (*trigon).* In plane trigonometry our subject of interest consists of plane angles and triangles. However, in trigonometric problems there may not always be an explicit mention of triangles. Solving trigonometric (trig.) problems will involve determining the measure of unknown angles or sides of triangles from given information. You may wonder why develop a whole subject about the property of triangles. To appreciate the importance of trigonometry, consider this: any plane geometrical shape can be ultimately broken up into triangular parts. Furthermore, any triangle can be thought of as consisting of no more than two right-angle triangles. Thus, trigonometry really consists of properties of a right-angle triangle. These properties can then be employed to study any figure in plane geometry.

In a triangle, as you have already learned, the sum of the internal angles equals 180^0 or π radians. Thus, knowing any two angles of a triangle, you know the shape of that triangle. In the case of a right-angle triangle, you only need one additional angle to determine the shape of the triangle. This means that all right angle triangles which have the same one acute angle are similar and, therefore the ratios of their corresponding sides will be only a function of this specified acute angle. These ratios and the relations among them is the subject of trigonometry.

The side opposite the specified acute angle is called the *altitude* which is perpendicular to the *base*. The remaining side (the longest side of the triangle) is called the *hypotenuse.* In the figure below the altitude, the base, and the hypotenuse are, respectively, represented by *a, b,* and *h.*

2. Trigonometric Functions



There are only two independent trig. functions:

$sine\theta = a/h$ (this is abbreviated to $sin\theta$), and

$cosine\theta = b/h$ (this is abbreviated to $cos\theta$)

all other trig functions can be obtained from these two functions.

$tangent\theta$, abbreviated to $tan\theta = a/b = (a/h)/b/h) = sin\theta/ cos\theta$

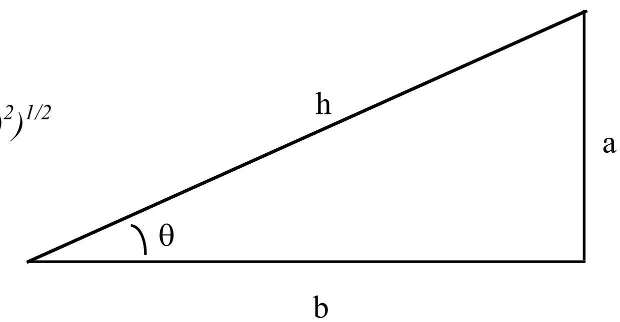
$cotangent\theta$, abbreviated to $cot\theta = 1/ tan\theta = b/a$

$cosecant\theta$, abbreviated to $cosec\theta = 1/ sin\theta = h/a$

secantθ, abbreviated to $sec\theta = 1/cos\theta = h/b$

cosine, cotangent, and cosecant are called co-functions , respectively, of *sine, tangent and the secant*.

3. Small Angle Approximation



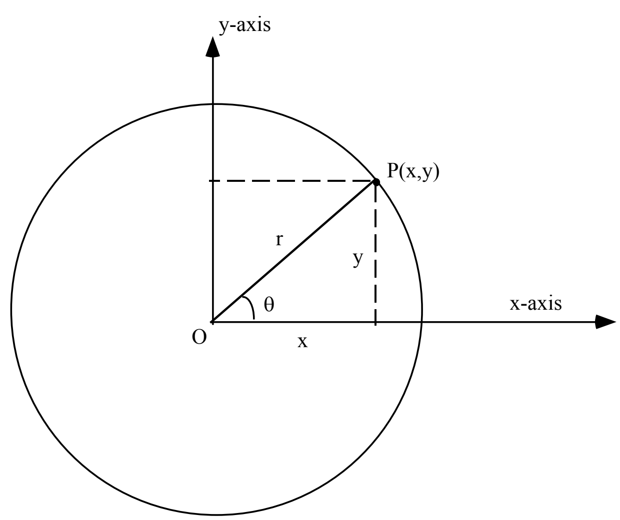
In the triangle (see figure)
$sin\theta = a/h = a/(a^2+b^2)^{1/2} = 1/(1+(b/a)^2)^{1/2}$
when the angle, θ is very small or $b>>a$, $(b/a)^2$ is much greater than *1*, which therefore can be neglected in comparison to $(b/a)^2$, thus
$sin\theta = 1/(1+(b/a)^2)^{1/2} = a/b = tan\theta$.

Furthermore, for small angles, $b \sim h$ and therefore $a/b = \theta$.

Thus, when angles are small, $sin\theta \sim tan\theta \sim \theta$ *(measured in units of radian)* You may be curious about exactly how small is a "small" angle. The term "small" is obviously a relative term. An angle can be considered small, for the purpose of the above approximation, as long as it is a small fraction of a radian.

Try Problem 2 in Sample Problems to get a better idea about what is meant by "small".

4. Signs of Trig. Functions



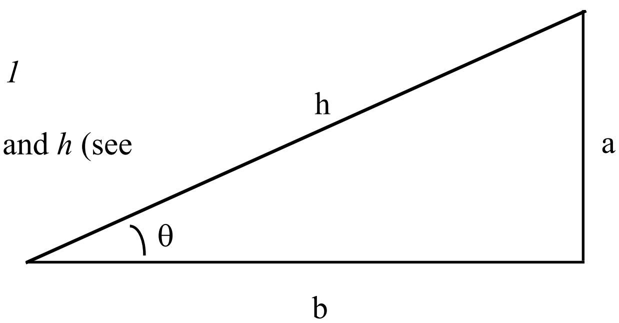
In the *xy*-coordinate system, the position of any point can be expressed in terms of two numbers called the *x*- and the *y*-coordinates. In this co-ordinate system, the *x*- and *y*-axes are perpendicular (just like the altitude and the base in a right-angle triangle) to each
other. Such a co-ordinate system is also called an orthogonal co-ordinate system. The orthogonality of the *xy*-coordinate system offers a convenient way to express the trig. functions in terms of the coordinates of a point on a circle.

$sin\theta$ = ordinate/radius = y/r
$cos\theta$ = abscissa/radius = x/r
$tan\theta$ = ordinate /abscissa = y/x

Since abscissas(x-coordinates) and the ordinate (y-coordinates) can be positive or negative depending upon which quadrant the point on the circle lies in, the numerical values of the various trig functions can be positive (+) or negative(−). For example, in the second quadrant(angles greater than 90^o but less than 180^o) , the x-coordinate of a point will be negative but the y-coordinate will be positive, therefore *sine* of angles in the second quadrant will be positive, but the cosine and the tangent functions will be negative. You can examine to convince yourself that only the tangent function is positive in the third quadrant and only the cosine function will be positive in the fourth quadrant.

5. Solved Examples

Example 1. Prove that $sin^2\theta + cos^2\theta = 1$



In the right-angel triangle of sides *a, b,* and *h* (see Figure)

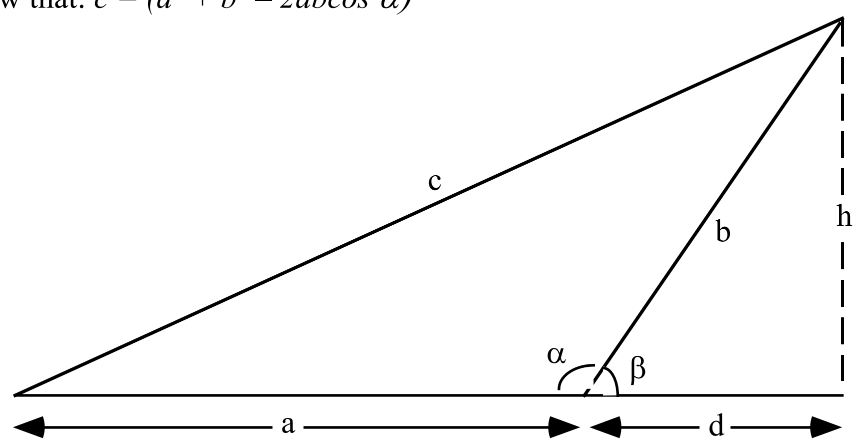
$sin\theta = a/h$, and $cos\theta = b/h$

Therefore,

$sin^2\theta + cos^2\theta = (a/h)^2 + (b/h)^2 = ((a^2 + b^2)/h^2 = 1$ (since $a^2 + b^2 = h^2$ from Pythagorean theorem)

Example 2. The Law of Cosines

Consider a triangle whose three sides are *a, b,* and *c* as shown. The angle between *a* and *b* is α. Show that: $c = (a^2 + b^2 - 2abcos\ \alpha)^{1/2}$



Solution:
From Pythagorean theorem:
$c^2 = h^2 + (a+d)^2 = h^2 + a^2 + d^2 + 2ad$
substitute, $d = b\ cos\beta$ and $h^2 + d^2 = b^2$ in the above equation to get

$c^2 = a^2 + b^2 + 2abcos\beta = a^2 + b^2 + 2abcos\ (180-a) = a^2 + b^2 - 2abcos\alpha$

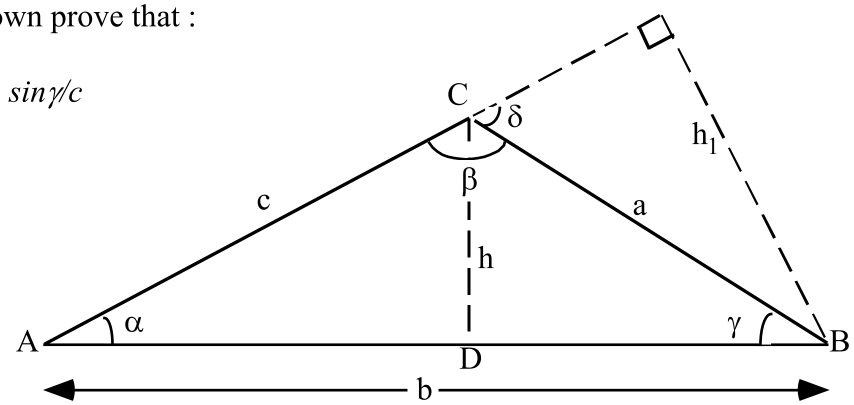
or,
$c = (a^2 + b^2 - 2abcos\ \alpha)^{1/2}$

(Note: the above equation is also known as the ***Law of Cosines***. The Law of Cosines states that *the square of any one side of a triangle is equal to the sum of the squares of the other two sides minus twice the product of these sides and the cosine of the included angle.*

Example 3. The Law of Sines:

In the triangle shown prove that :

$sin\alpha/a = sin\beta/b = sin\gamma/c$



(The equation above is also known as the ***Law of Lines*** which states that *in any triangle the lengths of the sides are proportional to the sines of the opposite sides.*

Solution:

Draw CD perpendicular to AB. Then, note that:

$h\ \ = a\ sin\gamma$ (from the right-angle triangle CDB)
$= c\ sin\alpha$ (from the right-angle triangle CAB)
The equality can be rearranged to read:
$\sin\alpha/a = \sin\gamma/c$ ……….[1]

Similarly, by drawing a perpendicular from B on to the extension of side AC , one can write:
$h_1 = a\ \sin\delta = b\ \sin\alpha$

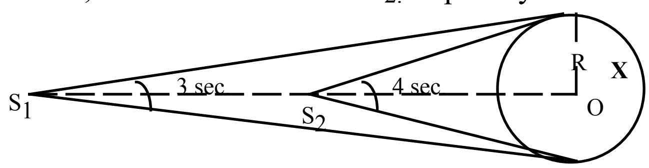
$\sin\alpha/a = \sin\delta/b = \sin(180 - \beta)/b = \sin\beta/b$ ……….[2]

Thus, combining eqn.[1] and [2],

$sin\alpha/a = sin\beta/b = sin\gamma/c$

Example 4.

An astronomical object, *X* of radius R is observed from two stars S_1 and S_2, 1.5 light-years apart. The angles subtended by *X* at the two stars are as shown. Determine the radius, R and the distance OS_2. Express your answer in meters.



Solution:

One light year equals the distance light travels in one year. The speed of light is $3x10^8$ m/s. Therefore the distance, d, between S_1 and S_2(traveled by light in 1.5years) is:

$d = 1.5*365*24*60*60*3*10^8 = 1.42*10^{16}$ m

Note: 1 deg = 60 min; 1min = 60 sec; 180 deg = π rad.
Thus, 1sec = π/60*60*180 rad.

Using the small angle approximation:

$R/OS_1 = R/(d+OS_2) = 1.5\ \pi/60*60*180$ rad ……………[1]
$R/OS_2 = 2\ \pi/60*60*180$ rad ……………[2]

Divide eqn.[2] by eqn.[1] to get:

$(d+OS_2)/ OS_2 = 4/3$, or $3d+3OS_2 = 4OS_2$

Thus, $OS_2 = 3d = 3*1.42*10^{16} = 4.26*10^{16}\,m$.

Now this value of OS_2 can be substituted in eqn.[2] to get :

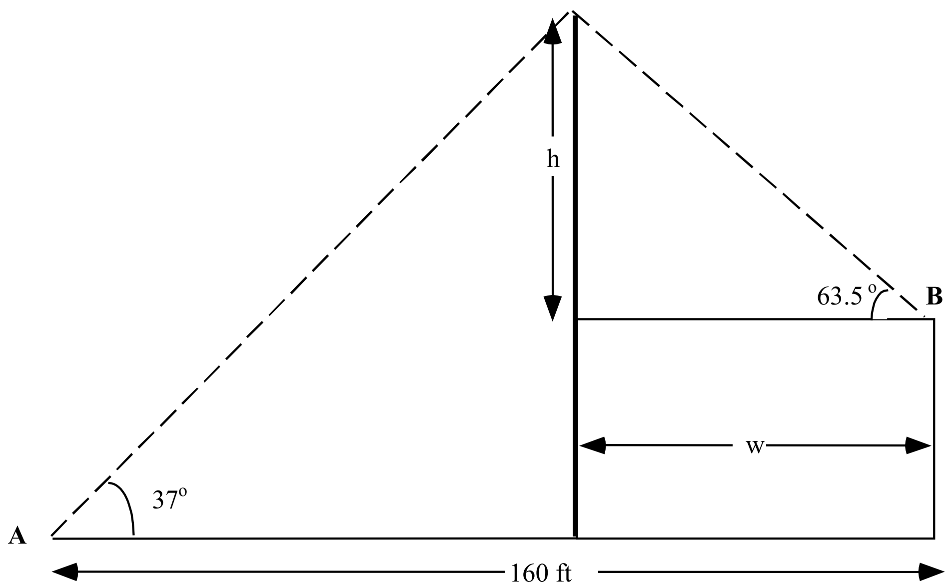
$R = OS_2*2\ \pi/60*60*180 = 4.26*10^{16}*2\ \pi/60*60*180 = 2.06*10^{11}m = 2.1*10^{11}m$

Example 5.

An antenna of height, ***h*** on the top of a *20 ft* tall building is viewed from point A on the ground and another point B on the roof of the building. The angles subtended by the antenna and the distances involved are shown in the diagram. Determine:

[a] the height ***h*** of the antenna.

[b] the width, *W* of the building.



Solution:

From the figure:

h/w = tan 63.5° = 2 , therefore

h = 2w, also

(h+20)/(160-w) = tan37° = 0.75,

(h+20) = 0.75*(160-w) ………….[1]

Also, substitute h = 2w in eqn.[1] to get

2w + 20 = 120 – 0.75w, which can be solved to get, w = 36.36 ft =36.4 ft, and finally

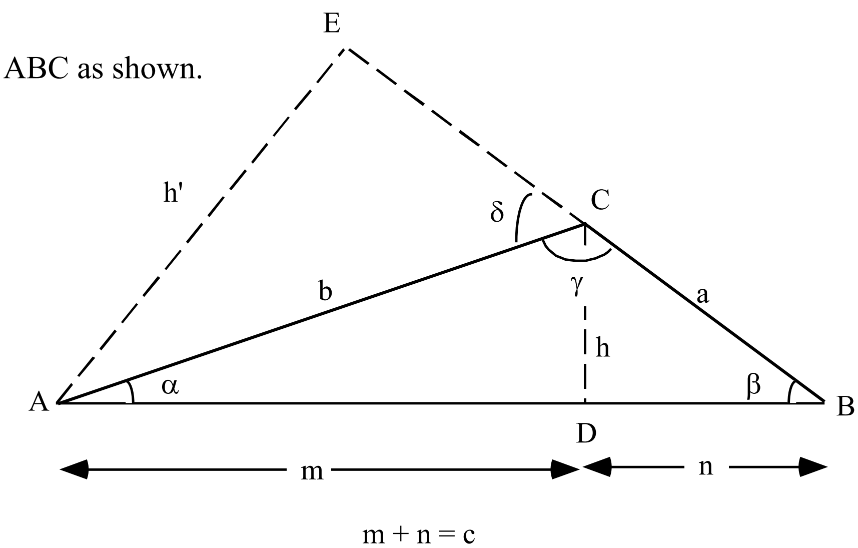
h = 2w = 2*36.36 = 72.7 ft

Example 6. Show that:

$sin(\alpha+\beta) = sin\alpha cos\beta + cos\alpha sin\beta$

Solution:

Consider a triangle ABC as shown.



Draw a perpendicular from C to the base AB. Similarly draw a perpendicular from A to BC extended.

Since $\delta+\gamma = 180 = \alpha+\beta+\gamma$,

angle $\delta = \alpha+\beta$

In the right-angle triangle AEC,

$h'/b = \sin(\alpha+\beta)$, or $h' = b\sin(\alpha+\beta)$

and in the right-angle triangle AEB,

$h'/(m+n) = \sin\beta$,

$h' = (m+n)\sin\beta = (b\cos\alpha + a\cos\beta)\sin\beta = b\sin(\alpha+\beta)$

$b\sin(\alpha+\beta) = b\cos\alpha \sin\beta + a\cos\beta \sin\beta$

$\sin(\alpha+\beta) = (a/b)\cos\beta \sin\beta + \cos\alpha \sin\beta$ ....[1]

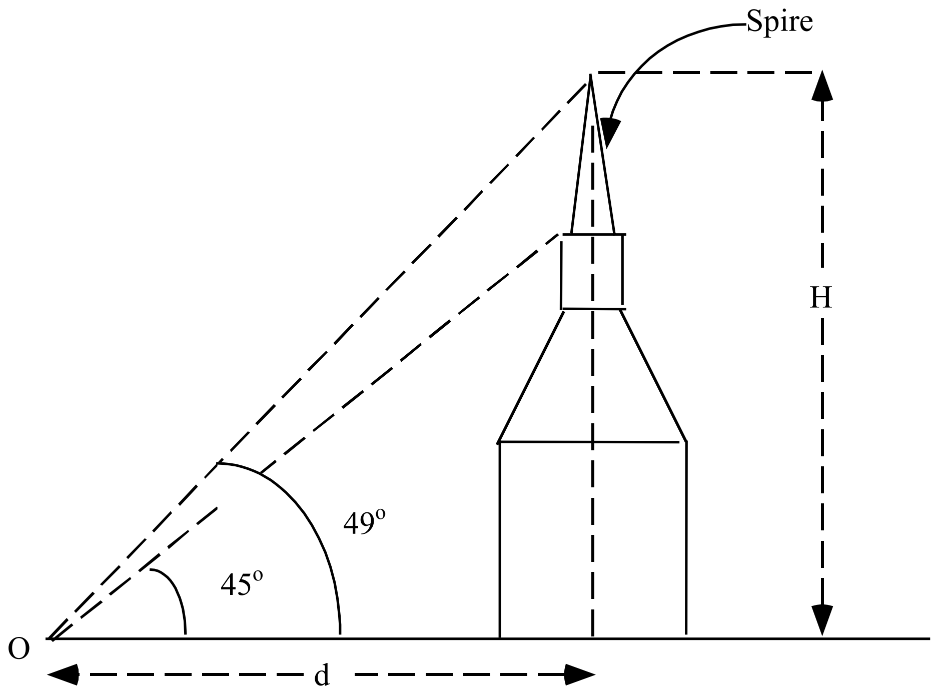
Applying law of sines to triangle ABC, gives, $\sin\beta/b = \sin\alpha/a$, which when substituted in the equation above gives:

$sin(\alpha+\beta) = sin\alpha\, cos\beta + cos\alpha\, sin\beta.$

Use of Trigonometric Functions

Interactive Problem 1:

An observer at O views a spire 10 ft tall, at the top of a church tower. The elevation angles subtended by the top and the bottom of the spire are as shown in the diagram to the right.



Question 1:

What is the relation between "H" and "d?

Choose from the following.

a. H = d

b. H = d tan 45°

c. H = d tan 49°

Answer 1:

The correct choice is (c).

Feedback:

H is the height of a right angled triangle whose base is "d". Angle made by the hypotenuse to the base is 49° and this is opposite to the vertical side "H". Thus H/d = tan 49°.

Question 2:

Write down the relation between the angle 45°, "H" and "d?

Choose from the following.

a. H – 10 = d
b. (H – 10)/d = d tan 45°
c. H = d tan 45°

Answer 2: The correct choice is (b).

Feedback:
(H–10) is the height of a right-angled triangle whose base is "d". Angle made by the hypotenuse to the base is 45° and this is opposite to the vertical side "(H–10)". Thus (H–10)/d = tan 45°.

Question 3:

From your answers to 1 and 2 find the value of H. Your answer is

a. H = 67 ft
b H= 77 ft
c H = 134 ft

Answer 3: The correct choice is (b).

Feedback:
From our answers to 1 and 2 we have,

d = H/tan49° = (H–10)/tan45°

Thus, H = (H – 10)*(tan 49°/tan45°) = (H– 10)*1.15 .

Thus, 0.15H = 11.5 and H = 77 ft.

Question 4:

From your answers to 1, 2 and 3 find the value of d. Your answer is

a. d = 67 ft

b d = 77 ft

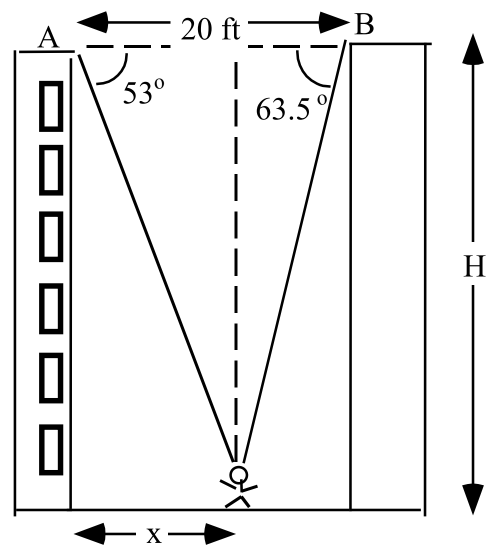
c d = 134 ft

Answer 3: The correct choice is (a).

Feedback: d =H/tan49° = 77 ft/1.15 = 67 ft.

Interactive Problem 2:

Two persons A and B on the roofs of two opposite buildings separated by 20 ft see a child on the street below in the directions shown. The height of the building "H" and the horizontal distance "x" of the child from A are unknown.



Question 1:

What is the relation between "H" and "x?

Choose from the following.

a. H = x

b. H = x tan 63.5°

c. H = x tan 53°

Answer 1: The correct choice is (c).

Feedback:

H is the height of a right angled triangle whose base is "x". Angle made by the hypotenuse to the base is 53° and this is opposite to the vertical side "H". Thus, H/d = tan 53°.

Question 2:

Write down the relation between the angle 63.5°, "H" and "x?

Choose from the following.

a. 20 – x = H

b. (20– x)/H = tan 63.5°

c. H/(20–x) = tan 63.5°

Answer 2: The correct choice is (c).

Feedback:

(20–x) is the horizontal distance of the child from B. Also, it is the base of a right angled triangle whose height is "H". Angle made by the hypotenuse to the base is 63.5° and this angle is opposite to the vertical side "H".

Thus H/(20–x)) = tan 63.5°.

Question 3:

From your answers to 1 and 2 find the value of x. Your answer is

a. x = 8 ft

b x = 4 ft

c x = 12 ft

Answer 3: The correct choice is (c).

Feedback:

From our answers to 1 and 2 we have,

H = x tan 53° = (20–x) tan 63.5°

Thus, x = (20 – x)*(tan 63.5°/tan53°)

= (20 – x)*(1.5) .

Thus, 2.5*x = 30 and x = 12 ft.

Question 4:

From your answers to 1, 2 and 3 find the value of H. Your answer is

a. H = 8 ft

b H = 12 ft

c H = 16 ft

Answer 3: The correct choice is (c).

Feedback: H = x tan53° = 16 ft.

Use of Trigonometric Functions

Sample Problems:

1. Show that

$cos(\alpha+\beta) = cos\alpha cos\beta - sin\alpha sin\beta$

2. Using your calculator, fill in the entries in the following table.

θ in degrees	θ in radians	sinθ	tanθ
2			
4			
6			
10			
15			
30			

3. Find h in Figure. 1. Δabc is a right-angle triangle.
(Ans., $h = 7.2$)

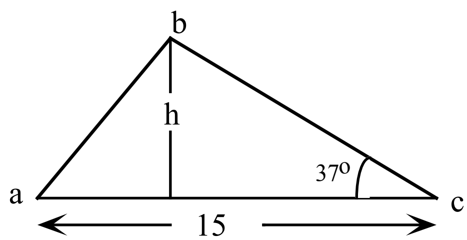


Figure 1

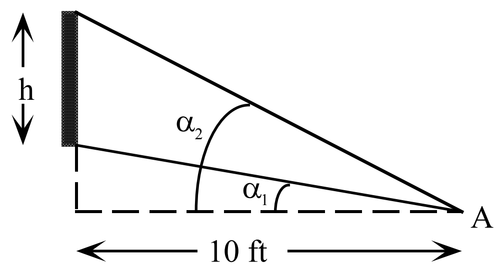


Figure 2

4. An observer's eye is at point A (see Figure 2) observing a mural of an unknown height h. The eye is 10 ft. from the wall, and the angles of elevation from the eye to the bottom and the top of the mural are α_1 and α_2, respectively. Write an expression for h in terms of α_1 and α_2.

(Ans. $h = 10\ (tan\ \alpha_2 - tan\ \alpha_1)$)

5. From two points east of a hill on level ground and 2000 ft apart, the angles of elevation of the hill are 12^o and 70^o (see Figure 3). Determine the height, h, of the hill.

(Ans. $h = 460.7\ ft$. or 461 to the nearest ft.)

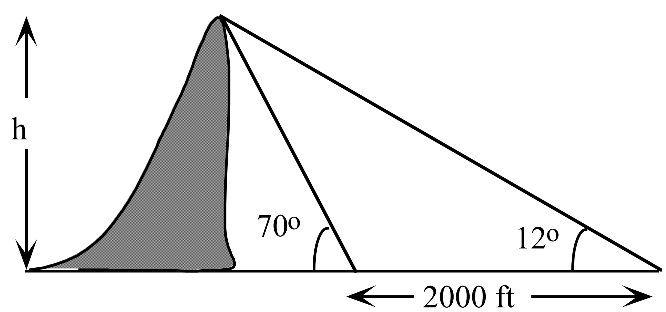


Figure 3

6. An observer stands on level ground, 300 m from the base of a TV tower, and looks up at an angle of 26^{o} to see the top of the TV tower. How high is the tower above the observer's eye level ?

(Ans. *Height of TV tower = 146.3 ft or 146 ft to the nearest foot*)

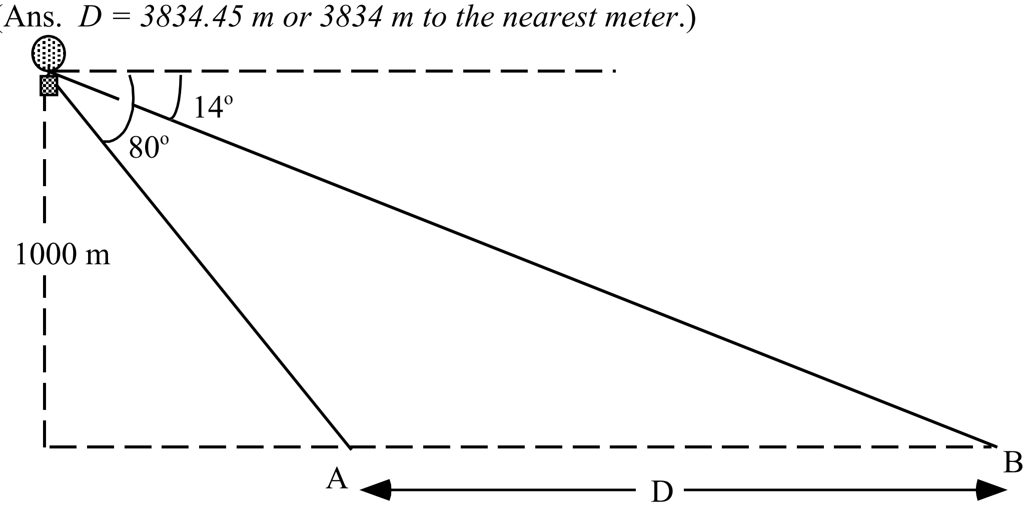
7. What is the angle of elevation of the sun when a 35-ft mast casts a 20-ft shadow ?

(Ans. *Elevation,* $\theta = 60.3^{o}$)

8. A weather balloon is directly west of two observing stations 10 km apart. The angles

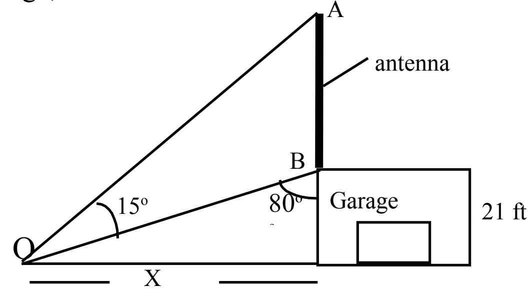
of elevation of the balloon from the two stations are 18^{o} and 75^{o}. How high is the balloon ?
(Ans. *height = 3559.2 m or 3559 m to the nearest meter.*)

9. From a balloon 1000 m high, the angles of depression to two artillery posts, in line with the balloon, are 14^{o} and 80^{o}. Determine the distance, D between the artillery posts.
(Ans. *D = 3834.45 m or 3834 m to the nearest meter*.)



10. A TV antenna is located on the top of a garage. The garage is 21 ft high.
From a point O as shown, the antenna subtends an angle of 15^{o}.
Calculate:
[a] *X*, the distance from *O* to the edge of the garage, and
[b] the height AB of the antenna,.

(Ans. *[a] X= 119.1 m, [b] 55.5 m*)



To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

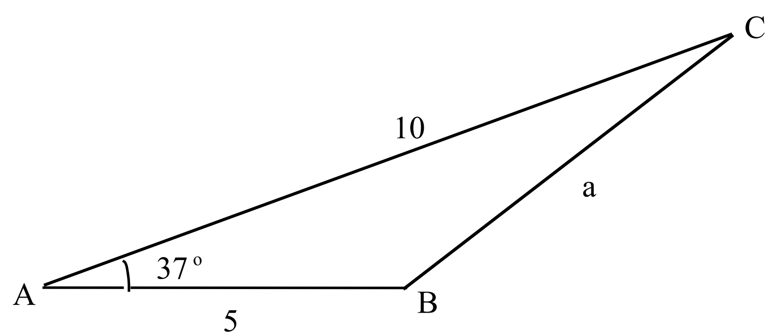
Unit 03 Trigonometric Functions: TEST-01

Problem 1. Determine the value of α and β in the following equation:

$$\sin(\alpha+\beta) = 0.6 \cos\beta + 0.5 \cos\alpha.$$

Ans. $\alpha = 37^0$; $\beta = 30^\circ$

Problem 2. Determine the unknown length of the side BC = a in the triangle ABC shown below. Given, AC = 10, and AB = 5.



Ans. a = 6.7

Problem 3. Two airplanes flying 1.0 km apart at the same altitude spot a target on the ground. The angles of depression subtended by the target at the two airplanes are 45^o and 37^o. Determine the altitude at which the planes are flying. (For the distances involved, the earth can be considered flat.)

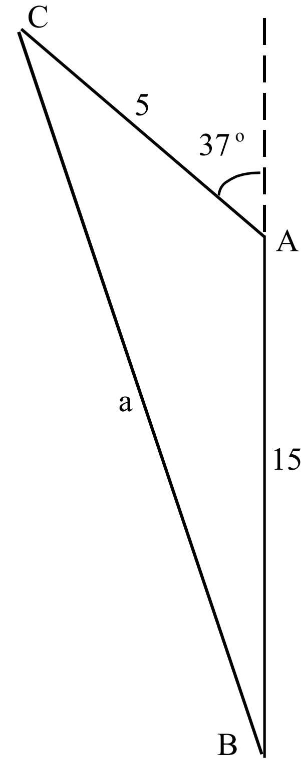
Ans. The altitude, a = 3000 m

Unit 03 Trigonometric Functions: TEST-02

Problem 1.
Determine the value of α and β in the following equation:

$cos\ (\alpha+\beta) = 0.6(\ cos\beta - \ sin\alpha)$

Ans. $\alpha = 53^0$; $\beta = 37^o$



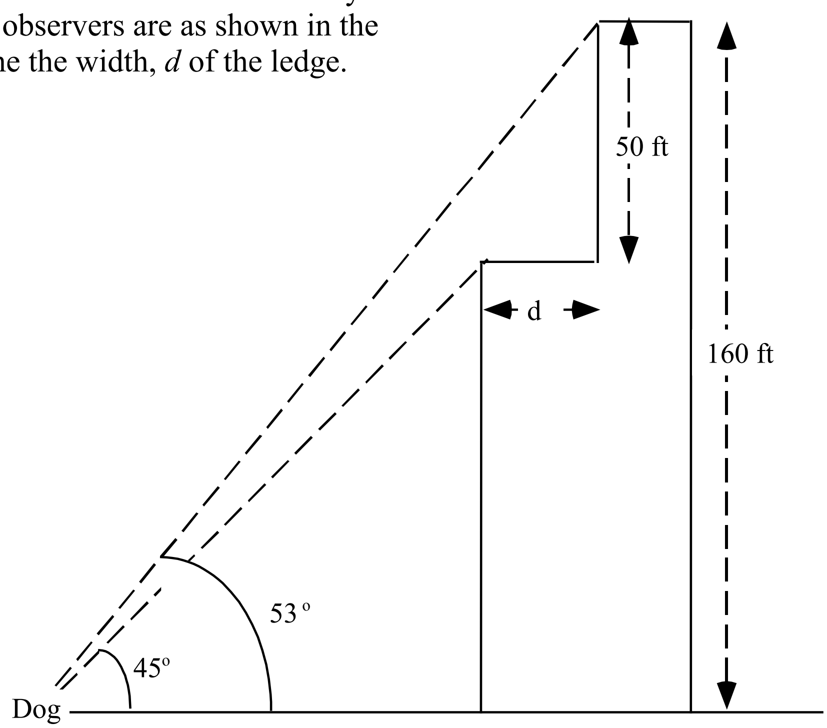
Problem 2.

Determine the length of the unknown side BC = *a* in the triangle ABC shown on the right. Given, AC = 5, and AB = 15.

Ans. a = 19.2

Problem 3.

A small dog (neglect its physical dimensions) is spotted from a *16th* floor window *160 ft* high, and a ledge of width, *d, 50 ft* below the 16th floor window. The angles of elevation subtended by the dog at the two observers are as shown in the diagram. Determine the width, *d* of the ledge.



Fundamentals of Solid Geometry - Shapes and Volumes

Introduction.

Skills you will learn:

a. Classify simple 3-dimensional geometrical figures.

b. Calculate surface areas of simple 3-dimensional figures and complex figures that can be obtained by combining such simple geometrical figures.
c. Calculate volumes of simple 3-dimensional figures and complex figures that can be obtained by combining such simple geometrical figures.

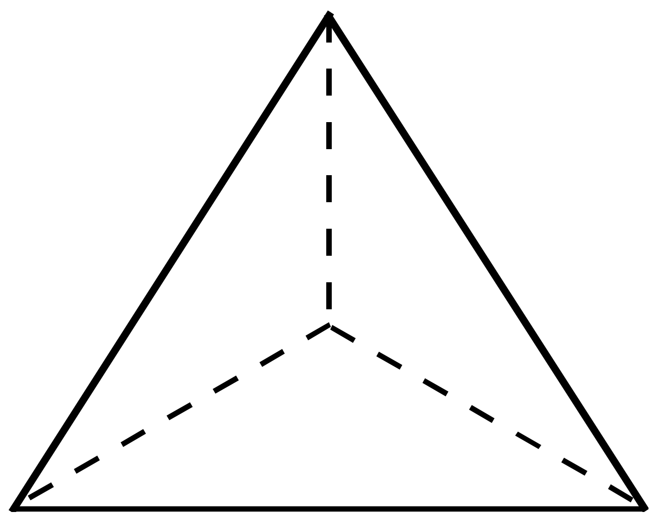
d. Solve real world problems involving surface areas and volumes.

Some Concepts.

Polyhedron

A *polyhedron* is a solid bounded by planes, for example, a *cube* is bounded by six planes and is a *hexahedron*.
A *tetrahedron* (see figure on the right) is formed by four triangular faces. When three or more planes meet at a point, it is called a *vertex*. The intersections of planes are called the *edges* and the sections of planes are called the *faces* of the polyhedron.

We will be concerned mostly with two types of polyhedrons- the *prism* and the *pyramid.*



Tetrahedron

Prism

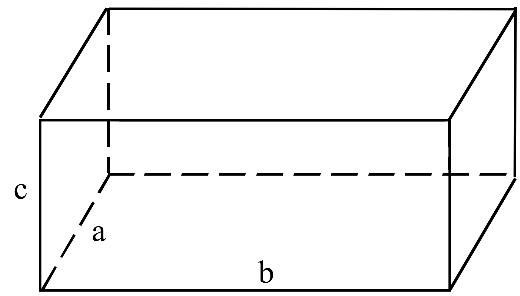
Two of the prism's faces are congruent polygons contained in two parallel planes. Planes through the sides of the congruent polygons form the other faces of a prism. A rectangular prism with some relevant terms is shown below as an example. A parallelepiped is a prism whose bases are parallelograms. A right circular cylinder can be viewed as a prism whose base is a circle-a polygon with infinite number of sides.

A Rectangular Prism.

In figure:

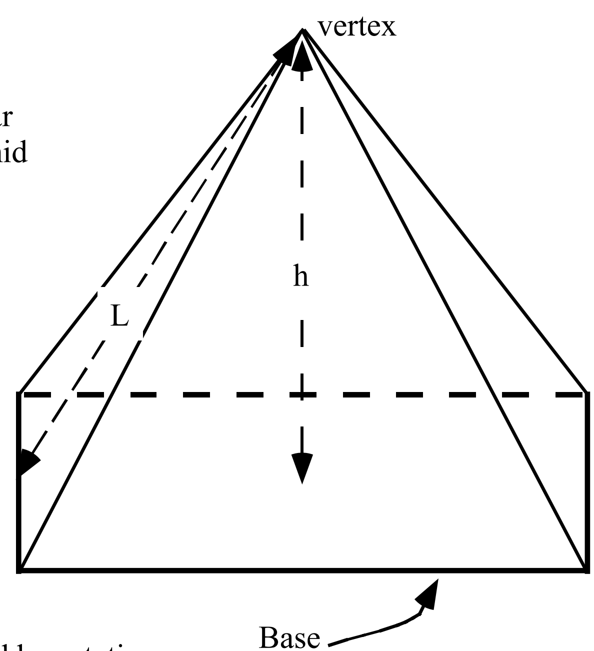
Altitude = c and

base of dimensions "a" and "b".



Pyramid.

A pyramid is a 3-dimensional figure formed by a polygon (called the base) and three or more triangular planes meeting at a common point (called the vertex).
The perpendicular from the vertex to the base is called the altitude (h) of the pyramid. If the base is a regular polygon and the altitude meets the base at the center, the pyramid called regular or right. The slant height (L) of a regular pyramid is the altitude of one of the lateral faces.
A right circular cone can be viewed as a pyramid whose base is a circle – a polygon with infinite sides.

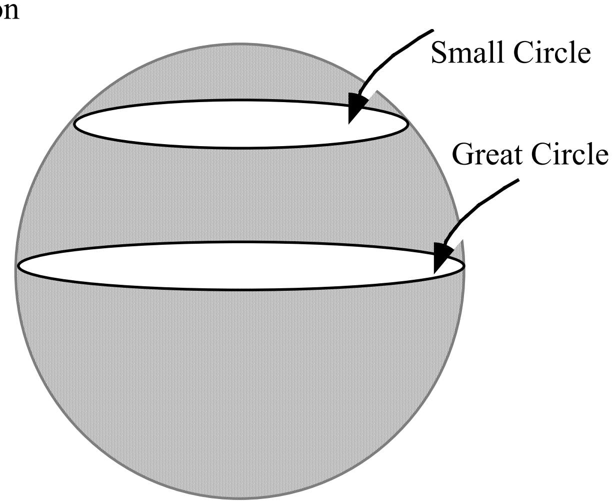


Sphere

A *spherical surface* is a curved surface obtained by rotating a semicircle about its diameter. All points on a spherical surface are equidistant from a common point called the *center*.

A sphere is a solid bounded by a spherical surface.

A *great circle* of a sphere is any circle obtained by the intersection of the sphere by a plane containing the center of the sphere. A *small circle* is obtained when the intersecting plane does not pass through the center.



Surface Areas of Solids

Area of a Prism

The lateral area S of a prism is the sum of the areas of the lateral faces and equals the product of the base perimeter P and the altitude h:

S = Ph. Thus, for a cylinder, S = 2πrh.

For a rectangular prism (see figure on previous page) altitude *c* and base dimensions of *a* and *b* the lateral area is given by, $S = 2(a+b)c$.

The total area *A* of a prism is simply the sum of the lateral area *S* and the two basal areas, B_1 and B_2.

$A = S + B_1 + B_2$

For a right prism, $B_1 = B_2 = B$, therefore $A = S+2B$.

For a cylinder $A = 2\pi rh + 2\pi r^2 = 2\pi r(h + r)$

For a right rectangular prism of edges, a, b, and c,
the total area $A = 2(a+b)c + 2ab = 2ab + 2bc + 2ca$.

Area of a Pyramid

Lateral area, S = ½ LP, where L is the slant height and P is the base perimeter.
Thus, S = πrL for a cone of base of radius r.
The total area A for a pyramid, $A = S + B$, where *B* is the basal area.
For a cone $A = \pi rL + \pi r^2 = \pi r(L + r)$

Area of a Sphere

The surface area, *A* of a sphere of radius *R* is given by , $A = 4\pi R^2$

Volumes of Solids

Volume of a Prism

The volume of a prism is the product of its basal area *B* and altitude *h*. For a cylinder $V = Bh = \pi R^2 h$. Similarly for a right rectangular prism $V = abc$, where a, b, and c are edge lengths of the prism.

Volume of a Pyramid

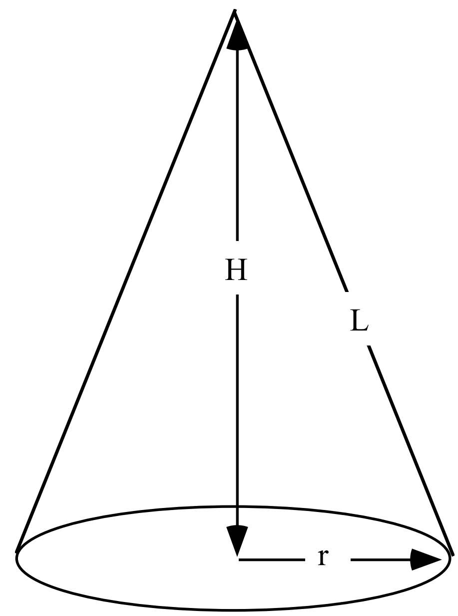
For a pyramid, the volume V = (1/3)*B*h. Thus, for a cone $V = \pi R^2 h/3$.

Volume of a Sphere

Volume of a sphere, $V = (4/3)\pi R^3$ where, *R* is the radius of the sphere.

Solved Examples.

Example 1.



A right circular cone has a base of radius r = 12 cm and an altitude of H = 16 cm.

Determine the cone's
[a] Lateral area
[b] Total area
[c] Volume

Solution:

The cone's slant height forms the hypotenuse of the right angle triangle whose base is the radius and altitude is the altitude of the cone. Therefore, using Pythagorean theorem, the slant height, $L = (12^2 + 16^2)^{1/2} = 20$cm

a. The total area $S = \pi rL = (3.14)*(12)*(20) = 753.6$ cm^2 (754 cm^2 rounded off to three significant figures.

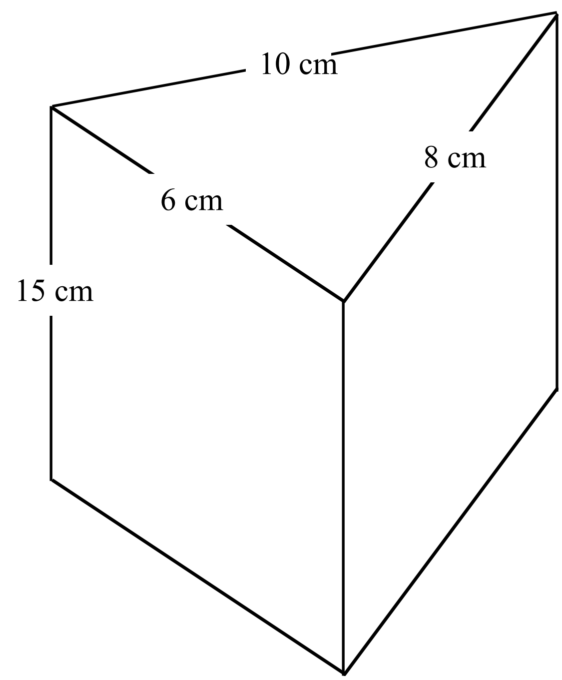
b. The total area = $\pi r(L + r) = (3.14)*(12)*(20+12) = 1205.76$ cm^2 (1206 cm^2 when approximated to the nearest whole number or to four significant digits)

c. The volume, $V = \pi R^2 h/3 = (1/3)*[(3.14)(12^2)(16)] = 2411.523$ (2412 cm^3 when approximated to the nearest whole number)

Example 2.

Consider a right triangular prism of basal edges 6.0 cm , 8.0 cm, and 10.0cm and an altitude of 15 cm. Note that the base of the prism is a right angle triangle.

Calculate the prism's
a. lateral area
b. total area
c. volume



Solution

a. The perimeter of the base of the prism is

$P = 6.0 + 8.0 + 10.0 = 24.0$ cm.

Therefore the lateral area

$S = Ph = (24)(15) = 360\ cm^2$.

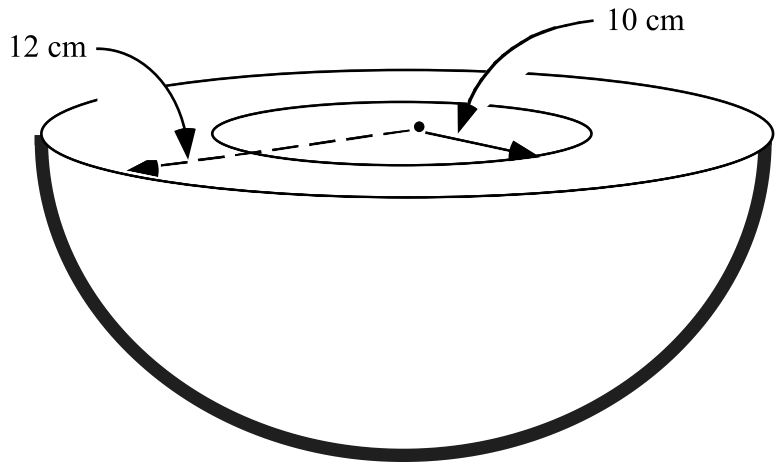
b. The total area,

A = S + area of the two triangular bases
= 360 + 2*(1/2)(base)(altitude)
= 360 + 2*(1/2)*(6)(8)
= $360 + 48 = 408\ cm^2$.

Example 3.

Find the mass of a spherical wooden shell whose inner and outer radii are, respectively, 10, and 12 cm. The mass density ρ of the wood is 20 kg/m^3 (Note: the mass density is defined as mass per unit volume $\rho = M/V$). Express your answer in *kg*.

Solution.



The spherical shell can be viewed as a sphere of radius 12 cm from which a concentric sphere of radius 10 cm has been removed. [One-half of a spherical shell is depicted in the diagram on the right]

The volume of the spherical shell :

$V = (4/3)\,\pi R^3 - (4/3)\,\pi r^3$
$= (4/3)\,\pi(R^3 - r^3)$
$= (4/3)(3.14)[(12)^3 - (10)^3\}$
$= 3047.9\ cm^3 = 3047.9 \times 10^{-6} m^3$
$= 3.05 \times 10^{-3}\ m^3$

(Note: Answer is rounded off to the second decimal place. Also, $1 cm^3 = 10^{-6}\ m^3$)

The mass $M = \rho V = (20\ kg/m^3)(3.05 \times 10^{-3}\ m^3) = 0.061$ kg.

Example 4.

The mass of the earth is given to be 6×10^{24} kg. Determine the average mass density ρ of the earth assuming it to be a perfect sphere of radius $R = 6.4 \times 10^3$ km.

(Express your answer both in kg/m^3 and gm/cm^3).

Solution.
$M = 6 \times 10^{24}$ kg. ; $V = (4\pi/3)(6.4 \times 10^6)^3$

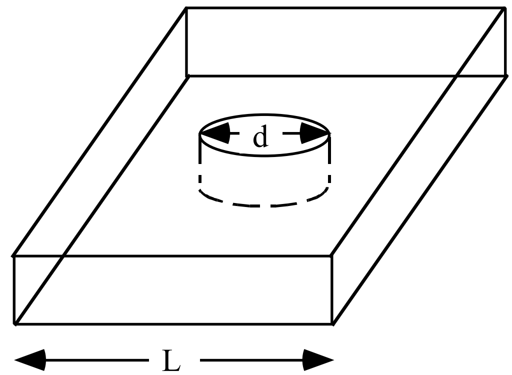
Substitute these values of M and V in $\rho = M/V$ to get,
$R = 5.5 \times 10^3\ kg/m^3 = 5.5\ gm/cm^3$.

Note: we have used the following conversion factors:
$1\ kg = 10^3$ gm, and $1 m^3 = 10^6\ cm^3$.

Fundamentals of Solid Geometry - Shapes and Volumes

Interactive Problem 1:

A hole of diameter d = 20 cm is cut through a square metal plate of side L = 1.2 m and thickness t = 8 cm.



Question 1:

What is the surface area of the metal plate before the hole is drilled through it?

Choose from the following.

a. 1.44 m^2

b. 1.536 m^2.

c. 3.264 m^2.

Answer 1: The correct choice is (c).

Feedback: It has a top and bottom surface each of area L^2 and four side surfaces

each of area L*t .

Thus, total surface area is $2L^2 + 4*L*t$ $= 2(1.2\text{ m})^2 + 4*(1.2\text{ m})*(0.08\text{ m})$

$= 2.88\text{ m}^2 + 4*(0.096\text{ m}^2) = 3.264\text{ m}^2$.

Question 2:

What is the surface area of the hole drilled through the metal plate ?

Choose from the following.

a. 0.05 m^2

b. 5.0 m^2.

c. 500 m^2.

Answer 3: The correct choice is (a).

Feedback: The hole is cylinder of height "t" and radius r = d/2. Its surface area is $2\pi rt$ = 2π*1 m)*(0.08 m) = 0.050 m^2.

Question 3:

After the hole is drilled, how much volume of the metal is left?

Choose from the following.

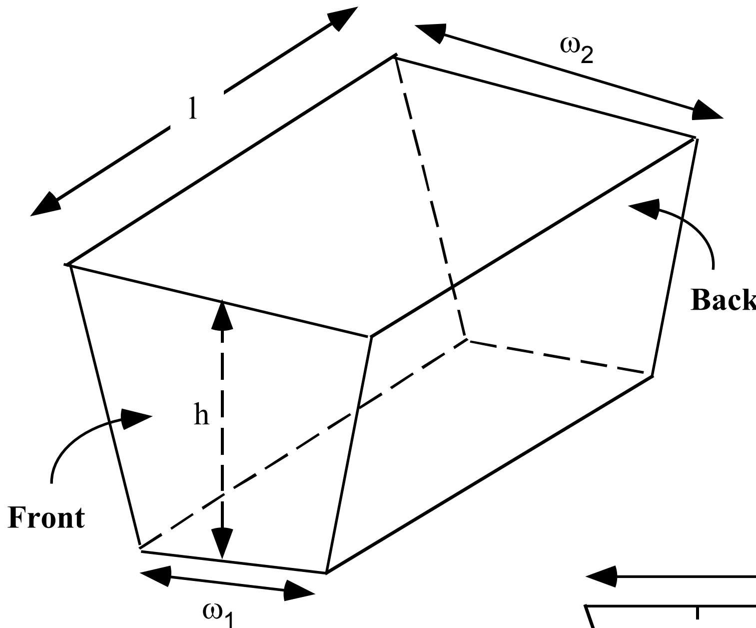
a. 115.2×10^{-3} m^3.
b. 2.5×10^{-3} m^3.
c. 112.7×10^{-3} m^3.

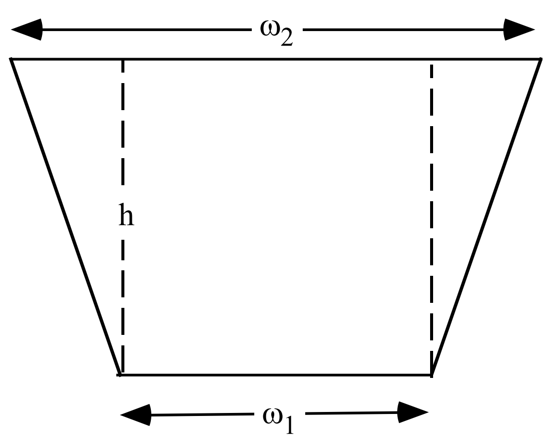
Answer 3: The correct choice is (c).

Feedback: The hole is cylinder of height "t" and radius r = d/2. Its volume is $\pi r^{2*} t = \pi (1\ m)^2 * (0.08\ m) = 2.5 \times 10^{-3}\ m^3$. Volume of the plate before the hole is drilled $L^2 * t = (1.2\ m)^2 * (0.08\ m) = 115.2 \times 10^{-3}\ m^3$.
Volume of metal remaining is $115.2 \times 10^{-3}\ m^3 - 2.5 \times 10^{-3}\ m^3 = 112.7 \times 10^{-3}\ m^3$.

Interactive Problem 2:

An above-ground swimming pool is shaped as shown in the figure below; i.e., its length l = 80 ft (same at the top and bottom), its width ω_1 = 40 ft at the bottom and increases to ω_2 = 52 ft at the top and its depth h = 8 ft throughout. Note that the front and back surfaces are vertical whereas, the side surfaces are inclined.





Question 1:

What is the area of the front surface?
Choose from the following.

a. 320 ft^2
b. 368 ft^2.
c. 2080 ft^2.

Answer 1: The correct choice is (b).
Feedback: Front surface consists of a rectangle of sides ω_1 = 40 ft and h =8 ft and two identical right angled triangles each of base $(\omega_2 - \omega_1)/2$ = 6 ft and height h= 8 ft.
Total area is (40 x 8) ft^2 + 2*(1/2)(6 x 8) ft^2 = 368 ft^2 .

Or you can consider the front surface as a trapezoid –see Unit 02-
area = $h*(\omega_1 + \omega_2)/2$ = 8*(50+40)/2 =368 ft^2 .

Question 2:

The length of each slant side surface is l = 80 ft, what is the width of this slant surface?

Choose from the following.

a. 6 ft
b. 8 ft.
c. 10 ft.

Answer 2: The correct choice is (c).
Feedback: Slant surface has a length of 80 ft and a width the same as the hypotenuse of a right-angled triangle of base 6 ft and altitude 8 ft. Thus, its width is 10 ft. See figure in question 1.

Question 3:

What is the volume of the above ground swimming pool ?

Choose from the following.

a. 29440 ft^3
b. 25600 ft^3
c. 3840 ft^3

Answer 3: The correct choice is (a).
Feedback:

Recall the pool can be thought of as a prism.

Volume of the pool is V = Area of the front surface*l = 368 ft^2*80 ft = 29440 ft^3.

Another Method:

Volume of the pool is $V = \omega_1 * l * h + (1/2)*(\omega_2 - \omega_1)* l * h$

$= 40*80*8 + (1/2)*(52-40)*80*8 \text{ ft}^3 = (25600 + 3840) \text{ ft}^3.$
$= 29440 \text{ ft}^3.$

Fundamentals of Solid Geometry - Shapes and Volumes

Sample Problems

1. The volume of a rectangular prism is 1080 cm^3. If the length is 12 cm and the height is 18 cm, determine the: [a] the width of the prism, [b] the area of the smallest face.

Ans. [a] w = 5cm [b] A= 60cm^2

2. If the edge of a cube is doubled, determine by what factor will [a] the diagonal of a face, [b] the surface area, and [c] the volume change ?

Ans. [a] 2 [b] 4 [c] 8.

3. You have to fabricate a 5m section of a concrete sewer pipe of inner and outer diameters, respectively, of 1.8 and 2.0 m.

a. Determine the volume of concrete needed.
b. Determine the mass of the pipe if the density of concrete is 4000 kg/m^3. (Note: the density is defined as mass/volume)

Answer: [a] 2.98m^3 approximated to 3m^3 ; [b] 12000 kg.

4. Consider an insulated cylindrical copper cable. The copper conductor is 4.0 cm in diameter surrounded by insulating polymer (plastic) sheathing 2 cm thick. If the densities of copper and polymer are, respectively, 9000 kg/m^3 and 1500 kg/m^3, what is the mass of a 1km length of such a cable?

Answer: Total mass = 16956 kg.

5. You have some pizza dough in the shape of a circle of radius 4 in. You roll it out (maintaining the circular shape) until the radius increases to 8 in. As a result of rolling, by what factor has the:

a. circumference of the original dough increased ?
b. the flat area of the original dough increased ?

Answer: [a] 2 [b] 4

6. **a.** A foundry was ordered by the defense minister to cast a lead cannonball of diameter 20 cm. The material for this will be provided by melting smaller, identical lead shots spherical in shape, each with a diameter of 1 cm. How many lead shots will be needed to cast a single cannonball ?

b. "Sorry, not solid cannonballs, we made a mistake" said the defense minister. We would like a spherical shell of uniform wall thickness of 1 cm. This way we can pack the shell with explosives and have some fun. If the foundry were now to recast the solid ball (of part 4a above) into one shell to please the minister, what would the outer diameter of such a shell be ?

Answer: [a] $(20)^3$ =8000 ;[b] 37.5 cm

7. You have a helium-filled balloon in the shape of a perfect sphere of radius 1.5 m. You
blow it up to a radius of 4.5m by introducing more high pressure helium. Assume that the balloon expands uniformly. As a result of increasing the size of the balloon, determine by what factor would the:

a. the surface area of the balloon increase ?
b. the volume of the balloon increase ?

Answer: [a] 9 ; [b] 27

8. Determine the length of a copper wire of circular cross section and diameter 0.4 cm that can be fabricated from a 20-kg copper ingot. The density of copper is 90 kg/m^3.

Ans. Length, L = 176.9 m

To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

Unit 04- Fundamentals of Solid Geometry - Shapes and Volumes

Test-01

Note: in the problems below you must use $\pi = 3.14$.

Problem 1.

The inner dimensions of an in-ground swimming pool are, width = 13ft, length = 18 ft, and depth = 9 ft. The pool walls and the bottom are uniformly one foot thick and made of concrete. Determine the weight, W (in pounds) of the concrete used to construct the pool, if one cubic inch of concrete weighs 0.25 pounds.

Answer: W = 386208 lbs.

Problem 2.

Consider a semi-spherical bowl machined out of wood. The outer radius of the bowl is 20 cm and it is uniformly 2cm thick. Determine the total surface area, A of the bowl. Express your answer in whole cm^2 (drop any fractional cm^2 from your final answer).

Answer: A = 4785 cm^2

Problem 3.

A 100 m long polymer cable of circular cross section of diameter 0.4*cm* has a mass of 1884 *gm*. Determine the mass density, ρ of the polymer in kg/m^3.

Answer: $\rho = 1500\ kg/m^3$.

To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

Unit 04- Fundamentals of Solid Geometry - Shapes and Volumes

TEST-02

Note: in the problems below you must use $\pi = 3.14$.

Problem 1.

The inner dimensions of an in-ground swimming pool are, width = 13ft, length = 18 ft, and depth = 9ft. The pool walls and the bottom are uniformly one foot thick and made of concrete. Determine the total surface area, A of the concrete pool in units of ft^2.

Answer: A = 2158 ft^2

Problem 2.

Consider a semi-spherical bowl machined out of wood. The outer radius of the bowl is 20 cm and it is uniformly 2cm thick. You now fill the bowl to the top with water. Determine the mass, M of the filled bowl , given that a cubic meter (m^3) of water and wood, respectively, have a mass of 1000 kg, and 500 kg. Express your answer in whole grams, i.e. drop any fraction of a gram from your answer.

Answer: M = 14477 gm.

Problem. 3.

A *1 km* long copper wire of circular cross- section of diameter 2cm is to be melted and recast into a wire of circular cross-section of radius 5 cm. What is the length, L (in whole meters) of the recast wire. The density of copper is 9000 kg/m^3.

Answer: L.= 40 m.

Physical Quantities - Scalars and Vectors

Introduction:

To define most of the physical quantities precisely, we need to give them a **magnitude**, expressed in appropriate units and, for some of them, we also need to give the **direction** associated with that magnitude. For example, consider how to define a force of 100 Newton (which would be the magnitude) on an object vertically upward (the direction then would be upward). This can be shown graphically along positive y-axis of a rectangular coordinate system. These conditions can be represented most conveniently by a vector, which gives the magnitude and direction with respect to a set of coordinate axes. However, note that some quantities like hydrostatic pressure (force per unit area) have the same magnitude in all directions; i.e., the pressure at a specific point in a fluid is the same in all directions. So, a single vector cannot represent it; while the force (that produced the pressure) can be represented by a vector. (This pressure/force distinction is due to the changing direction/orientation of the area upon which the force acts.) Since many engineering situations require us to represent quantities in terms of magnitudes and directions, engineering students must acquire the ability to represent the appropriate quantities as vectors and be able to manipulate these vector quantities.

Important concepts from this chapter:

- Scalars represent simple magnitudes with no directions involved;
- Vector representations of directed physical quantities
  - Free vector, Sliding vector, Fixed vector
- Combinations of Vectors
  - Parallel, Concurrent, Collinear, Coplanar
- Projections of a Vector
  - Components in rectangular coordinate frames
  - Resolution in non-orthogonal directions
  - Unit vectors in specified directions
  - Law of Sines and Cosines
- Vector Addition
  - Graphical
  - Parallelogram Law
  - Analytical

What mathematical background is needed?

- Symbolic, algebraic manipulation.
- Algebraic addition, multiplication and division.

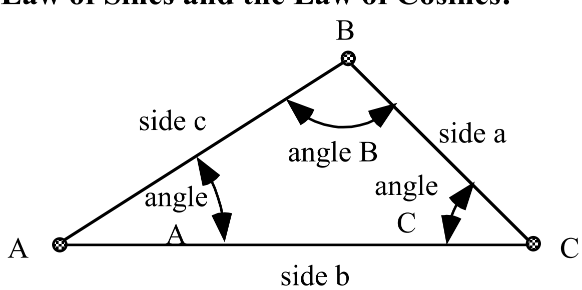
What are the objectives of this unit and what will we be able to do after mastering this unit?

1. Utilize the characteristics of a vector quantity;
2. Represent a vector quantity in terms of its magnitude and direction, given a description of the physical situation;
3. Utilize the characteristics of two or more vectors that are concurrent, or collinear, or coplanar;
4. Combine (add or subtract) two or more vectors into a single, resultant vector;
5. Resolve (break-up) a vector into components in specified directions.
6. Combine vector components into a resultant vector, giving magnitude and direction in terms of orthogonal unit vectors in the reference frame;
7. Combine vectors given in any reference frame by graphical methods;
8. Learn to use the Parallelogram Law to add similar vector quantities;
9. Utilize algebraic and trigonometric relationships between vectors and the reference frame to combine vectors;
10. Find the unit vector in a given direction in a given reference frame;
11. Represent a vector in terms of its magnitude and a unit vectors in the principal directions of a given reference frame;

As a starting point, we will deal with the basic graphical, algebraic and trigonometric operations associated with 1- and 2- dimensional vectors.

In the following sections we will look at examples of the vector addition of forces, and show how to find their resultants using graphical methods along with simple trigonometric relationships, such as the Law of Sines and the Law of Cosines.

Law of Sines and the Law of Cosines:



Law of Sines:

$$a/\sin A = b/\sin B = c/\sin C$$

Law of Cosines:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = c^2 + a^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Representation of a vector: In this unit, vectors are represented by bold-faced letters i.e. **A** where "A" is the magnitude of **A** and it is represented graphically by a straight line drawn to a scale with the length representing the magnitude (A) and the arrow giving the direction of that vector (**A**).

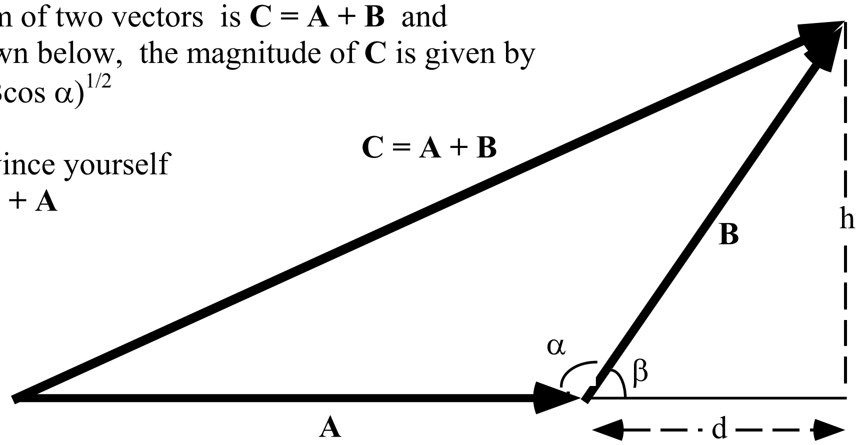
In figure, **A** (vector A) is represented by a horizontal straight line whereas, **B** (vector B) is inclined to **A** at an angle β. Direction of **B** is at an angle β to that of **A** .

Addition of vectors: Only similar vectors (vectors representing the same physical quantity can be added – Velocity to velocity, force to force and so on) can be added.

Graphical Method of addition: To add two vectors **A** and **B**, place the tail of the second vector (**B**) at the head of the first vector (**A**). A third vector – let us say **C** drawn from the tail of the first (**A**) to the head of the second (**B**) gives the sum of the two vectors **A** and **B** .

Graphically, the sum of two vectors is **C = A + B** and analytically, as shown below, the magnitude of **C** is given by
$C = (A^2 + B^2 - 2AB\cos\alpha)^{1/2}$

Note: You can convince yourself that **C = A + B = B + A**



From Pythagorean theorem:

$C^2 = h^2 + (A+d)^2 = h^2 + A^2 + d^2 + 2Ad$

substitute, $d = B\cos\beta$ and $h^2 + d^2 = B^2$ in the above equation to get

$C^2 = A^2 + B^2 + 2AB\cos\beta = A^2 + B^2 + 2AB\cos(180-\alpha) = A^2 + B^2 - 2AB\cos\alpha$
or,
$C = (A^2 + B^2 - 2AB\cos\alpha)^{1/2}$.

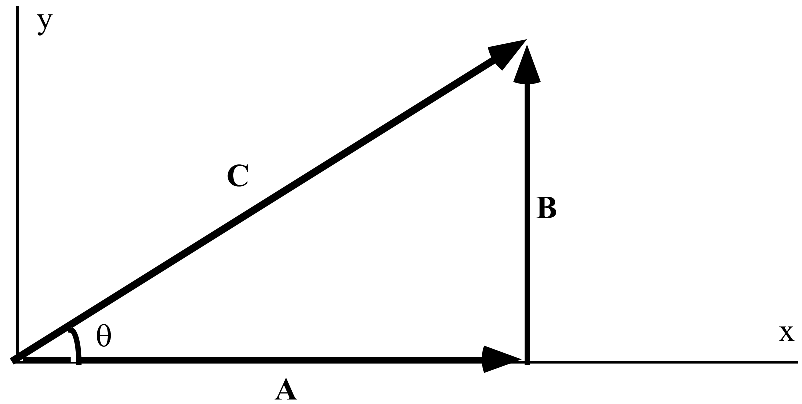
We identify **C** as the sum of vectors **A** and **B** or alternately we can also identify **A** and **B** as components vectors of **C.**

By a similar argument, we see that "d" is the component of **B** along the direction of **A** whereas, "h" is the component of **B** along a direction perpendicular to **A**.

When the two vectors **A** and **B** are at right angles, $\alpha = 90^0$, we get

$C = (A^2 + B^2 - 2AB\cos 90^0)^{1/2} = (A^2 + B^2 - 2AB*\text{zero})^{1/2} = (A^2 + B^2)^{1/2}$

In this case, **A** is along x-axis and **B** is along y-axis, we call **A** and **B** as rectangular component vectors of **C**. See figure below.



From the property of a right-angled triangle, magnitudes are related by

A/C = cosθ whereas, B/C = sinθ.

Thus, A (= C*cosθ) along x-axis is called the x-component of C (= C_x) whereas, B (= C*sinθ) along y-axis is the y-component of C (= C_y).

Magnitude of **C** is obtained as follows:

$C = (C_x^2 + C_y^2)^{1/2}$ since $(C_x^2 + C_y^2) = C^2[(\cos^2\theta) + (\sin^2\theta)] = C^2$.

Also, the direction of **C** is given by the angle θ made by **C** with x-axis.

We see that $(C_y / C_x) = C^*\sin\theta / C^*\cos\theta. = \tan\theta$.

Thus, if we know the rectangular components C_x and C_y of a vector **C**, we can determine the magnitude $C = (C_x^2 + C_y^2)^{1/2}$ and direction from $\tan\theta = (C_y / C_x)$.

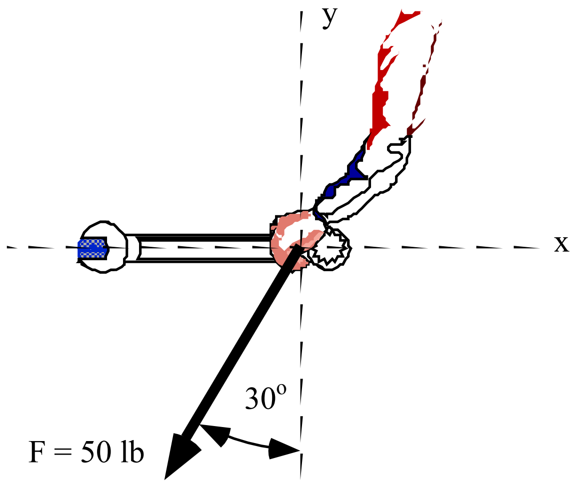
While adding many similar vectors, say **A, B, C**……, we can resolve each one into rectangular components (x and y components) and add all the x-components as if we add scalar quantities to find the resultant x-component ($R_x = A_x + B_x + C_x + \ldots$) and add all the y-components to find the resultant y-component ($R_y = A_y + B_y + C_y + \ldots$). Once we know these resultant x and y components, we can get the sum or resultant vector using $R = (R_x^2 + R_y^2)^{1/2}$ and direction by $\tan\theta = (R_y / R_x)$.

Unit Vector Representation: Vector of one unit in magnitude along x-axis is **i** whereas that along y-axis is **j**. Thus, C = A **i** + B **j** or C = C_x **i** + C_y **j**. In this representation, we can identify the components C_x and C_y and the direction directly from the equation itself. For example, a displacement vector **d** = (10 **i** + 12 **j**) m implies it has x-component 10 m and y-component 12 m. You will see more examples below.

Example 1:

Loosening a nut on a bolt is a common experience and we see how a force applied may be split into various **components**. In order to loosen a nut, a person holding a horizontal wrench exerts a downward force **F** = 50 lb at an angle of 30° to the vertical.

a) What are the **horizontal and vertical components** of the force **F** ?



Solution:

Vertical

$F_V = -50 \cos 30^\circ$

$F_V = -50(0.866)$

$F_V = -43.3$ lb

Horizontal

$F_H = -50 \sin 30^\circ$

$F_H = -50(0.5)$

$F_H = -25$ lb

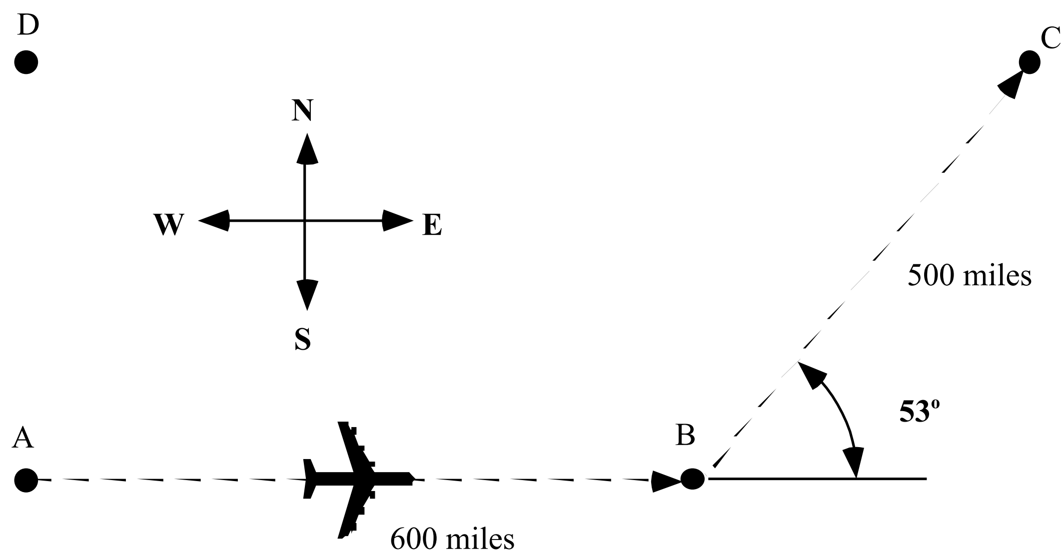
Negative sign indicates that they are along negative x and y-axes.

b) Express **F** in the unit vector **i** and **j** notation.

$F = F_H\, \mathbf{i} + F_V\, \mathbf{j} = [-43.3\, \mathbf{i} - 25\, \mathbf{j}]$ lb

Example 2:

In this problem, an airplane flies from one city to two other cities. We can use simple vector methods to find the magnitude and direction of its displacements during various legs of its flight: An airplane flies from city A to city B, in a direction due east for 600 miles. In the next leg of its trip, it flies from city B to city C, in a direction 53° north of east for 500 miles.



a. Determine the **components**, along the easterly and northerly directions, of the resultant displacement of the plane from city **A** to city **C**.

East direction : 600 miles, from A to B, and 500•(cos 53°) miles, from B to C:
= 600 + 500(0.6) = 600 + 300 = 900 miles **Due East**

North direction: 500•(sin 53°) miles, from B to C =500(0.8)=400 miles **Due North**

b. What are the **magnitude** and **direction** of the resultant displacement of the plane from city **A** to city **C**?

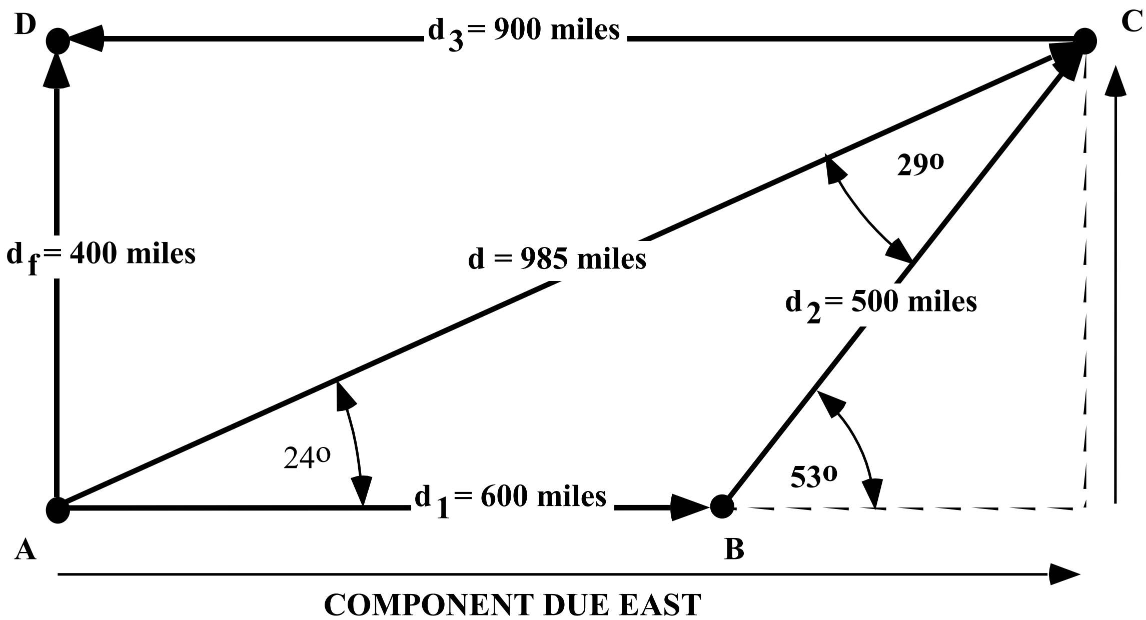
Magnitude: $d = (900^2 + 400^2)^{1/2} = 984.9$ miles
Direction: $\tan\theta = 400/900 = 4/9$, so $\theta = \tan^{-1}(4/9) = 24.0°$.

This direction is 24.0° **North of East**.

The plane then flies directly from city **C** to city **D** — directly north of city **A**, a distance of 400 miles in the last segment of its flight.

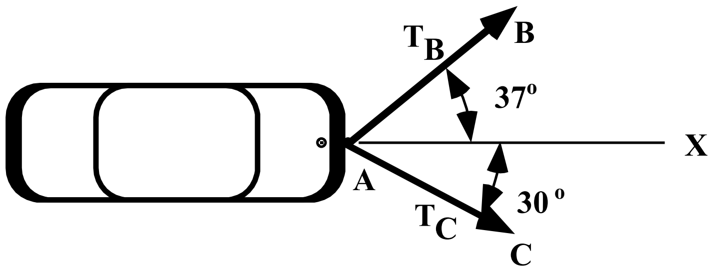
c. Determine the **magnitude** and the **direction** of the displacement of the plane from city **C** to city **D** ?

Magnitude = 900 miles. This direction is westerly; see the vector representation below.



Example 3:

A disabled automobile is pulled to the right by means of two cables AB and AC as shown. The tension in the cable AC is $\mathbf{T_C}$ = 6000 N. If it has to be pulled along the direction AX, the axis of the automobile, determine the magnitude of the resultant force, **R**, in that direction and the tension, $\mathbf{T_B}$, in cable AB by trigonometry.

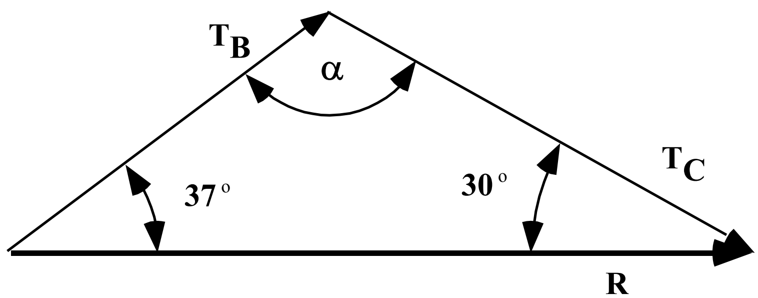


We note that the sum of the interior angles of a triangle is 180°, so the angle α = 180° - (30° + 37°) = 113° ;

then, using the law of sines: [a/sin A = b/sin B = c/sin C]

$$R/\sin 113° = T_C/\sin 37° = T_B/\sin 30° ;$$

so we obtain : $\mathbf{T_B} = (T_C \sin 30^\circ)/\sin 37^\circ = 6000(0.5)/0.6 = 5000$ N



$$\mathbf{R} = (T_C \sin 113^\circ)/\sin 37^\circ = 6000(0.92)/0.6 = 9200 \text{ N}$$

The following example also illustrates the addition of vectors. Use the law of sines and the law of cosines, and sketches of the force polygons, to solve the following problem:

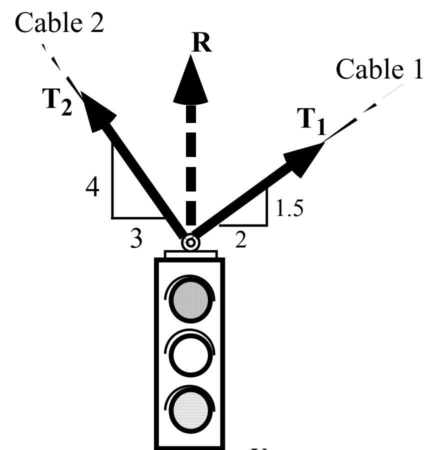
Example 3: Another method:

a. Given: $\mathbf{T_C}$ = 6000 N and sin 30° = 0.5 so $T_C(\sin 30^\circ)$ = 3000 N = tension in the negative y-direction.
   This is balanced by $T_B \sin 37^\circ$ (component of T_B in the positive y-direction).

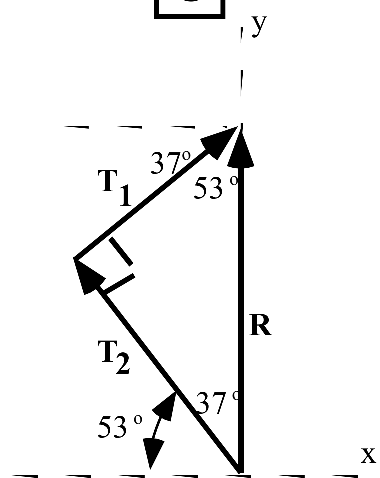
   $T_B \sin 37^\circ = T_B (0.6) = 3000$ N ⇒ $T_B = 3000 \text{ N}/0.6 = 5000$ N.

b. Resultant must be in the + x direction and have the magnitude of
$R = T_C (\cos 30^{\circ}) + T_B (\cos 37^{\circ}) = (6000\ N)(0.866)+(5000\ N)(0.8) = 9200\ N.$

Example 4:



Use Law of Sines method **only** *to solve this problem.* A traffic light is supported by two cables as shown. It is required that the magnitude of the resultant, **R** , of the tension forces exerted by these two cables be vertical, and be equal to 120 pounds. Determine the magnitudes of the two tensions, $\mathbf{T_1}$ and $\mathbf{T_2}$, in order to satisfy these requirements.



By the law of cosines:

$R^2 = T_1^2 + T_2^2 - 2\, T_1 T_2 \cos 90^{\circ}$

By the law of sines :

$T_1/ \sin 37^{\circ} = T_2/ \sin 53^{\circ} = R / \sin 90^{\circ}$

Thus, $T_1/ \sin 37^{\circ} = 120/ \sin 90^{\circ}$

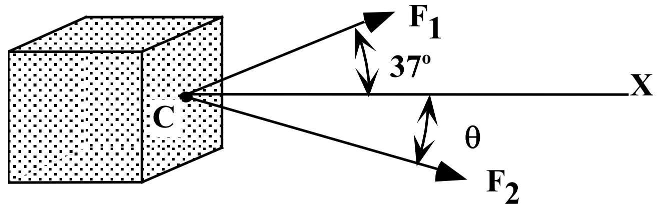
$T_1 = 120 * \sin 37^{\circ}$ **= 72 lb.**

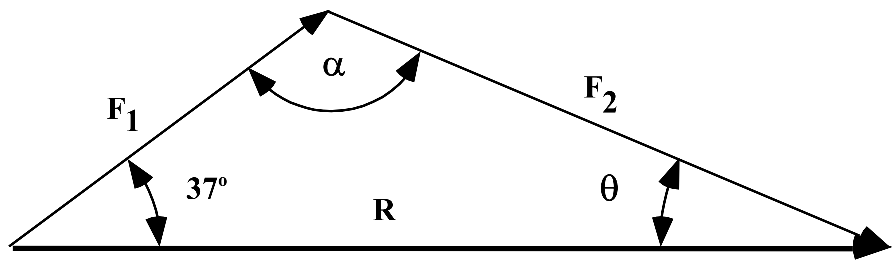
and $T_2/ \sin 53^{\circ} = 120/ \sin 90^{\circ} \Rightarrow$
$T_2 = 120 * \sin 53^{\circ}$ **= 96 lb.**

NOTATION: Vectors are represented by **bold type**, e.g., **A** stands for vector **A .** A in plain script stands for **|A|** read as " magnitude of vector **A** "

Example 5:

Two horizontal forces $\mathbf{F_1}$ and $\mathbf{F_2}$ act on a crate placed on a level floor. If the resultant force on the crate is 920 lb, acting along the direction CX, find the **magnitude of force** $\mathbf{F_1}$ and the **angle** θ made by force $\mathbf{F_2}$ = 600 lb Note that forces $\mathbf{F_1}$ and $\mathbf{F_2}$ and line CX all lie in a horizontal plane.





Using the Law of sines:

We note that the sum of the interior angles of a triangle is 180°, so the angle $\theta = 180° - (37° + \alpha) \Rightarrow \alpha = (143° - \theta)$

Then, $F_1/\sin\theta = F_2/\sin 37° = R/\sin(143° - \theta)$

$\sin(143° - \theta) = R \sin 37°/F_2 = 920(0.6)/600 = 0.92$.

If we choose $\alpha = (143° - \theta)$, $\sin\alpha = 0.92$, then the two possible answers are $\alpha = 67°$ or $113°$

Now, if $\alpha = 67°$, $\theta = 76°$ and $F_1 = 970$ lb.
On the other hand, if $\alpha = 113°$, $\theta = 30°$ and $F_1 = 500$ lb

As similar vectors are added to get the sum, we can split a vector into its orthogonal (perpendicular) components. A simple example of splitting a vector is given in the problems that follow. We also can use the concept of vector resolution (into arbitrary directions) to solve some problems.

Example 5 - Another method:

Since the resultant is along CX, the components of F_1 and F_2 perpendicular to CX must cancel out.

Thus, $F_1 \sin 37^\circ - F_2 \sin\theta = 0$ and so, $0.6 F_1 = 600 \sin\theta \Rightarrow F_1 = 1000*\sin\theta$

Parallel to CX we have, $F_1 \cos 37^\circ + F_2 \cos\theta = R = 920$
$F_1 = (920 - 600 \cos\theta)/ 0.8 = 1150 - 750*\cos\theta$

Thus, we get $1000 \sin\theta = 1150 - 750 \cos\theta$

$\sin\theta = 1.15 - 0.75*\cos\theta$

We know that $(\sin\theta)^2 + (\cos\theta)^2 = 1$ so we can write:
$\sin\theta = [1 - (\cos\theta)^2]^{1/2}$

Thus, $\sin\theta = 1.15 - 0.75*\cos\theta$ becomes $[1 - (\cos\theta)^2]^{1/2} = 1.15 - 0.75*\cos\theta$.

Squaring both sides yields:

$1 - (\cos\theta)^2 = [1.15 - 0.75 \cos\theta]^2 \Rightarrow 1.56(\cos\theta)^2 - 1.73(\cos\theta) + 0.323 = 0$

This is a quadratic equation in $\cos\theta$, of the form ,

$1.56 x^2 - 1.73 x + 0.323 = 0$ where $\cos\theta = x$;

Solving, we then get : $\cos\theta = 0.24$ or 0.866

$\theta = 76^\circ$ or $\theta = 30^\circ$ and using these values of θ, we have $\mathbf{F_1} = \mathbf{970}$ lb, or **500** lb.

Note that both the values are physically meaningful**.**

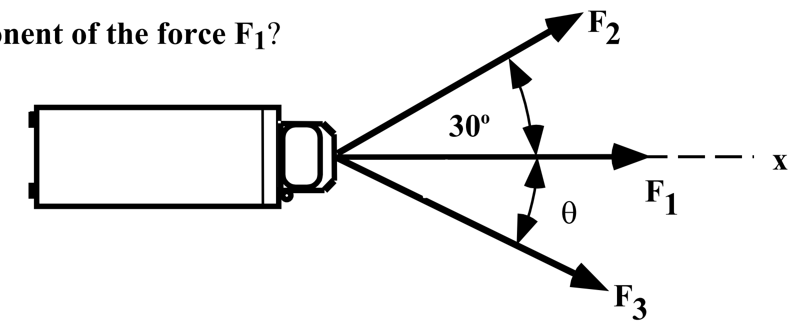
Physical Quantities - Scalars and Vectors

Interactive Problem 1:

A truck in trouble is pulled on a level road by applying three forces as shown in this figure. The forces are $\mathbf{F_1}$ = 1600 lb and $\mathbf{F_2}$ = 500 lb. Magnitude and direction of force $\mathbf{F_3}$ are yet to be determined. The resultant of the forces is **R** = 2500 lb along the x-axis to the right. Determine the **magnitude** of $\mathbf{F_3}$ and the **angle** θ made by $\mathbf{F_3}$ with the x-axis .

Question 1:

What is the y-component of the force $\mathbf{F_1}$?



a. 500 lb.
b. 1600 lb.
c. zero lb.

Answer is (c)

Feedback:

Force $\mathbf{F_1}$ acts along x-axis. Its x-component is 1600 lb. and since it makes 0° with x-axis its y-component is $F_1 \sin 0°$.= zero.

Question 2:

What are x and y-components of the force $\mathbf{F_2}$?

a. 500 and 1600 lb.
b. 433 lb. and 250 lb.
c. 250 lb. and 433 lb.

Answer is (b)

Feedback:

Force $\mathbf{F_2}$ makes 30° with x-axis. Its x-component is $F_2 \cos 30°$= (500 lb.)(0.866)= 433 lb. and its y-component is $F_2 \sin 30°$ = (500 lb.)(0.5) = 250 lb.

Question 3:

If the sum of the x-components of all the forces is the resultant, R = 2500 lb along x-axis , what is the x-component of $\mathbf{F_3}$?

a. 500 lb.
b. 433 lb.
c. 467 lb.

Answer is (c)

Feedback:

Since $F_{1x} + F_{2x} + F_{3x} = 2500$ lb ; so $1600 + 433 + F_{3x} = 2500 \Rightarrow F_{3x} = 467$ lb

Question 4:

If F_3 makes an angle θ below x-axis as shown, from your answers to questions 1, 2 and 3, find θ.

a. 30°
b. 28.2°.
c. 37°.

Answer is (b)

Feedback:

Since $F_{3x} = 467$ lb $= F_3 \cos\theta = 467$ lb and we have $F_3 = 467/\cos\theta$.

Because there is no component of the resultant in the y-direction, the component of $\mathbf{F_3}$ in the y-direction balances the y-component of $\mathbf{F_2}$; this gives

$F_{2y} = F_2 \sin 30^\circ = 250 \text{ lb} = F_{3y} = F_3 \sin\theta = [467/\cos\theta] \sin\theta$
and hence, we get $\sin\theta / \cos\theta = \tan\theta = 250/467 = 0.54$.

Thus, $\theta = \tan^{-1}(0.54) = 28.2^\circ$.

Question 5:

What is the magnitude of the force F_3?

a. 500 lb.
b. 467 lb.
c. 530 lb.

Answer is (c)

Feedback:

Now we can find F_3 since
$F_{3x} = F_3 \cos 28.2^\circ = 467 \text{ lb} \Rightarrow F_3 = 467 \text{ lb} / \cos 28.2^\circ \Rightarrow F_3 = 530 \text{ lb}$

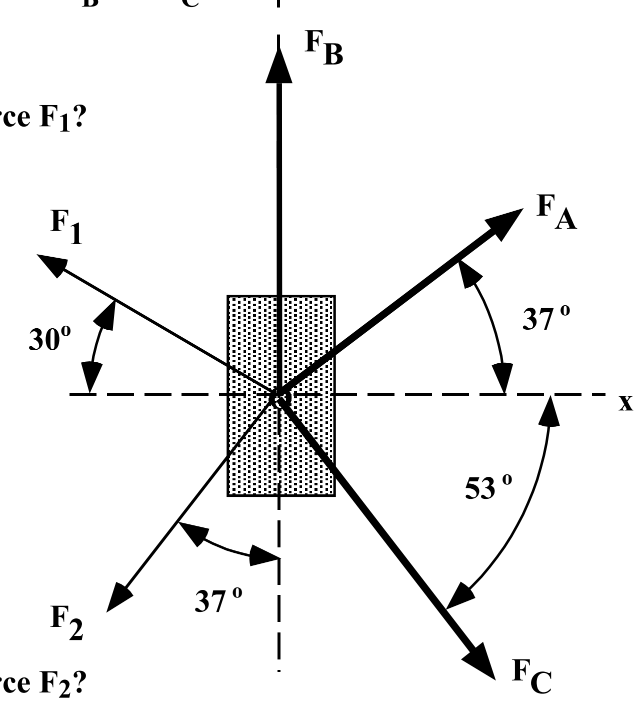
Checking the results using y-component:

From the y-component for F_3 we get,
$F_{3y} = F_3 \sin 28.2^\circ = 250$ lb, which yields $F_3 = 250/\sin 28.2^\circ = 530$ lb.

Thus, F_3 is 530 lb at $\theta = 28.2^\circ$.

Interactive Problem 2:

A plate is pinned at point O and is supported by three rods along the directions of $\mathbf{F_A}$, $\mathbf{F_B}$ and $\mathbf{F_C}$ as shown in the figure. Forces $\mathbf{F_1}$ = 40 lb and $\mathbf{F_2}$ = 60 lb are applied to the plate through a smooth pin at O. The forces along the rods $\mathbf{F_A}$, $\mathbf{F_B}$ and $\mathbf{F_C}$ add up to zero. If the force $\mathbf{F_A}$ = 50 lb, determine the forces $\mathbf{F_B}$ and $\mathbf{F_C}$.



Question 1:

What are the x and y components of the force F_1?

a. 34.6 lb. and – 20 lb
b. – 34.6 lb. and + 20 lb.
c. 40 lb. and 20 lb.

Answer is (b)

Feedback: Force F_1 is in the third quadrant and makes 30° with negative x-axis. Hence it has a negative x-component = – 40 cos 30° and positive y-component = + 40 sin 30°.

Question 2:

What are the x and y components of the force F_2?

a. 36 lb. and – 48 lb
b. – 36 lb. and + 48 lb.
c. – 36 lb. and – 48 lb.

Answer is (c)

Feedback: Force F_2 is in the fourth quadrant and makes 37° with negative y-axis or 53° below negative x-axis. Hence it has a negative x-component = – 60 cos53° and negative y-component = – 60 sin53°

Question 3:

What are the x and y components of the force F_A?

a. + 40 lb. and + 30 lb
b. – 40 lb. and + 30 lb.
c. + 40 lb. and – 30 lb.

Answer is (a)

Feedback: Force F_A is in the first quadrant and makes 37° with positive x-axis. Hence it has positive x-component = 50 cos37° and positive y-component = 50 sin37°.

Question 4:

All the forces add up to zero. All x-components add up to zero.

What is the value of F_C ?

a. 31 lb.
b. 41 lb.
c. 51 lb.

Answer is (c)

Feedback: Force F_B has no x-component since it acts along y-axis. Force makes F_C 53° below positive x-axis and hence it has a positive x-component = + $F_C \cos 53^\circ$ = $0.6F_C$. Adding all x-components, we get

$-34.6 - 36 + 40 + 0.6\,F_C = 0 \Rightarrow 0.6\,F_C = 30.6$ lb. Thus, $F_C = 30.6$ lb./0.6 = **51** lb.

Question 5:

All y-components add up to zero.

What is the value of F_B ?

a. 39 lb.
b. 49 lb.
c. 51 lb.

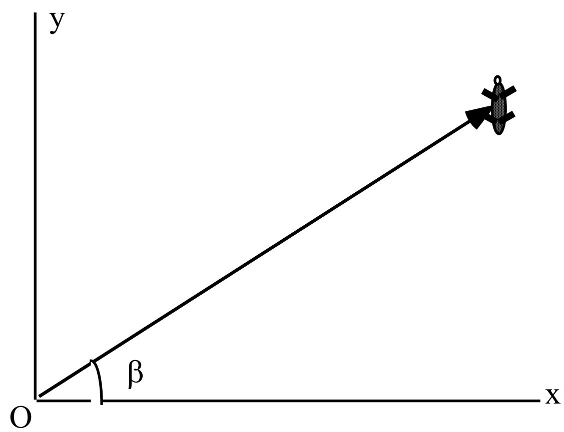
Answer is (a)

Feedback: Force F_B has no x-component since it acts along y-axis.
Force makes F_C 53° below positive x-axis and hence it has a negative y-component $= -F_C \sin 53^\circ = -0.8F_C = -(0.8)*51 = -41$ lb. Adding all y-components, we get
$20 - 48 + 30 - 41 + F_B = 0 \Rightarrow -39 + F_B = 0$. Thus, $F_B = 39$ lb.

Physical Quantities - Scalars and Vectors

Sample Problems

1. [a] An ant crawls on a tabletop. The position vector **A** of the ant's position has components x= 40mm ; y = 30 mm with respect to the origin of the coordinate system. Determine the magnitude and direction of **A**.



[b] The ant changes its direction and keeps crawling in a straight line and ends up with a position vector, **B,** with components
x = 60 mm; y = - 20 mm.
[i] express **B** in the **i**, **j** notation.
[ii] find the direction and magnitude of **B**.
[iii] find the direction and magnitude of **B - A**

If the ant started from the origin, what is the:
[iv] total distance traveled by the ant ?
[v] magnitude of the net displacement of the ant ?

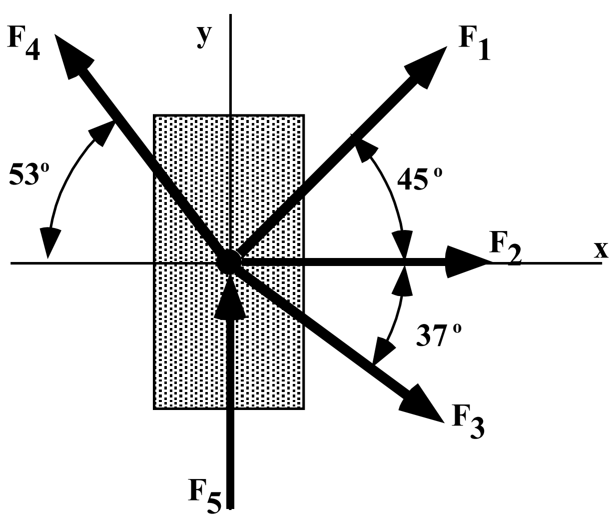
2. [a] Determine the orientation (with respect to positive x-axis) and magnitude of the following vectors : **A** = 2**i** + 4**j** ; **B** = - 2**i** + 4**j** and **C** = - 2**i** - 4**j.**

[b] If **A** - **B** + **D** = 0, determine the direction and magnitude of **D.**

3. What are the x and y components of the unit vector **i** ?

4. Two vectors of lengths *a* and *b* make an angle θ with each other when placed tail to tail. Prove by taking components along two perpendicular axes, that the length of their sum is
$r = [\, a^2 + b^2 + 2ab\cos\theta]^{1/2}$

5. Five forces $\mathbf{F_1}$ = 71N, $\mathbf{F_2}$ = 60 N, $\mathbf{F_3}$ = 50 N, $\mathbf{F_4}$ = 80 N and $\mathbf{F_5}$ = 60 N act on a steel plate as shown. Use the unit vector (**i**, **j**, **k**) notation to express each of the forces on the plate. Determine the magnitude of the resultant **R** and the angle θ between the x-axis and the line of action of the resultant force using the rectangular resolution method.

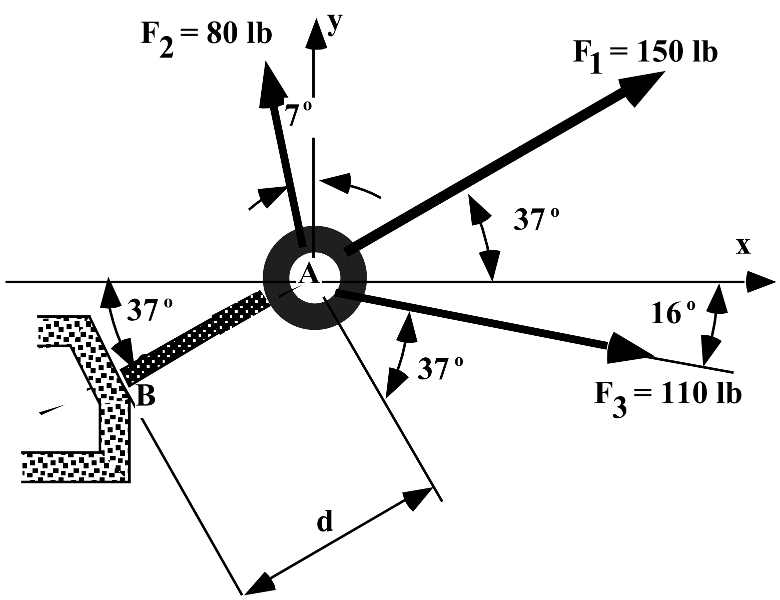


To demonstrate a mastery of concepts and skills in this unit, you have to take a test and solve all the problems in the test correctly. You may work your solutions on any paper and enter your answers in the spaces provided. Correct answers will be given to you after you have entered your answers.

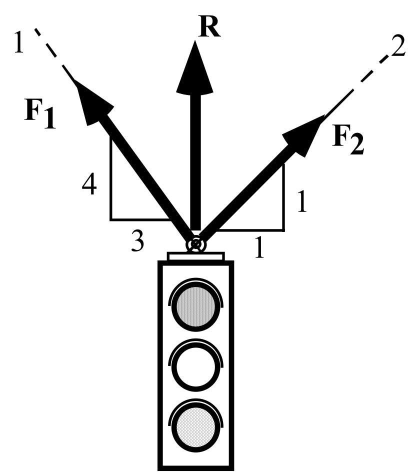
Physical Quantities - Scalars and Vectors

Test 01:

1. Three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$ all intersecting at "A" - the origin of coordinate system, act on the screw eye AB as shown. Given: sin7° = cos83° = 0.122; cos7° = sin 83° = 0.993; sin16°= cos74° = 0.276; cos16°= sin 74° = 0.961; sin37° = cos53° = 0.6 ; cos37° = sin 53° = 0.8. Resolve the three forces into x and y components and find R_x and R_y (the x and y components of the resultant force). Determine **the magnitude** and **direction** of the resultant force**, R**, on the eye.



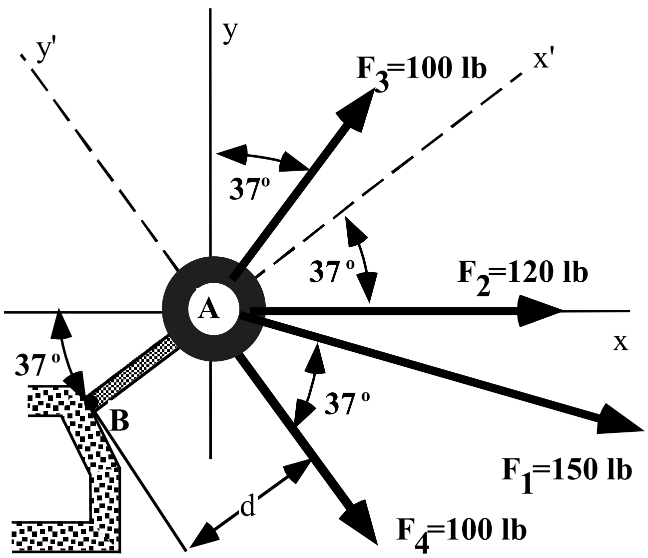
2. In this problem, a traffic light is supported by two cables as shown. It is required that the magnitude of the resultant, **R** , of the forces exerted by these two cables be vertical, and be equal to 360 pounds. Determine the magnitudes of the two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, in order to satisfy these requirements.

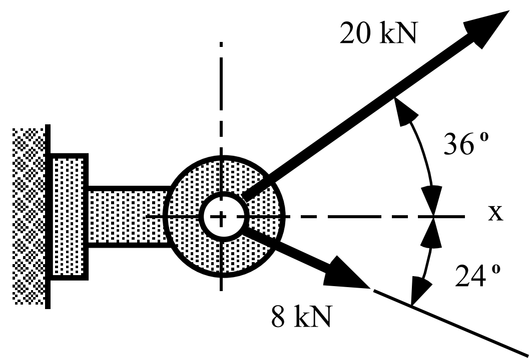


3. Use the Law of Sines and the Law Cosines, and sketches of the force polygons, to solve the following problem. Two forces F_1 = 60 kN and F_2 = 45 kN act on a disc as in the figure. Determine the magnitude of the resultant force **R** and the angle θ between the x-axis and the line of action of the resultant force.

Test 02:

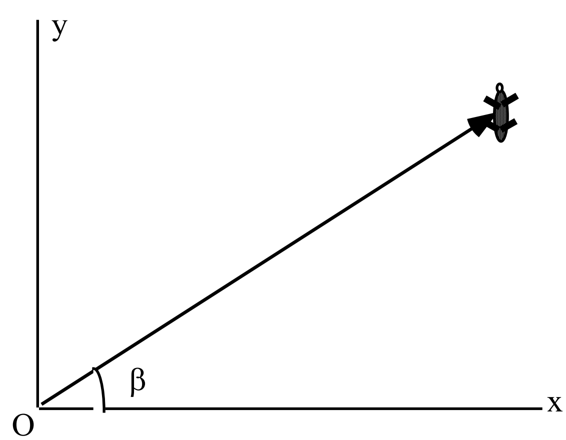
1. Four forces F_1, F_2, F_3 and F_4 all intersecting at "A" - the origin of coordinate system, act on the screw eye AB as shown. There are two sets of co-ordinates axes (x-y and x'-y') shown below. Resolve the four forces into x and y components and find R_x and R_y (the x and y components of the resultant force). Use a **separate sketch** of R_x and R_y and find **the magnitude** and **direction** of the resultant force (**R**) of these forces.





2. Find the magnitude of the resultant force, **R**, for the forces shown in the figure above (on the right), and the angle θ between the line of action of the resultant **R** and the x-axis. Use Law of Sines and Law of Cosines method or the rectangular resolution method.

3. [a] An ant crawls on a tabletop. The position vector **A** of the ant's position has components x= 80mm ; y = 60 mm with respect to the origin of the coordinate system. Determine the magnitude and direction of **A**.



[b] The ant changes its direction and keeps crawling in a straight line and ends up with a position vector, **B,** with components
x = 40 mm; y = – 30 mm.
[i] express **B** in the **i**, **j** notation.
[ii] find the direction and magnitude of **B**.
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If the ant started from the origin, what is the:
[iv] total distance traveled by the ant ?
[v] magnitude of the net displacement of the ant ?