Engineering Mechanics Studio Classroom
Dynamics Modules

Gateway Funded Research
Spring 2001

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Professor Vito Guido
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Executive Summary

The Albert Nerken School of Engineering intends to develop an interactive method of instruction for Engineering Mechanics (ESC 100). This new approach to undergraduate education adapts and furthers the "studio" concept of instruction. In this system of learning, student workstations, tabletop experiments, computer software, videotapes and traditional textbooks will be combined to create a dynamic teaching environment, which integrates the traditional separate activities of instruction, lecture, recitation and laboratory. The current research effort focused on the development of teaching modules for the dynamics portion of the course.

Introduction

This research effort was performed by Professors Grossman, Guido and Li, all of whom have taught Engineering Mechanism (ESC 100) numerous times. There are currently three sections of ESC 100, which is required to be taken by the Civil Engineering, Electrical Engineering and Mechanical Engineering students in the fall semester of their sophomore year. All three investigators concurred that the course is currently taught as two-thirds statics and one-third dynamics. Teaching modules for statics were developed in the Summer of 2000 and this report contains the teaching modules for dynamics that were developed in the Spring of 2001.

It was decided that five teaching modules would be developed for the dynamics portion of Engineering Mechanics (ESC 100). These modules can be found in Table 1. In the development of each module: Prof. Grossman researched the lecture material; Prof. Guido the table top experiments; and Prof. Li the available CD Roms, software, videos, etc.

Table 1
ESC 100 - Engineering Mechanics
Teaching Modules for Dynamics

Topics	Dynamics Modules	Chapters*
Kinematics (1 module)	Rectilinear and Curvilinear Motion	11
Kinetics I (1 module)	Second Law and Space Dynamics	12
Kinetics II (2 modules)	Work-Energy Impulse-Momentum	13.10-15
Vibrations (1 module)	Free, Forced, Damped	19

Note: Each module is 3 hours or 1 week and 1-2 hour exam is included in this frame, after the module on vibrations.

* The textbook currently used for ESC 100 is "Vector Mechanics for Engineers: Statics and Dynamics," by F. P. Beer and E.R. Johnston, Jr., Sixth Edition. McGraw-Hill, 1997.

Teaching Modules

Professor Grossman has suggested that the Review and summary sections at the end of each chapter of the textbook be used as the lecture material for each of the five modules. See Table 2.

Table 2
Lecture Material

Topics	Modules	Lecture Material*
VI. Kinematics	11. Rectilinear and Curvilinear Motion	pp. 659-662
VII. Kinetics I	12. Second Law and Space Dynamics	pp. 720-723
VIII. Kinctics II	13. Work-Energy 14. Impulse-Momentum	pp. 816-819 pp. 819-821
IX. Vibrations	15. Free, Forced, Damped	pp. 1234-1238

* Text: "Vector Mechanics for Engineers: Statics and Dynamics," by F. P. Beer and E.R. Johnston, Jr., Sixth Edition. McGraw-Hill, 1997. See Appendix 1 for actual pages.

Professor Guido contacted PASCO Scientific, the company that manufactures the table top equipment needed for the table top experiments, and obtained a web site (www2.PASCO.com/TECHSUPP) from which he obtained experiments for the dynamics equipment needed. This equipment was specified in the summer 1999 report entitled "Engineering Mechanics Studio Classroom." The main table top equipment needed for dynamics is "The Dynamics Cart Accessory Track Set – 2.2M Version" (ME 9490) and the "Projectile Launcher Short/Long Version" (ME 6800, ME 6801). Professor Guido has specified which table top experiments should be performed by the instructor as class demonstrations and which experiments should be performed by the students as homework assignments. See Table 3.

Professor Li has searched the web and has obtained websites containing information pertaining to each of the five modules. These websites can be gone to during regular class time or visited by the students at their leisure. In addition, Professor Li has reviewed the 2 CD-roms now available with the textbook. They are "Working Model @ 3D Simulations CD-Rom," and "Student Resources CD-Rom." She has reviewed both sample problems and homework problems. See page 4.

Table 3
Table Top Experiments

Module 11		**Rectilinear and Curvilinear Motion**
		Using "Projectile Launcher Short/Long Version" from PASCO:
	(Class demo)	Experiment No. 1 – Projectile Motion
	(HW)	Experiment No. 4 – Projectile Path
Module 12		**Second Law and Space Dynamics**
		Using "Dynamics Cart Accessory Track Set (2.2M Version)" from PASCO:
	(Class demo)	Experiment No. 6 – Newton's Second Law
	(HW)	Experiment No. 9 – Newton's Second Law II
Module 13		**Work-Energy**
		Using "Dynamics Cart Accessory Track Set (2.2M Version)" from PASCO:
	(Class demo)	Experiment No. 9 – Conservation of Energy
		Using "Projectile Launcher Short/Long Version" from PASCO:
	(HW)	Experiment No. 5 – Conservation of Energy
Module 14		**Impulse-Momentum**
		Using "Dynamics Cart Accessory Track Set (2.2M Version)" from PASCO:
	(Class Demo)	Experiment No. 2 – Conservation of Momentum in Collisions
Module 15		**Free, Forced, Damped Vibrations**
		Using "Dynamics Cart Accessory Track Set (2.2M Version)" from PASCO:
	(Class demo)	Experiment No. 3 – Simple Harmonic Oscillator
	(HW)	Experiment No. 5 – Springs in Series and Parallel

*Note: See Appendix 2 for the experiments associated with the "Dynamics Cart Accessory Track Set (2.2M Version)", and Appendix 3 for the experiments associated with the "Projectile Launcher Short/Long Version".

CD-Rom Accompanying "Vector Mechanics for Engineers: Statics and Dynamics"

<u>CD-ROMs</u>: all CD-ROMs are attached with the textbook
New Media Version – Vector Mechanics for Engineers:
CD1 Working Model, 3D Simulations
CD2 Student Resources

Pertinent Websites

Module 11: Rectilinear & Curvilinear Motions

From CD1

Sample Problem 11.8 (p.629) - Interact with initial launch speed and trajectory angle in a classic projectile motion problem

Homework Problem 11.36 (p.604) - See the effect of acceleration and initial speed on a car's displacement

Homework Problem 11.41 (p.606) - Correlate acceleration with displacement in a boat race

From CD2:

Dynamics Tutorial→Analysis Tools:
Rectilinear & Curvilinear Motions

<u>Resource from Web</u>:
From the textbook publisher
http://www.mhhe.com/engcs/engmech/beerjohnston/slide.mhtml
Chapter Reviews
http://www.mhhe.com/engcs/engmech/beerjohnston/ppreview/CHAP11_files/v3_document.htm
Figures
http://www.mhhe.com/engcs/engmech/beerjohnston/ppfigures/CHAP11_files/v3_document.htm
Problems
http://www.mhhe.com/engcs/engmech/beerjohnston/chapterp.mhtml

Module 12: Newton 2^{nd} Law, Linear Momentum (Chapter12)

From CD1:

Sample Problem 12.3 (p.677) - Notice that acceleration and pulley tension are functions of block mass

Sample Problem 12.4 (p.678) - Experiment with the mass of a sliding-block-on-sliding-wedge problem to learn about relative acceleration

Homework Problem 12.45 (p.690) - See the effects of angle and wrecking ball mass on cable tension

From CD2:

Dynamics Tutorial→Particle Kinetics:
Newton's 2nd Law

Resource from Web:
From the textbook publisher
http://www.mhhe.com/engcs/engmech/beerjohnston/slide.mhtml
Chapter Reviews
http://www.mhhe.com/engcs/engmech/beerjohnston/ppreview/CHAP12_files/v3_document.htm
Figures
http://www.mhhe.com/engcs/engmech/beerjohnston/ppfigures/CHAP12_files/v3_document.htm
Problems
http://www.mhhe.com/engcs/engmech/beerjohnston/chapterp.mhtml

Module 13: Work-Energy

From CD1:

Sample Problem 13.6 (p.761) - Form engineering insights into the role of spring constant on amplitude and frequency of collar motions

Sample Problem 13.17 (p.805) - Learn about the effect of spring stiffness on the maximum deflection of a dropped block

From CD2:

Dynamics Tutorial→Particle Kinetics:
Work-Energy (13.1-13.8)

Resource from Web:
From the textbook publisher
http://www.mhhe.com/engcs/engmech/beerjohnston/slide.mhtml
Chapter Reviews
http://www.mhhe.com/engcs/engmech/beerjohnston/ppreview/CHAP13_files/v3_document.htm
Figures
http://www.mhhe.com/engcs/engmech/beerjohnston/ppfigures/CHAP13_files/v3_document.htm
Problems
http://www.mhhe.com/engcs/engmech/beerjohnston/chapterp.mhtml

Module 14: Impulse-Momentum

From CD1:

Homework Problem 13.187 (p.815) - Observe post-impact velocity and energy loss due to:
a. inclination angle
b. coefficient of restitution

Homework Problem 13.188 (p.815) - View the effect of coefficient of restitution on post-impact speed and energy loss

From CD2:

Dynamics Tutorial→Particle Kinetics:
Impulse-Momentum (13.10-13.15)

Resource from Web:
From the textbook publisher
http://www.mhhe.com/engcs/engmech/beerjohnston/slide.mhtml
Chapter Reviews
http://www.mhhe.com/engcs/engmech/beerjohnston/ppreview/CHAP13_files/v3_document.htm
Figures
http://www.mhhe.com/engcs/engmech/beerjohnston/ppfigures/CHAP13_files/v3_document.htm
Problems
http://www.mhhe.com/engcs/engmech/beerjohnston/chapterp.mhtml

Module 15: Vibration

From CD1:

Sample Problem 19.1 (p.1178) - Compare and contrast the motion of a system connected by springs in parallel with a system connected by springs in series

Sample Problem 19.5 (p.1212) - Develop intuition about amplitude amplification and resonance caused by a small revolving particle

Resource from Web:
From the textbook publisher
http://www.mhhe.com/engcs/engmech/beerjohnston/slide.mhtml
Chapter Reviews
http://www.mhhe.com/engcs/engmech/beerjohnston/ppreview/CHAP19_files/v3_document.htm
Figures
http://www.mhhe.com/engcs/engmech/beerjohnston/ppfigures/CHAP19_files/v3_document.htm
Problems
http://www.mhhe.com/engcs/engmech/beerjohnston/chapterp.mhtml

Conclusions and Recommendations

The integration of table top experiments for the demonstration of relevant ideas, and the use of the internet and other available multimedia resources into the traditional lecture mode of teaching Engineering Mechanics (ESC 100) will take a concerted effort on the part of the instructors. When the Engineering Mechanics Studio/Classroom is made available and the necessary equipment purchased (See Gateway Report Summer 1999), a familiarity and expertise with the equipment will be essential by the instructors for the development of the new Engineering Mechanics course. Professors Grossman, Guido and Li should be the development team. Additional funding should be made available for this very important phase of the project.

Appendix 1:

Lecture Material

MODULE NO. 11

REVIEW AND SUMMARY FOR CHAPTER 11

Position coordinate of a particle in rectilinear motion.

In the first half of the chapter, we analyzed the *rectilinear motion of a particle,* i.e., the motion of a particle along a straight line. To define the position P of the particle on that line, we chose a fixed origin O and a positive direction (Fig. 11.27). The distance x from O to P, with the appropriate sign, completely defines the position of the particle on the line and is called the *position coordinate* of the particle [Sec. 11.2].

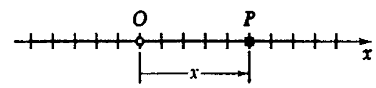

Fig. 11.27

Velocity and acceleration in rectilinear motion

The *velocity* v of the particle was shown to be equal to the time derivative of the position coordinate x,

$$v = \frac{dx}{dt} \tag{11.1}$$

and the *acceleration* a was obtained by differentiating v with respect to t,

$$a = \frac{dv}{dt} \tag{11.2}$$

or

$$a = \frac{d^2x}{dt^2} \tag{11.3}$$

We also noted that a could be expressed as

$$a = v\frac{dv}{dx} \tag{11.4}$$

We observed that the velocity v and the acceleration a were represented by algebraic numbers which can be positive or negative. A positive value for v indicates that the particle moves in the positive direction, and a negative value that it moves in the negative direction. A positive value for a, however, may mean that the particle is truly accelerated (i.e., moves faster) in the positive direction, or that it is decelerated (i.e., moves more slowly) in the negative direction. A negative value for a is subject to a similar interpretation [Sample Prob. 11.1].

Determination of the velocity and acceleration by integration

In most problems, the conditions of motion of a particle are defined by the type of acceleration that the particle possesses and by the initial conditions [Sec. 11.3]. The velocity and position of the particle can then be obtained by integrating two of the equations (11.1) to (11.4). Which of these equations should be selected depends upon the type of acceleration involved [Sample Probs. 11.2 and 11.3].

Uniform rectilinear motion

Two types of motion are frequently encountered: the *uniform rectilinear motion* [Sec. 11.4], in which the velocity v of the particle is constant and

$$x = x_0 + vt \tag{11.5}$$

Uniformly accelerated rectilinear motion

and the *uniformly accelerated rectilinear motion* [Sec. 11.5], in which the acceleration a of the particle is constant and we have

$$v = v_0 + at \tag{11.6}$$

$$x = x_0 + v_0t + \tfrac{1}{2}at^2 \tag{11.7}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{11.8}$$

Relative motion of two particles

When two particles A and B move along the same straight line, we may wish to consider the *relative motion* of B with respect to A

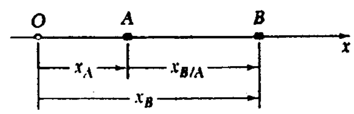

Fig. 11.28

[Sec. 11.6]. Denoting by $x_{B/A}$ the *relative position coordinate* of B with respect to A (Fig. 11.28), we had

$$x_B = x_A + x_{B/A} \tag{11.9}$$

Differentiating Eq. (11.9) twice with respect to t, we obtained successively

$$v_B = v_A + v_{B/A} \tag{11.10}$$

$$a_B = a_A + a_{B/A} \tag{11.11}$$

where $v_{B/A}$ and $a_{B/A}$ represent, respectively, the *relative velocity* and the *relative acceleration* of B with respect to A.

Blocks connected by inextensible cords

When several blocks are *connected by inextensible cords*, it is possible to write a *linear relation* between their position coordinates. Similar relations can then be written between their velocities and between their accelerations and can be used to analyze their motion [Sample Prob. 11.5].

Graphical solutions

It is sometimes convenient to use a *graphical solution* for problems involving the rectilinear motion of a particle [Secs. 11.7 and 11.8]. The graphical solution most commonly used involves the x–t, v–t, and a–t curves [Sec. 11.7; Sample Prob. 11.6]. It was shown that, at any given time t,

$$v = \text{slope of } x\text{–}t \text{ curve}$$
$$a = \text{slope of } v\text{–}t \text{ curve}$$

while, over any given time interval from t_1 to t_2,

$$v_2 - v_1 = \text{area under } a\text{–}t \text{ curve}$$
$$x_2 - x_1 = \text{area under } v\text{–}t \text{ curve}$$

Position vector and velocity in curvilinear motion

In the second half of the chapter, we analyzed the *curvilinear motion of a particle*, i.e., the motion of a particle along a curved path. The position P of the particle at a given time [Sec. 11.9] was defined by the *position vector* **r** joining the origin O of the coordinates and point P (Fig. 11.29). The *velocity* **v** of the particle was defined by the relation

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \qquad (11.15)$$

and was found to be a *vector tangent to the path of the particle* and of magnitude v (called the *speed* of the particle) equal to the time derivative of the length s of the arc described by the particle:

$$v = \frac{ds}{dt} \qquad (11.16)$$

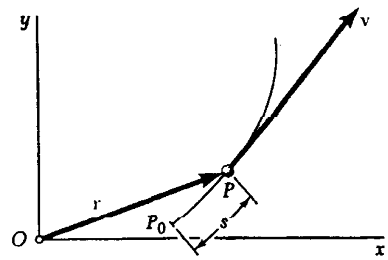

Fig. 11.29

Acceleration in curvilinear motion

The *acceleration* **a** of the particle was defined by the relation

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} \qquad (11.18)$$

and we noted that, in general, *the acceleration is not tangent to the path of the particle.*

Derivative of a vector function

Before proceeding to the consideration of the components of velocity and acceleration, we reviewed the formal definition of the derivative of a vector function and established a few rules governing the differentiation of sums and products of vector functions. We then showed that the rate of change of a vector is the same with respect to a fixed frame and with respect to a frame in translation [Sec. 11.10].

Rectangular components of velocity and acceleration

Denoting by x, y, and z the rectangular coordinates of a particle P, we found that the rectangular components of the velocity and acceleration of P equal, respectively, the first and second derivatives with respect to t of the corresponding coordinates:

$$v_x = \dot{x} \qquad v_y = \dot{y} \qquad v_z = \dot{z} \qquad (11.29)$$
$$a_x = \ddot{x} \qquad a_y = \ddot{y} \qquad a_z = \ddot{z} \qquad (11.30)$$

Component motions

When the component a_x of the acceleration depends only upon t, x, and/or v_x, and when similarly a_y depends only upon t, y, and/or v_y, and a_z upon t, z, and/or v_z, Eqs. (11.30) can be integrated independently. The analysis of the given curvilinear motion can thus be reduced to the analysis of three independent rectilinear component motions [Sec. 11.11]. This approach is particularly effective in the study of the motion of projectiles [Sample Probs. 11.7 and 11.8].

Relative motion of two particles

For two particles A and B moving in space (Fig. 11.30), we considered the relative motion of B with respect to A, or more precisely, with respect to a moving frame attached to A and in translation with A [Sec. 11.12]. Denoting by $\mathbf{r}_{B/A}$ the *relative position vector* of B with respect to A (Fig. 11.30), we had

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \qquad (11.31)$$

Denoting by $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$, respectively, the *relative velocity* and the *relative acceleration* of B with respect to A, we also showed that

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad (11.33)$$

and

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} \qquad (11.34)$$

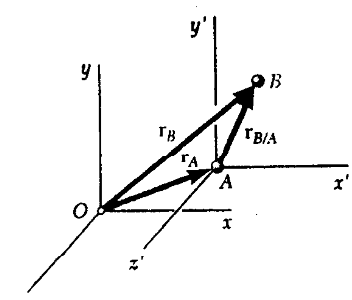

Fig. 11.30

It is sometimes convenient to resolve the velocity and acceleration of a particle P into components other than the rectangular x, y, and z components. For a particle P moving along a path contained in a plane, we attached to P unit vectors $\mathbf{e}_t$ tangent to the path and $\mathbf{e}_n$ normal to the path and directed toward the center of curvature of the path [Sec. 11.13]. We then expressed the velocity and acceleration of the particle in terms of tangential and normal components. We wrote

Tangential and normal components

$$\mathbf{v} = v\mathbf{e}_t \tag{11.36}$$

and

$$\mathbf{a} = \frac{dv}{dt}\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \tag{11.39}$$

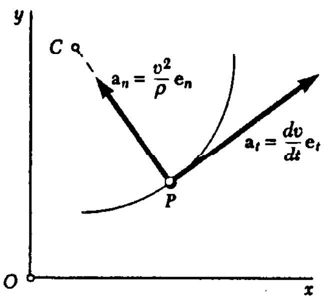

Fig. 11.31

where v is the speed of the particle and ρ the radius of curvature of its path [Sample Probs. 11.10 and 11.11]. We observed that while the velocity $\mathbf{v}$ is directed along the tangent to the path, the acceleration $\mathbf{a}$ consists of a component $\mathbf{a}_t$ directed along the tangent to the path and a component $\mathbf{a}_n$ directed toward the center of curvature of the path (Fig. 11.31).

Motion along a space curve

For a particle P moving along a space curve, we defined the plane which most closely fits the curve in the neighborhood of P as the *osculating plane*. This plane contains the unit vectors $\mathbf{e}_t$ and $\mathbf{e}_n$ which define, respectively, the tangent and principal normal to the curve. The unit vector $\mathbf{e}_b$ which is perpendicular to the osculating plane defines the *binormal.*

Radial and transverse components

When the position of a particle P moving in a plane is defined by its polar coordinates r and θ, it is convenient to use radial and transverse components directed, respectively, along the position vector $\mathbf{r}$ of the particle and in the direction obtained by rotating $\mathbf{r}$ through 90° counterclockwise [Sec. 11.14]. We attached to P unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ directed, respectively, in the radial and transverse directions (Fig. 11.32). We then expressed the velocity and acceleration of the particle in terms of radial and transverse components

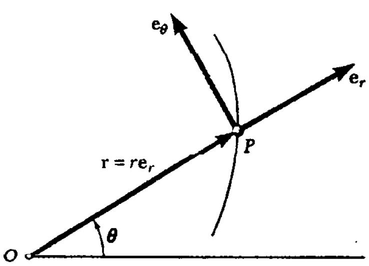

Fig. 11.32

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \tag{11.43}$$
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \tag{11.44}$$

where dots are used to indicate differentiation with respect to time. The scalar components of the velocity and acceleration in the radial and transverse directions are therefore

$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta} \tag{11.45}$$
$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \tag{11.46}$$

It is important to note that a_r is *not* equal to the time derivative of v_r, and that a_θ is *not* equal to the time derivative of v_θ [Sample Prob. 11.12].

The chapter ended with a discussion of the use of cylindrical coordinates to define the position and motion of a particle in space.

MODULE NO. 12

REVIEW AND SUMMARY FOR CHAPTER 12

This chapter was devoted to Newton's second law and its application to the analysis of the motion of particles.

Newton's second law

Denoting by m the mass of a particle, by $\Sigma\mathbf{F}$ the sum, or resultant, of the forces acting on the particle, and by $\mathbf{a}$ the acceleration of the particle relative to a *newtonian frame of reference* [Sec. 12.2], we wrote

$$\Sigma\mathbf{F} = m\mathbf{a} \tag{12.2}$$

Linear momentum

Introducing the *linear momentum* of a particle, $\mathbf{L} = m\mathbf{v}$ [Sec. 12.3], we saw that Newton's second law can also be written in the form

$$\Sigma\mathbf{F} = \dot{\mathbf{L}} \tag{12.5}$$

which expresses that *the resultant of the forces acting on a particle is equal to the rate of change of the linear momentum of the particle.*

Consistent systems of units

Equation (12.2) holds only if a consistent system of units is used. With SI units, the forces should be expressed in newtons, the masses in kilograms, and the accelerations in m/s^2; with U.S. customary units, the forces should be expressed in pounds, the masses in $lb \cdot s^2/ft$ (also referred to as *slugs*), and the accelerations in ft/s^2 [Sec. 12.4].

Equations of motion for a particle

To solve a problem involving the motion of a particle, Eq. (12.2) should be replaced by equations containing scalar quantities [Sec. 12.5]. Using *rectangular components* of $\mathbf{F}$ and $\mathbf{a}$, we wrote

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \tag{12.8}$$

Using *tangential and normal components*, we had

$$\Sigma F_t = m\frac{dv}{dt} \qquad \Sigma F_n = m\frac{v^2}{\rho} \tag{12.9'}$$

Dynamic equilibrium

We also noted [Sec. 12.6] that the equations of motion of a particle can be replaced by equations similar to the equilibrium equations used in statics if a vector $-m\mathbf{a}$ of magnitude ma but of sense opposite to that of the acceleration is added to the forces applied to the particle; the particle is then said to be in *dynamic equilibrium.* For the sake of uniformity, however, all the Sample Problems were solved by using the equations of motion, first with rectangular components [Sample Probs. 12.1 through 12.4], then with tangential and normal components [Sample Probs. 12.5 and 12.6].

Angular momentum

In the second part of the chapter, we defined the *angular momentum* $\mathbf{H}_O$ of a particle about a point O as the moment about O of the linear momentum $m\mathbf{v}$ of that particle [Sec. 12.7]. We wrote

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v} \qquad (12.12)$$

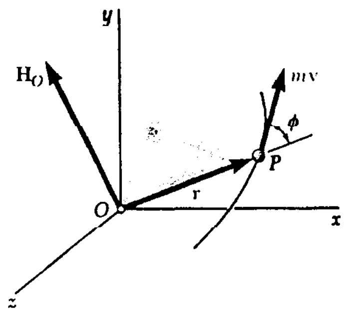

Fig. 12.24

and noted that $\mathbf{H}_O$ is a vector perpendicular to the plane containing $\mathbf{r}$ and $m\mathbf{v}$ (Fig. 12.24) and of magnitude

$$H_O = rmv \sin \phi \qquad (12.13)$$

Resolving the vectors $\mathbf{r}$ and $m\mathbf{v}$ into rectangular components, we expressed the angular momentum $\mathbf{H}_O$ in the determinant form

$$\mathbf{H}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix} \qquad (12.14)$$

In the case of a particle moving in the xy plane, we have $z = v_z = 0$. The angular momentum is perpendicular to the xy plane and is completely defined by its magnitude. We wrote

$$H_O = H_z = m(xv_y - yv_x) \qquad (12.16)$$

Rate of change of angular momentum

Computing the rate of change $\dot{\mathbf{H}}_O$ of the angular momentum $\mathbf{H}_O$, and applying Newton's second law, we wrote the equation

$$\Sigma\mathbf{M}_O = \dot{\mathbf{H}}_O \qquad (12.19)$$

which states that *the sum of the moments about O of the forces acting on a particle is equal to the rate of change of the angular momentum of the particle about O.*

Radial and transverse components

In many problems involving the plane motion of a particle, it is found convenient to use *radial and transverse components* [Sec. 12.8, Sample Prob. 12.7] and to write the equations

$$\Sigma F_r = m(\ddot{r} - r\dot{\theta}^2) \qquad (12.21)$$
$$\Sigma F_\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \qquad (12.22)$$

Motion under a central force

When the only force acting on a particle P is a force $\mathbf{F}$ directed toward or away from a fixed point O, the particle is said to be moving *under a central force* [Sec. 12.9]. Since $\Sigma\mathbf{M}_O = 0$ at any given instant, it follows from Eq. (12.19) that $\dot{\mathbf{H}}_O = 0$ for all values of t and, thus, that

$$\mathbf{H}_O = \text{constant} \qquad (12.23)$$

We concluded that *the angular momentum of a particle moving under a central force is constant, both in magnitude and direction,* and that the particle moves in a plane perpendicular to the vector $\mathbf{H}_O$.

Recalling Eq. (12.13), we wrote the relation

$$rmv \sin \phi = r_0 m v_0 \sin \phi_0 \qquad (12.25)$$

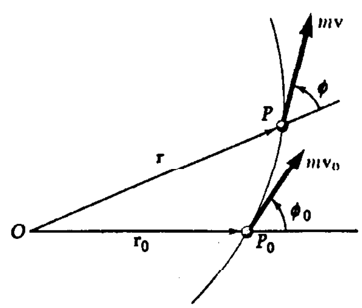

Fig. 12.25

for the motion of any particle under a central force (Fig. 12.25). Using polar coordinates and recalling Eq. (12.18), we also had

$$r^2 \dot{\theta} = h \qquad (12.27)$$

where h is a constant representing the angular momentum per unit mass, H_O/m, of the particle. We observed (Fig. 12.26) that the infinitesimal area dA swept by the radius vector OP as it rotates through $d\theta$ is equal to $\frac{1}{2}r^2\, d\theta$ and, thus, that the left-hand member of Eq. (12.27) represents twice the *areal velocity* dA/dt of the particle. Therefore, *the areal velocity of a particle moving under a central force is constant.*

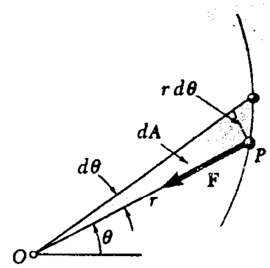

Fig. 12.26

Newton's law of universal gravitation

An important application of the motion under a central force is provided by the orbital motion of bodies under gravitational attraction [Sec. 12.10]. Acording to *Newton's law of universal gravitation*, two particles at a distance r from each other and of masses M and m, respectively, attract each other with equal and opposite forces $\mathbf{F}$ and $-\mathbf{F}$ directed along the line joining the particles (Fig. 12.27). The common magnitude F of the two forces is

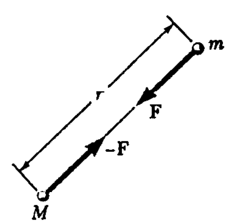

Fig. 12.27

$$F = G\frac{Mm}{r^2} \qquad (12.28)$$

where G is the *constant of gravitation*. In the case of a body of mass m subjected to the gravitational attraction of the earth, the product GM, where M is the mass of the earth, can be expressed as

$$GM = gR^2 \qquad (12.30)$$

where $g = 9.81\ \text{m/s}^2 = 32.2\ \text{ft/s}^2$ and R is the radius of the earth.

Orbital motion

It was shown in Sec. 12.11 that a particle moving under a central force describes a trajectory defined by the differential equation

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2} \qquad (12.37)$$

where $F > 0$ corresponds to an attractive force and $u = 1/r$. In the case of a particle moving under a force of gravitational attraction [Sec. 12.12], we substituted for F the expression given in Eq. (12.28). Measuring θ from the axis OA joining the focus O to the point A of the trajectory closest to O (Fig. 12.28), we found that the solution to Eq. (12.37) was

$$\frac{1}{r} = u = \frac{GM}{h^2} + C \cos \theta \qquad (12.39)$$

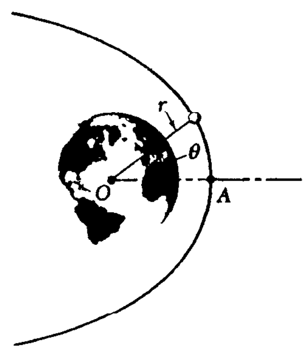

Fig. 12.28

This is the equation of a conic of eccentricity $\varepsilon = Ch^2/GM$. The conic is an *ellipse* if $\varepsilon < 1$, a *parabola* if $\varepsilon = 1$, and a hyperbola if $\varepsilon > 1$. The constants C and h can be determined from the initial conditions; if the particle is projected from point A ($\theta = 0$, $r = r_0$) with an initial velocity $\mathbf{v}_0$ perpendicular to OA, we have $h = r_0 v_0$ [Sample Prob. 12.9].

Escape velocity

It was also shown that the values of the initial velocity corresponding, respectively, to a parabolic and a circular trajectory were

$$v_{esc} = \sqrt{\frac{2GM}{r_0}} \qquad (12.43)$$

$$v_{circ} = \sqrt{\frac{GM}{r_0}} \qquad (12.44)$$

and that the first of these values, called the *escape velocity*, is the smallest value of v_0 for which the particle will not return to its starting point.

Periodic time

The *periodic time* τ of a planet or satellite was defined as the time required by that body to describe its orbit. It was shown that

$$\tau = \frac{2\pi ab}{h} \qquad (12.45)$$

where $h = r_0 v_0$ and where a and b represent the semimajor and semiminor axes of the orbit. It was further shown that these semiaxes are respectively equal to the arithmetic and geometric means of the maximum and minimum values of the radius vector r.

Kepler's laws

The last section of the chapter [Sec. 12.13] presented *Kepler's laws of planetary motion* and showed that these empirical laws, obtained from early astronomical observations, confirm Newton's laws of motion as well as his law of gravitation.

MODULES NO. 13 AND 14

REVIEW AND SUMMARY FOR CHAPTER 13

This chapter was devoted to the method of work and energy and to the method of impulse and momentum. In the first half of the chapter we studied the method of work and energy and its application to the analysis of the motion of particles.

Work of a force

We first considered a force **F** acting on a particle A and defined the *work of* **F** *corresponding to the small displacement* $d\mathbf{r}$ [Sec. 13.2] as the quantity

$$dU = \mathbf{F} \cdot d\mathbf{r} \qquad (13.1)$$

or, recalling the definition of the scalar product of two vectors,

$$dU = F\, ds \cos \alpha \qquad (13.1')$$

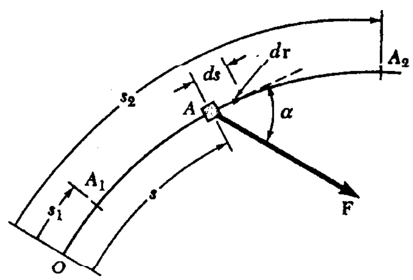

Fig. 13.29

where α is the angle between **F** and $d\mathbf{r}$ (Fig. 13.29). The work of **F** during a finite displacement from A_1 to A_2, denoted by $U_{1\rightarrow 2}$, was obtained by integrating Eq. (13.1) along the path described by the particle:

$$U_{1\rightarrow 2} = \int_{A_1}^{A_2} \mathbf{F} \cdot d\mathbf{r} \qquad (13.2)$$

For a force defined by its rectangular components, we wrote

$$U_{1\rightarrow 2} = \int_{A_1}^{A_2} (F_x\, dx + F_y\, dy + F_z\, dz) \qquad (13.2'')$$

Work of a weight

The work of the weight **W** of a body as its center of gravity moves from the elevation y_1 to y_2 (Fig. 13.30) was obtained by substituting $F_x = F_z = 0$ and $F_y = -W$ into Eq. (13.2″) and integrating. We found

$$U_{1\rightarrow 2} = -\int_{y_1}^{y_2} W\, dy = Wy_1 - Wy_2 \qquad (13.4)$$

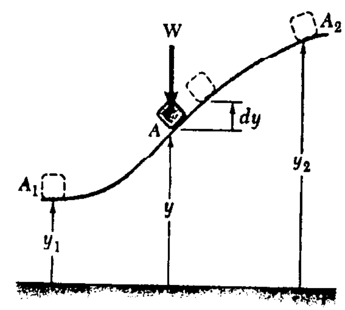

Fig. 13.30

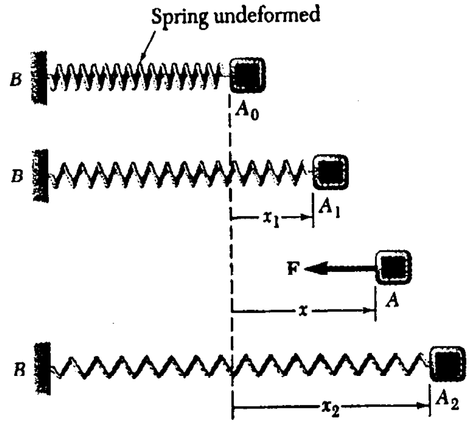

Fig. 13.31

Work of the force exerted by a spring

The work of a force $\mathbf{F}$ exerted by a spring on a body A during a finite displacement of the body (Fig. 13.31) from $A_1(x = x_1)$ to $A_2(x = x_2)$ was obtained by writing

$$dU = -F\,dx = -kx\,dx$$
$$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 \qquad (13.6)$$

The work of $\mathbf{F}$ is therefore positive *when the spring is returning to its undeformed position.*

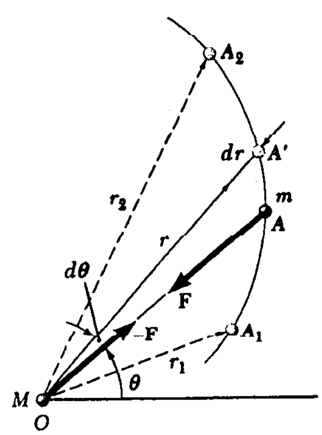

Fig. 13.32

Work of the gravitational force

The *work of the gravitational force* $\mathbf{F}$ exerted by a particle of mass M located at O on a particle of mass m as the latter moves from A_1 to A_2 (Fig. 13.32) was obtained by recalling from Sec. 12.10 the expression for the magnitude of $\mathbf{F}$ and writing

$$U_{1\to2} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}\,dr = \frac{GMm}{r_2} - \frac{GMm}{r_1} \qquad (13.7)$$

Kinetic energy of a particle

The *kinetic energy of a particle* of mass m moving with a velocity $\mathbf{v}$ [Sec. 13.3] was defined as the scalar quantity

$$T = \tfrac{1}{2}mv^2 \qquad (13.9)$$

Principle of work and energy

From Newton's second law we derived the *principle of work and energy,* which states that *the kinetic energy of a particle at* A_2 *can be obtained by adding to its kinetic energy at* A_1 *the work done during the displacement from* A_1 to A_2 *by the force* **F** *exerted on the particle:*

$$T_1 + U_{1\to 2} = T_2 \qquad (13.11)$$

Method of work and energy

The method of work and energy simplifies the solution of many problems dealing with forces, displacements, and velocities, since it does not require the determination of accelerations [Sec. 13.4]. We also note that it involves only scalar quantities and that forces which do no work need not be considered [Sample Probs. 13.1 and 13.3]. However, this method should be supplemented by the direct application of Newton's second law to determine a force normal to the path of the particle [Sample Prob. 13.4].

Power and mechanical efficiency

The power developed by a machine and its mechanical efficiency were discussed in Sec. 13.5. Power was defined as the time rate at which work is done:

$$\text{Power} = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v} \qquad (13.12, 13.13)$$

where **F** is the force exerted on the particle and **v** the velocity of the particle [Sample Prob. 13.5]. The *mechanical efficiency,* denoted by η, was expressed as

$$\eta = \frac{\text{power output}}{\text{power input}} \qquad (13.15)$$

Conservative force. Potential energy

When the work of a force **F** is independent of the path followed [Secs. 13.6 and 13.7], the force **F** is said to be a *conservative force,* and its work is equal to *minus the change in the potential energy* V associated with **F**:

$$U_{1\to 2} = V_1 - V_2 \qquad (13.19')$$

The following expressions were obtained for the potential energy associated with each of the forces considered earlier:

Force of gravity (weight): $V_g = Wy$ (13.16)

Gravitational force: $V_g = -\dfrac{GMm}{r}$ (13.17)

Elastic force exerted by a spring: $V_e = \frac{1}{2}kx^2$ (13.18)

Substituting for $U_{1\to2}$ from Eq. (13.19′) into Eq. (13.11) and rearranging the terms [Sec. 13.8], we obtained

$$T_1 + V_1 = T_2 + V_2 \tag{13.24}$$

Principle of conservation of energy

This is the *principle of conservation of energy*, which states that when a particle moves under the action of conservative forces, *the sum of its kinetic and potential energies remains constant*. The application of this principle facilitates the solution of problems involving only conservative forces [Sample Probs. 13.6 and 13.7].

Motion under a gravitational force

Recalling from Sec. 12.9 that, when a particle moves under a central force **F**, its angular momentum about the center of force O remains constant, we observed [Sec. 13.9] that, if the central force **F** is also conservative, the principles of conservation of angular momentum and of conservation of energy can be used jointly to analyze the motion of the particle [Sample Prob. 13.8]. Since the gravitational force exerted by the earth on a space vehicle is both central and conservative, this approach was used to study the motion of such vehicles [Sample Prob. 13.9] and was found particularly effective in the case of an *oblique launching*. Considering the initial position P_0 and an arbitrary position P of the vehicle (Fig. 13.33), we wrote

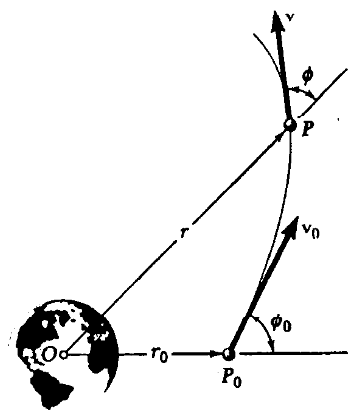

Fig. 13.33

$$(H_O)_0 = H_O: \qquad r_0 m v_0 \sin\phi_0 = rmv\sin\phi \tag{13.25}$$

$$T_0 + V_0 = T + V: \qquad \tfrac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \tag{13.26}$$

where m was the mass of the vehicle and M the mass of the earth.

Principle of impulse and momentum for a particle

The second half of the chapter was devoted to the method of impulse and momentum and to its application to the solution of various types of problems involving the motion of particles.

The *linear momentum of a particle* was defined [Sec. 13.10] as the product $m\mathbf{v}$ of the mass m of the particle and its velocity **v**. From Newton's second law, $\mathbf{F} = m\mathbf{a}$, we derived the relation

$$m\mathbf{v}_1 + \int_{t_1}^{t_2} \mathbf{F}\,dt = m\mathbf{v}_2 \tag{13.28}$$

where $m\mathbf{v}_1$ and $m\mathbf{v}_2$ represent the momentum of the particle at a time t_1 and a time t_2, respectively, and where the integral defines the *linear impulse of the force* **F** during the corresponding time interval. We wrote therefore

$$m\mathbf{v}_1 + \mathbf{Imp}_{1\to2} = m\mathbf{v}_2 \tag{13.30}$$

which expresses the principle of impulse and momentum for a particle.

When the particle considered is subjected to several forces, the sum of the impulses of these forces should be used; we had

$$m\mathbf{v}_1 + \Sigma\, \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2 \qquad (13.32)$$

Since Eqs. (13.30) and (13.32) involve *vector quantities*, it is necessary to consider their x and y components separately when applying them to the solution of a given problem [Sample Probs. 13.10 and 13.11].

Impulsive motion

The method of impulse and momentum is particularly effective in the study of the *impulsive motion* of a particle, when very large forces, called *impulsive forces*, are applied for a very short interval of time Δt, since this method involves the impulses $\mathbf{F}\,\Delta t$ of the forces, rather than the forces themselves [Sec. 13.11]. Neglecting the impulse of any nonimpulsive force, we wrote

$$m\mathbf{v}_1 + \Sigma\mathbf{F}\,\Delta t = m\mathbf{v}_2 \qquad (13.35)$$

In the case of the impulsive motion of several particles, we had

$$\Sigma m\mathbf{v}_1 + \Sigma\mathbf{F}\,\Delta t = \Sigma m\mathbf{v}_2 \qquad (13.36)$$

where the second term involves only impulsive, external forces [Sample Prob. 13.12].

In the particular case *when the sum of the impulses of the external forces is zero,* Eq. (13.36) reduces to $\Sigma m\mathbf{v}_1 = \Sigma m\mathbf{v}_2$; that is, *the total momentum of the particles is conserved.*

Direct central impact

In Secs. 13.12 through 13.14, we considered the *central impact* of two colliding bodies. In the case of a *direct central impact* [Sec. 13.13], the two colliding bodies A and B were moving along the *line of impact* with velocities $\mathbf{v}_A$ and $\mathbf{v}_B$, respectively (Fig. 13.34). Two equations could be used to determine their velocities $\mathbf{v}'_A$ and $\mathbf{v}'_B$ after the

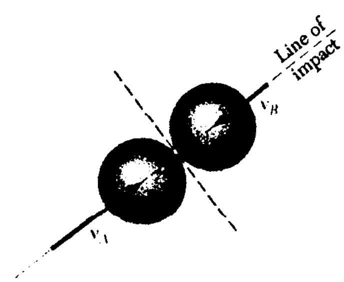

Fig. 13.34

impact. The first expressed conservation of the total momentum of the two bodies,

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \tag{13.37}$$

where a positive sign indicates that the corresponding velocity is directed to the right, while the second related the *relative velocities* of the two bodies before and after the impact,

$$v'_B - v'_A = e(v_A - v_B) \tag{13.43}$$

The constant e is known as the *coefficient of restitution;* its value lies between 0 and 1 and depends in a large measure on the materials involved. When $e = 0$, the impact is said to be *perfectly plastic;* when $e = 1$, it is said to be *perfectly elastic* [Sample Prob. 13.13].

Oblique central impact

In the case of an *oblique central impact* [Sec. 13.14], the velocities of the two colliding bodies before and after the impact were resolved into n components along the line of impact and t components along the common tangent to the surfaces in contact (Fig. 13.35). We observed that the t component of the velocity of each

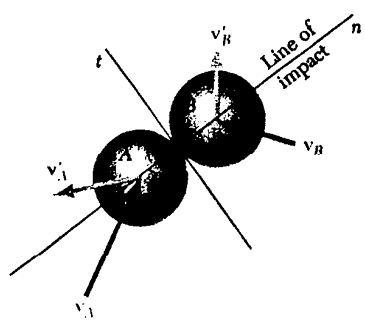

Fig. 13.35

body remained unchanged, while the n components satisfied equations similar to Eqs. (13.37) and (13.43) [Sample Probs. 13.14 and 13.15]. It was shown that although this method was developed for bodies moving freely before and after the impact, it could be extended to the case when one or both of the colliding bodies is constrained in its motion [Sample Prob. 13.16].

Using the three fundamental methods of kinetic analysis

In Sec. 13.15, we discussed the relative advantages of the three fundamental methods presented in this chapter and the preceding one, namely, Newton's second law, work and energy, and impulse and momentum. We noted that the method of work and energy and the method of impulse and momentum can be combined to solve problems involving a short impact phase during which impulsive forces must be taken into consideration [Sample Prob. 13.17].

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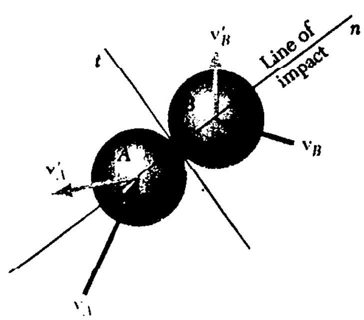

Fig. 13.35

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MODULE NO. 15

REVIEW AND SUMMARY FOR CHAPTER 19

This chapter was devoted to the study of *mechanical vibrations,* i.e., to the analysis of the motion of particles and rigid bodies oscillating about a position of equilibrium. In the first part of the chapter [Secs. 19.2 through 19.7], we considered *vibrations without damping,* while the second part was devoted to *damped vibrations* [Secs. 19.8 through 19.10].

Free vibrations of a particle

In Sec. 19.2, we considered the *free vibrations of a particle,* i.e., the motion of a particle P subjected to a restoring force proportional to the displacement of the particle—such as the force exerted by a spring. If the displacement x of the particle P is measured from its equilibrium position O (Fig. 19.17), the resultant $\mathbf{F}$ of the forces acting on P (including its weight) has a magnitude kx and is directed toward O. Applying Newton's second law $F = ma$ and recalling that $a = \ddot{x}$, we wrote the differential equation

$$m\ddot{x} + kx = 0 \qquad (19.2)$$

or, setting $\omega_n^2 = k/m$,

$$\ddot{x} + \omega_n^2 x = 0 \qquad (19.6)$$

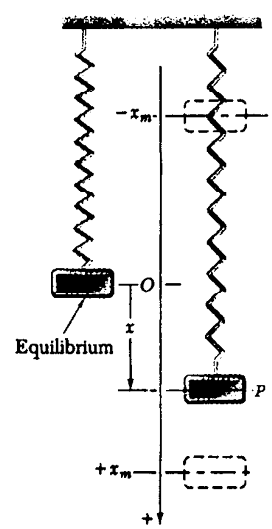


Fig. 19.17

The motion defined by this equation is called a *simple harmonic motion*.

The solution of Eq. (19.6), which represents the displacement of the particle P, was expressed as

$$x = x_m \sin(\omega_n t + \phi) \qquad (19.10)$$

where x_m = amplitude of the vibration
$\omega_n = \sqrt{k/m}$ = natural circular frequency
ϕ = phase angle

The *period of the vibration* (i.e., the time required for a full cycle) and its *natural frequency* (i.e., the number of cycles per second) were expressed as

$$\text{Period} = \tau_n = \frac{2\pi}{\omega_n} \qquad (19.13)$$

$$\text{Natural frequency} = f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi} \qquad (19.14)$$

The velocity and acceleration of the particle were obtained by differentiating Eq. (19.10), and their maximum values were found to be

$$v_m = x_m\omega_n \qquad a_m = x_m\omega_n^2 \qquad (19.15)$$

Since all the above parameters depend directly upon the natural circular frequency ω_n and thus upon the ratio k/m, it is essential in any given problem to calculate the value of the constant k; this can be done by determining the relation between the restoring force and the corresponding displacement of the particle [Sample Prob. 19.1].

It was also shown that the oscillatory motion of the particle P can be represented by the projection on the x axis of the motion of a point Q describing an auxiliary circle of radius x_m with the constant angular velocity ω_n (Fig. 19.18). The instantaneous values of the velocity and acceleration of P can then be obtained by projecting on the x axis the vectors $\mathbf{v}_m$ and $\mathbf{a}_m$ representing, respectively, the velocity and acceleration of Q.

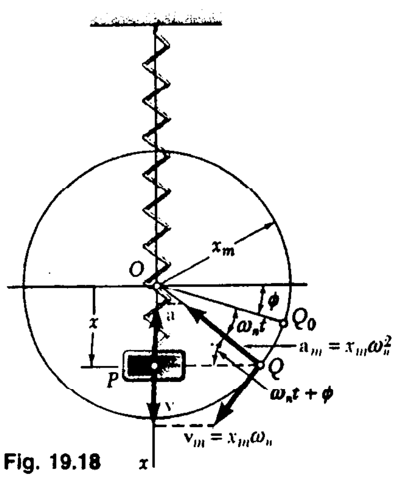


Fig. 19.18

Simple pendulum

While the motion of a *simple pendulum* is not truly a simple harmonic motion, the formulas given above can be used with $\omega_n^2 = g/l$ to calculate the period and natural frequency of the *small oscillations* of a simple pendulum [Sec. 19.3]. Large-amplitude oscillations of a simple pendulum were discussed in Sec. 19.4.

Free vibrations of a rigid body

The *free vibrations of a rigid body* can be analyzed by choosing an appropriate variable, such as a distance x or an angle θ, to define the position of the body, drawing a free-body-diagram equation to express the equivalence of the external and effective forces, and writing an equation relating the selected variable and its second derivative [Sec. 19.5]. If the equation obtained is of the form

$$\ddot{x} + \omega_n^2 x = 0 \qquad \text{or} \qquad \ddot{\theta} + \omega_n^2 \theta = 0 \tag{19.21}$$

the vibration considered is a simple harmonic motion and its period and natural frequency can be obtained *by identifying* ω_n and substituting its value into Eqs. (19.13) and (19.14) [Sample Probs. 19.2 and 19.3].

Using the principle of conservation of energy

The *principle of conservation of energy* can be used as an alternative method for the determination of the period and natural frequency of the simple harmonic motion of a particle or rigid body [Sec. 19.6]. Choosing again an appropriate variable, such as θ, to define the position of the system, we express that the total energy of the system is conserved, $T_1 + V_1 = T_2 + V_2$, between the position of maximum displacement ($\theta_1 = \theta_m$) and the position of maximum velocity ($\dot{\theta}_2 = \dot{\theta}_m$). If the motion considered is simple harmonic, the two members of the equation obtained consist of homogeneous quadratic expressions in θ_m and $\dot{\theta}_m$, respectively.† Substituting $\dot{\theta}_m = \theta_m \omega_n$ in this equation, we can factor out θ_m^2 and solve for the circular frequency ω_n [Sample Prob. 19.4].

Forced vibrations

In Sec. 19.7, we considered the *forced vibrations* of a mechanical system. These vibrations occur when the system is subjected to a periodic force (Fig. 19.19) or when it is elastically connected to a support which has an alternating motion (Fig. 19.20). Denoting by ω_f the forced circular frequency, we found that in the first case, the motion of the system was defined by the differential equation

$$m\ddot{x} + kx = P_m \sin \omega_f t \tag{19.30}$$

and that in the second case it was defined by the differential equation

$$m\ddot{x} + kx = k\delta_m \sin \omega_f t \tag{19.31}$$

The general solution of these equations is obtained by adding a particular solution of the form

$$x_{\text{part}} = x_m \sin \omega_f t \tag{19.32}$$

† If the motion considered can only be *approximated* by a simple harmonic motion, such as for the small oscillations of a body under gravity, the potential energy must be approximated by a quadratic expression in θ_m.

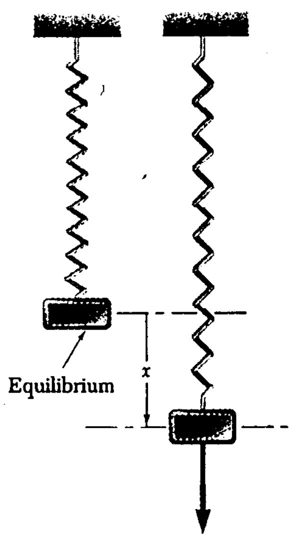


Fig. 19.19

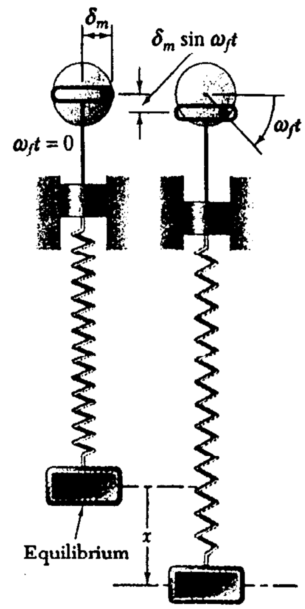


Fig. 19.20

to the general solution of the corresponding homogeneous equation. The particular solution (19.32) represents a *steady-state vibration* of the system, while the solution of the homogeneous equation represents a *transient free vibration* which can generally be neglected.

Dividing the amplitude x_m of the steady-state vibration by P_m/k in the case of a periodic force, or by δ_m in the case of an oscillating support, we defined the *magnification factor* of the vibration and found that

$$\text{Magnification factor} = \frac{x_m}{P_m/k} = \frac{x_m}{\delta_m} = \frac{1}{1 - (\omega_f/\omega_n)^2} \qquad (19.36)$$

According to Eq. (19.36), the amplitude x_m of the forced vibration *becomes infinite when* $\omega_f = \omega_n$, i.e., *when the forced frequency is equal to the natural frequency of the system*. The impressed force or impressed support movement is then said to be in *resonance* with the system [Sample Prob. 19.5]. Actually the amplitude of the vibration remains finite, due to damping forces.

Damped free vibrations

In the last part of the chapter, we considered the *damped vibrations* of a mechanical system. First, we analyzed the *damped free vibrations* of a system with *viscous damping* [Sec. 19.8]. We found that the motion of such a system was defined by the differential equation

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad (19.38)$$

where c is a constant called the *coefficient of viscous damping*. Defining the *critical damping coefficient* c_c as

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n \qquad (19.41)$$

where ω_n is the natural circular frequency of the system in the absence of damping, we distinguished three different cases of damping, namely, (1) *heavy damping*, when $c > c_c$; (2) *critical damping*, when $c = c_c$; and (3) *light damping*, when $c < c_c$. In the first two cases, the system when disturbed tends to regain its equilibrium position without any oscillation. In the third case, the motion is vibratory with diminishing amplitude.

Damped forced vibrations

In Sec. 19.9, we considered the *damped forced vibrations* of a mechanical system. These vibrations occur when a system with viscous damping is subjected to a periodic force **P** of magnitude $P = P_m \sin \omega_f t$ or when it is elastically connected to a support with an alternating motion $\delta = \delta_m \sin \omega_f t$. In the first case, the motion of the system was defined by the differential equation

$$m\ddot{x} + c\dot{x} + kx = P_m \sin \omega_f t \qquad (19.47)$$

and in the second case by a similar equation obtained by replacing P_m by $k\delta_m$ in (19.47).

The *steady-state vibration* of the system is represented by a particular solution of Eq. (19.47) of the form

$$x_{\text{part}} = x_m \sin(\omega_f t - \varphi) \qquad (19.48)$$

Dividing the amplitude x_m of the steady-state vibration by P_m/k in the case of a periodic force, or by δ_m in the case of an oscillating support, we obtained the following expression for the magnification factor:

$$\frac{x_m}{P_m/k} = \frac{x_m}{\delta_m} = \frac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2}} \qquad (19.53)$$

where $\omega_n = \sqrt{k/m}$ = natural circular frequency of undamped system
$c_c = 2m\omega_n$ = critical damping coefficient
c/c_c = damping factor

We also found that the *phase difference* φ between the impressed force or support movement and the resulting steady-state vibration of the damped system was defined by the relation

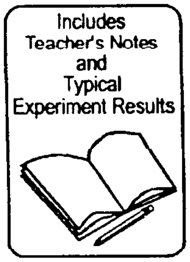
$$\tan \varphi = \frac{2(c/c_c)(\omega_f/\omega_n)}{1 - (\omega_f/\omega_n)^2} \qquad (19.54)$$

Electrical analogues

The chapter ended with a discussion of *electrical analogues* [Sec. 19.10], in which it was shown that the vibrations of mechnical systems and the oscillations of electrical circuits are defined by the same differential equations. Electrical analogues of mechanical systems can therefore be used to study or predict the behavior of these systems.

Appendix 2:

Dynamics Cart Accessory Track Set 2.2M Version



Instruction Manual and Experiment Guide for the PASCO scientific Model ME-9458 and ME-9452

012-05024E
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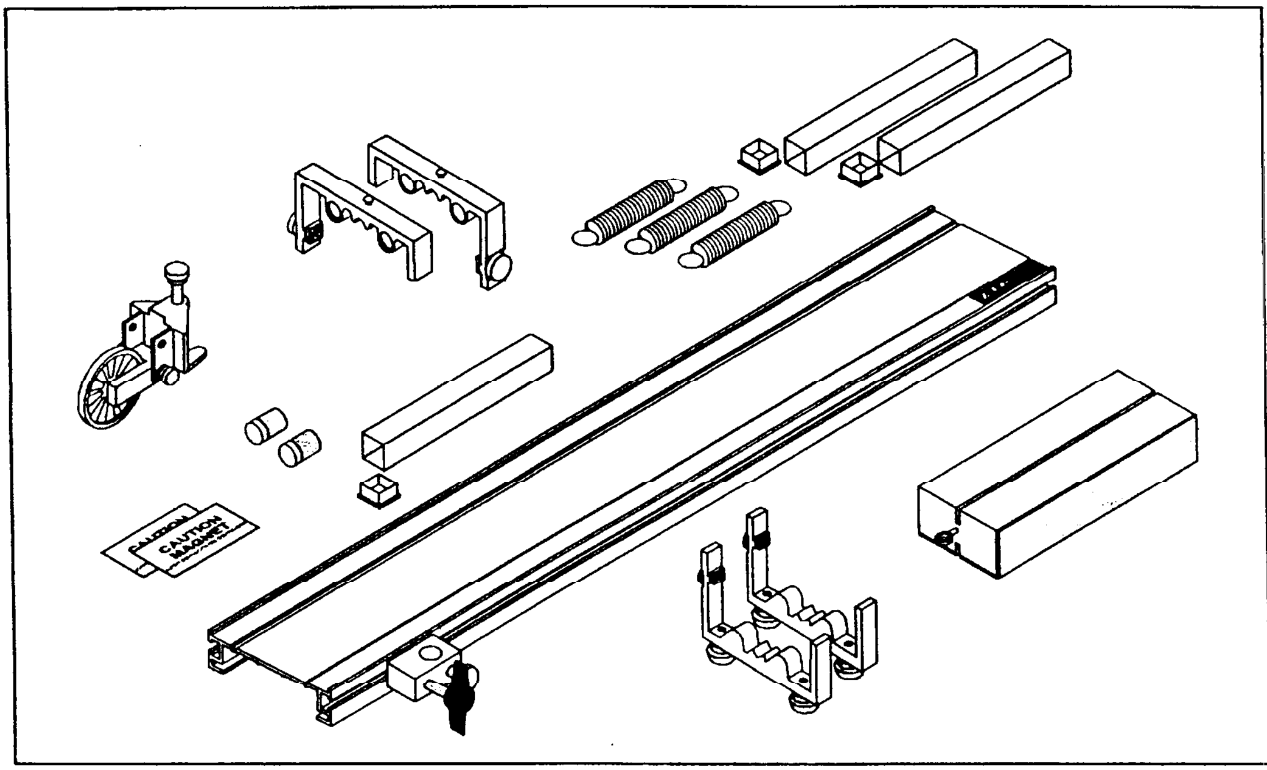
Dynamics Cart Accessory Track Set (2.2m version)



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This manual authored by: Ann & John Hanks

Teacher's guide written by: Eric Ayars

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① The carton must be strong enough for the item shipped.

② Make certain there is at least two inches of packing material between any point on the apparatus and the inside walls of the carton.

③ Make certain that the packing material can not shift in the box, or become compressed, thus letting the instrument come in contact with the edge of the box.

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Introduction

The PASCO Model ME-9458 Dynamics Cart Accessory Track Set enables the user to perform a wide variety of experiments when used with the Dynamics Cart (ME-9430) and the Collision Cart (ME-9454). The Track ensures easy setup and accurate alignment with the lowest possible friction, and it accomodates most linear motion experiments.

Features include:

- Adjustable leveling feet.
- Low friction wheel slots keep the carts aligned even after a collision.
- Mounted to a standard lab rod, the track adjusts to any angle for inclined plane experiments.
- Durable construction with Adjustable End Stops protects the cart.

Equipment

The ME-9458 Dynamics Cart Accessory Track Set includes the following:

- Dynamics Cart Track:

  2.2m (7.5') extruded aluminum track with alignment grooves in top surface, two leveling feet and two adjustable End Stops.

> ➤ **NOTE:** The End Stop has a round head screw on the top to allow easy attachment of springs, string, etc.

- Force Table Clamp with Super Pulley.
- (3) Springs for simple harmonic motion with storage tubes.

> ➤ **NOTE:** A small piece of double sided tape is attached to the ends of each storage tube so the tubes may be permanently attached to the underside of the Dynamics Cart Track.

- Friction Block
- Magnet Bumper Kit (includes 2 magnets) with storage tube.
- Pivot Clamp [for use with the Base and Support Rod (ME-9355)].
- (2) Labels: "CAUTION! MAGNET".

The ME-9452 Introductory Dynamics System (2.2m version) includes all the components of the ME-9458 plus the following:

- Dynamics Cart with Mass (ME-9340)
- Collision Cart (ME-9454)

The ME-9459 Introductory Dynamics Demonstration System includes all the components of the ME-9458 plus the following:

- Dynamics Cart with Mass (ME-9340)
- (2) Collision Carts (ME-9454)
- Additional Spring

The ME-9453 Dynamics Track Set (2.2m) includes the following:

- 2.2m Track
- (2) Leveling Feet (ME-9470)
- (2) Adjustable End Stops (ME-9469)

Additional Equipment Required for ME-9458

- Dynamics Cart with Mass (ME-9430)

Specific experiment requirements:

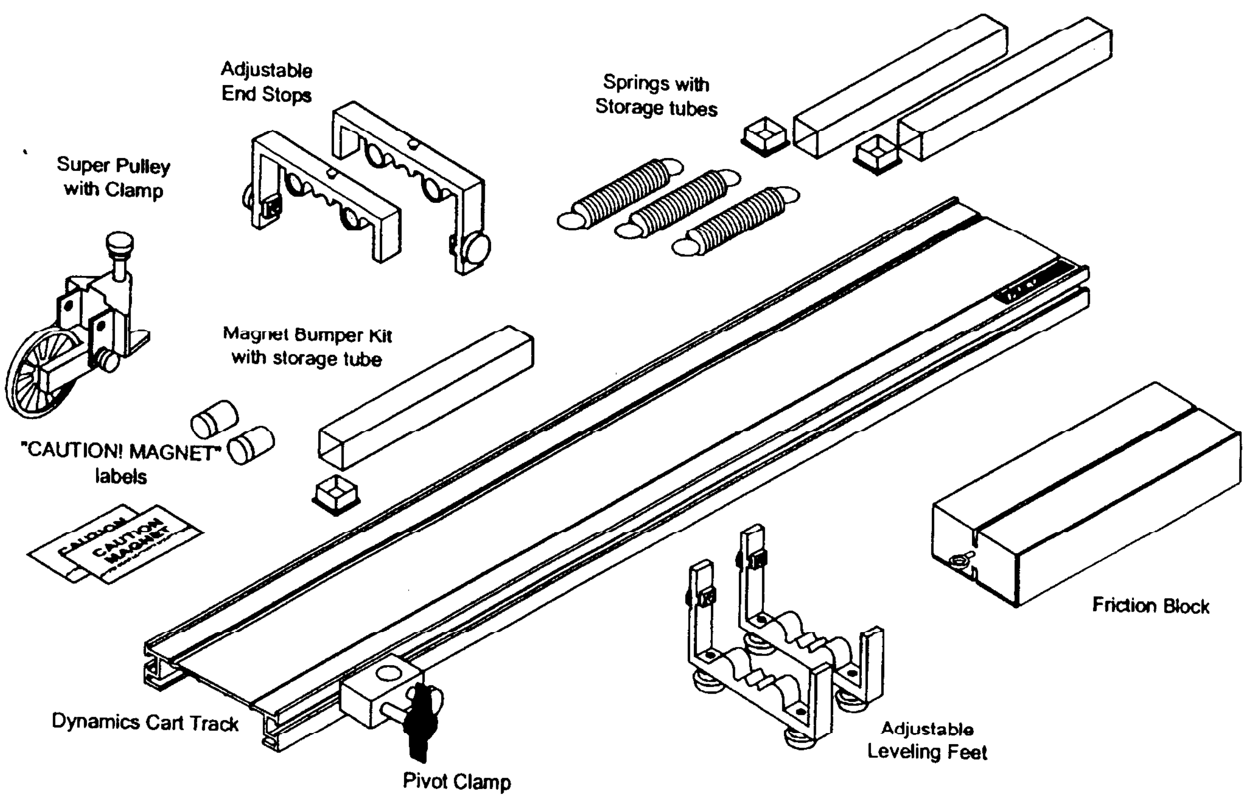
- Thread
- Mass Set
- Super Pulley with Clamp
- Base and Support Rod
- Metric Ruler
- Stopwatch
- Mass balance
- Wooden or metal block
- Graph paper

Additional Equipment Recommended

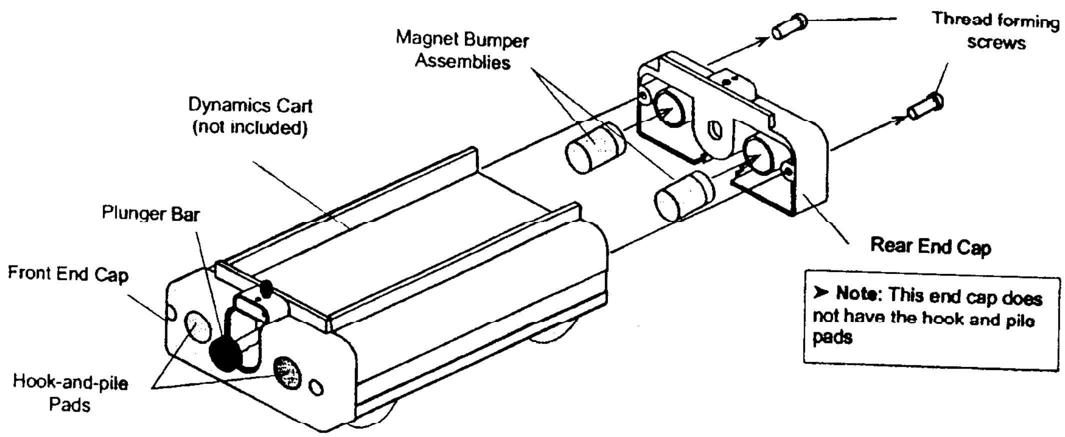
- Photogate Accessory Kit with Software, (Apple) (ME-9436) or (IBM PC) (ME-9437)

or

- Software Accessory Kit, (Apple) (ME-9438) or (IBM PC) (ME-9439).



Assembly



Installing the Magnet Bumpers

> ➤ **NOTE:** The ME-9454 Collision Cart comes with 2 sets of magnetic bumpers already installed. The ME-9430 Dynamics Cart comes without any magnetic bumpers.

① Detach the end cap at the rear of the cart by removing the two screws from the rear end cap as shown.

> ➤ **NOTE:** The screws that secure the end caps to either end of the Dynamics Cart are thread forming screws and may require substantial force to remove and reinstall. A #1 Phillips point screw driver is required.

② Insert the two magnet bumper assemblies, magnet end first, into the cavities on the inside of the end cap as shown.

> ➤ **CAUTION!**
>
> Each magnet assembly consists of a foam pad attached to a neodymium magnet. ***The neodymium magnets are extremely strong.*** Though only the south end of the magnet is exposed they can still be a hazard. When opposite poles are brought close to each other they will accelerate rapidly and can pinch fingers or be easily chipped. They can also erase computer disks and distort computer monitors and television sets.

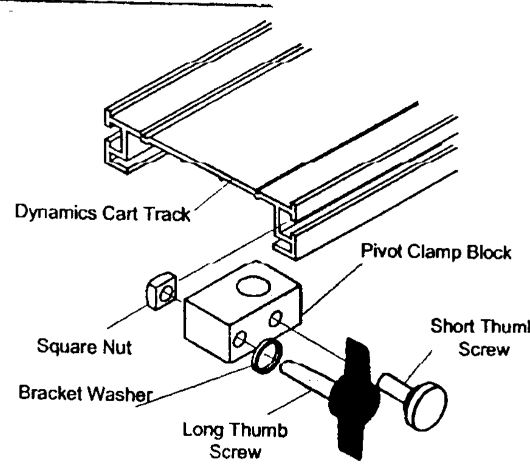
③ Replace the rear end cap with the two screws.

Installing the Pivot Clamp

① Remove Pivot Clamp Assembly from underneath the Dynamics Cart Track.

② Insert long thumb screw through the hole in the Pivot Clamp Block and thread ½ to ¾ turn into the hex nut.

> ➤ **NOTE:** Observe the orientation of the Pivot Clamp Block. Also note that the flat side of the square nut must face the outside of the Dynamics Cart Track as shown.



③ Align the square nut within the groove on the desired side of the Dynamics Cart Track. Locate and adjust Pivot Clamp to desired position and tighten thumb screw to secure.

Installing the Leveling Feet

The leveling feet serve 3 purposes: to level the track, to reduce any twist in the track, and to reduce any bow in the track. Assembly is as follows:

① Thread a locking nut onto each of the four long screws as shown in Figure 1.

② Thread two of the long screws in top the two holes in the bottom of each aluminum leveling foot. The heads of these screws form the feet which will rest on the table when the track is in use.

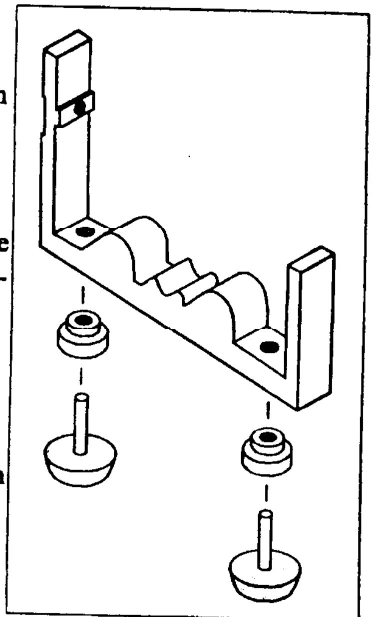


Fig. 1 - Attaching Feet

③ Place the washer on the short screw and insert the short screw through the hole in the side of the aluminum leveling foot as shown in Figure 2. Screw the square nut onto the end of the short screw just far enough to keep the short screw from falling out.

④ Align the square nut within the groove on the desired side of the Dynamics Cart Track. Slide the leveling foot down the track to the desired position. To minimize the bow in the track, it is best to place a leveling foot about 1/4 of the track length from each end of the track (see Figure 3).

⑤ To level the track, place a cart on the track to see which way it rolls. Then loosen the lock nuts and screw the leveling screws up or down to change the height of one end of the track until the cart when placed at rest will stay at rest. When the track is level, tighten the lock nuts against the aluminum foot.

⑥ It is also possible to take some twist out of the track by adjusting the leveling screws on one side of the track.

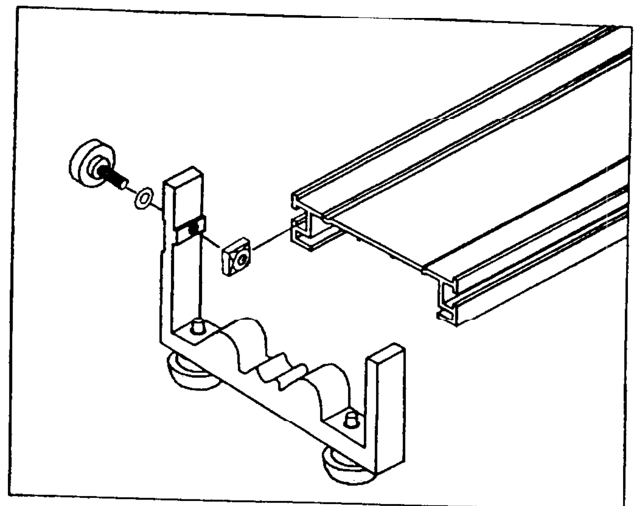


Fig. 2 - Attaching Leveling Bracket to Track

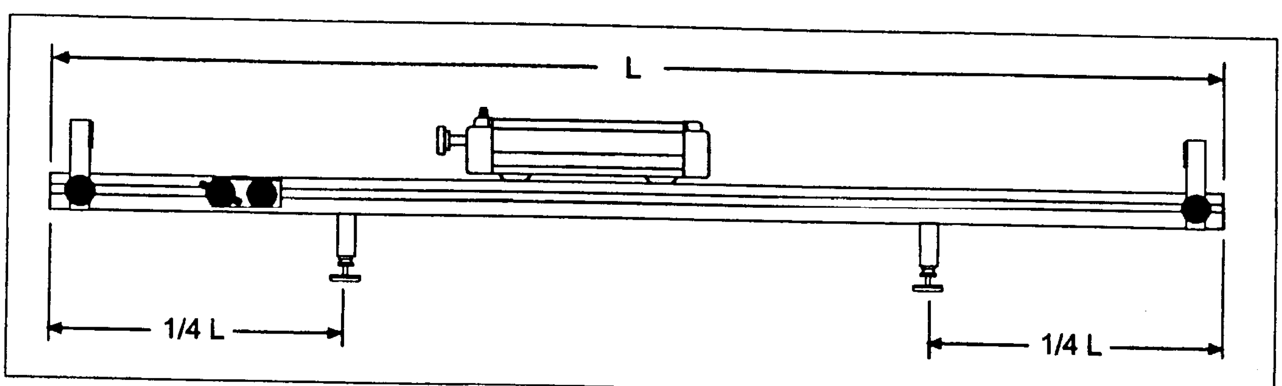


Fig. 3 - Optimum Position of Leveling Feet

Installing the Adjustable End Stop

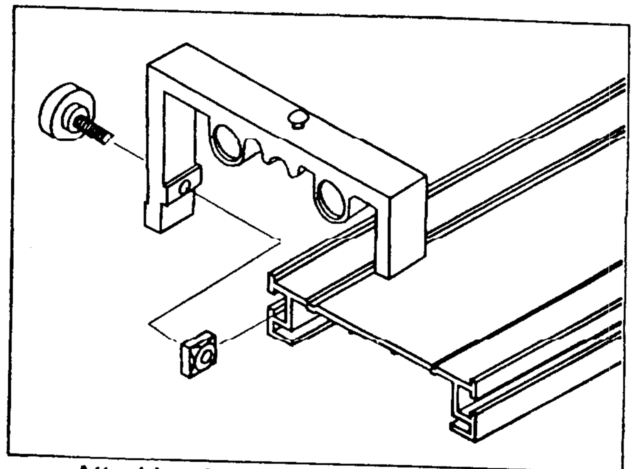
The Adjustable End Stop can be used at any point on the track as a bumper. Either the plunger bar on the cart or the cart's magnetic bumper can be used to rebound off the End Stop because the End Stop contains magnets. The cart can also be stopped against the End Stop when the velcro end of the cart hits the velcro side of the End Stop. This is useful when it is desired to keep the cart from rebounding. There is also a post on top of the End Stop to allow a string or spring to be attached. Assembly is as follows:

① The Adjustable End Stop Assembly consists of the end stop with two magnets installed, a black plastic thumb screw, and a square nut.

② It is best to install the End Stops in the groove opposite to the side being used for the leveling feet so the End Stops can slide past the leveling feet without interference.

③ Align the square nut within the groove on the desired side of the Dynamics Cart Track as shown. Locate and adjust the End Stop to the desired position and tighten the thumb screw to secure.

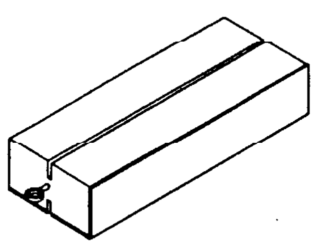
④ When storing the End Stop when it is not on the track, remember that it has two strong magnets in it. Keep the End Stop away from computers.



Attaching Adjustable End Stop to Track

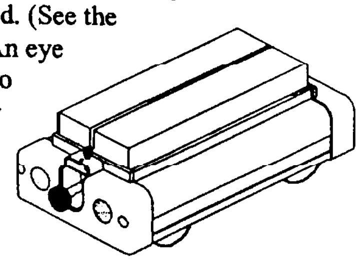
Using the Friction Block

The Friction Block is a wood rectangle that fits neatly on top of the Dynamics Cart (ME-9430).

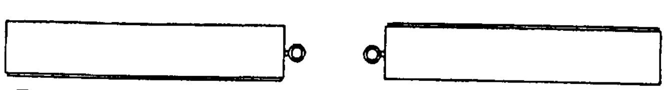


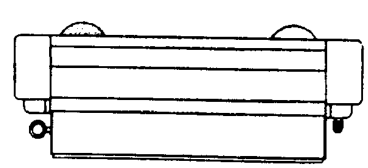
In experiments that use the Friction Block you will investigate some of the properties of sliding friction - the force that resists the sliding motion of two objects when they are already in motion.

The top and bottom surfaces of the Friction Block have a slot which allows a "picket fence" to be inserted. (See the PASCO catalog.) An eye screw is provided so that you may easily attach a string to the block.



The exposed wood on the top and one side of the block produce minimal friction. Felt pads attached to the bottom surface and the other side provide more friction. Mass can be placed on the top surface of the Friction Block as shown.





Replacement Parts (ME-9458)

Description	Part No.
Magnet Bumper Kit Assembly (4per)	003-05027
Super Pulley with Clamp (1ea)	ME-9448A
Friction Block (1ea)	003-04708
Label, Magnet Caution (1ea)	646-04445
Spring (3ea)	632-04978
Pivot Clamp Assembly:	003-05019
Pivot clamp (1ea)	648-04654
Long thumb screw (1ea)	610-183 & 620-047
Short thumb screw (1ea)	610-181 & 620-067
Washer	615-184
Square nut (1ea)	614-054
Adjustable End Stop (2ea)	ME-9469
Leveling Feet (2ea)	ME-9470

Experiment 2: Conservation of Momentum in Collisions

EQUIPMENT NEEDED:

- Dynamics Cart with Mass (ME-9430)
- Collision Cart (ME-9454)
- (2) Bumper magnet set (installed)
- Dynamics Cart Track
- Paper

Purpose

The purpose of this experiment is to qualitatively explore conservation of momentum for elastic and inelastic collisions.

Theory

When two carts collide with each other, the total momentum $\vec{p} = m\vec{v}$ of both carts is conserved regardless of the type of collision. An elastic collision is one in which the two carts bounce off each other with no loss of kinetic energy. In this experiment, magnetic bumpers are used to minimize the energy losses due to friction during the collision. In reality, this "elastic" collision is slightly inelastic. A completely inelastic collision is one in which the two carts hit and stick to each other. In this experiment, this is accomplished with the hook-and-pile tabs on the end caps of the carts.

Procedure

① Level the track by setting a cart on the track to see which way it rolls. Adjust the leveling feet at the end of the track to raise or lower that end until a cart placed at rest on the track will not move.

② Draw two diagrams (one for before the collision and one for after the collision) for each of the following cases. In each diagram, show a velocity vector for each cart with a length that approximately represents the relative speed of the cart.

Part I: Elastic Collisions

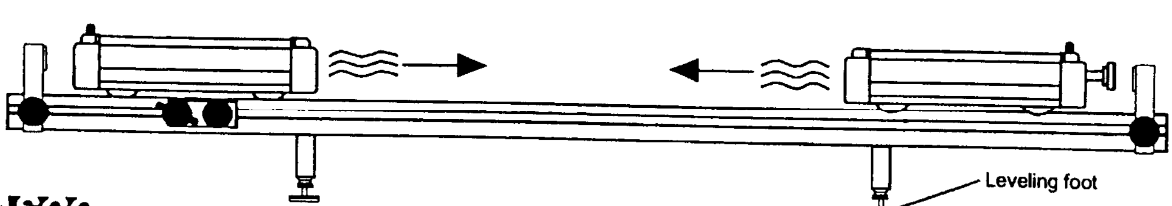
A. Carts with Equal Mass

Orient the two carts so their magnetic bumpers are toward each other.

Case 1: Place one cart at rest in the middle of the track. Give the other cart an initial velocity toward the cart at rest.

Case 2: Start the carts with one at each end of the track. Give each cart approximately the same velocity toward each other.

Case 3: Start both carts at one end of the track. Give the first cart a slow velocity and the second cart a faster velocity so that the second cart catches the first cart.



B. Carts with Unequal Mass

Put two mass bars in one of the carts so that the mass of one cart is approximately three times the mass (3M) of the other cart (1M).

Case 1: Place the 3M cart at rest in the middle of the track. Give the other cart an initial velocity toward the cart at rest.

Case 2: Place the 1M cart at rest in the middle of the track. Give the 3M cart an initial velocity toward the cart at rest.

Case 3: Start the carts with one at each end of the track. Give each cart approximately the same velocity toward each other.

Case 4: Start both carts at one end of the track. Give the first cart a slow velocity and the second cart a faster velocity so that the second cart catches the first cart. Do this for both cases: with the 1M cart first and then for the 3M cart first.

Part II: Completely Inelastic Collisions:

③ Orient the two carts so their hook-and-pile ends are toward each other. Make sure the plunger bar is pushed in completely so it won't interfere with the collision.

④ Repeat the same procedures listed in **Part I** for carts with equal mass **and** carts with unequal mass.

Questions

① When two carts having the same mass and the same speed collide and stick together, they stop. What happened to each cart's momentum? Is momentum conserved?

② When two carts having the same mass and the same speed collide and bounce off of each other elastically, what is the final total momentum of the carts?

Experiment 3: Simple Harmonic Oscillator

EQUIPMENT NEEDED:

- Dynamics Cart with Mass (ME-9430)
- (2) Springs
- Mass hanger and mass set (ME-9348)
- String
- Graph paper
- Dynamics Cart Track
- Super Pulley with clamp
- Stopwatch
- Mass balance (SE-8723)

Purpose

The purpose is to measure the period of oscillation of a spring and mass system and compare it to the theoretical value.

Theory

For a mass attached to a spring, the theoretical period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where **T** is the time for one complete back-and-forth motion, **m** is the mass that is oscillating, and **k** is the spring constant.

According to Hooke's Law, the force exerted by the spring is proportional to the distance the spring is compressed or stretched, **F = kx**, where **k** is the proportionality constant. Thus the spring constant can be experimentally determined by applying different forces to stretch the spring different distances. Then the force is plotted versus distance and the slope of the resulting straight line is equal to **k**.

Procedure

Measurements to Find the Theoretical Period

① Use the balance to find the mass of the cart. Record this value at the top of Table 3.1.

② Level the track by setting the cart on the track to see which way it rolls. Adjust the leveling feet at the ends of the track to raise or lower the ends until the cart placed at rest on the track will not move. Put the pulley with the table clamp at one end of the track.

③ Set the cart on the track and attach a spring to each end of the cart by inserting the end of the spring in the hole provided in the cart. Then attach the other ends of the springs to the endstops (See Figure 3.1).

④ Attach a string to the end of the cart and hang a mass hanger over the pulley as shown.

⑤ Record the equilibrium position in Table 3.1.

⑥ Add mass to the mass hanger and record the new position. Repeat this for a total of 5 different masses, being careful not to over-stretch the springs. Because both springs are acting on the mass, this method will give the effective spring constant for both springs.

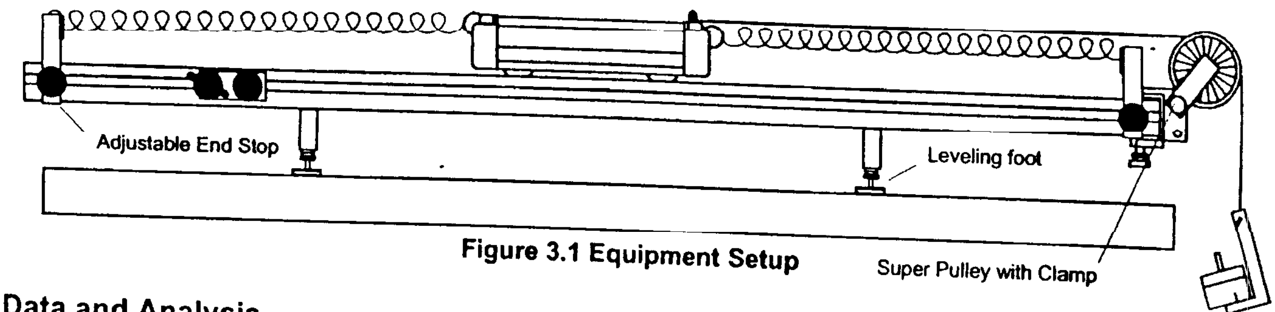


Figure 3.1 Equipment Setup

Data and Analysis

Table 3.1

Mass of cart =__________

Equilibrium position =__________

Added Mass	Position	Displacement from Equilibrium	Force (mg)

Measuring the Experimental Period

⑦ Displace the cart from equilibrium a specific distance and let it go. Time 5 oscillations and record the time in Table 3.2.

⑧ Repeat this measurement at least 5 times, using the same initial displacement (amplitude).

⑨ Add a 500 g mass to the cart. Measure the time for 5 oscillations 5 times and record this data in Table 3.2.

Calculations

Theoretical Period

① Using the data in Table 3.1, plot force versus displacement. Draw the best-fit straight line through the data points and determine the slope of the line. The slope is equal to the effective spring constant, **k**.

k = __________

② Using the mass of the cart and the spring constant, calculate the period using the theoretical formula. Also calculate the theoretical period for the cart with the 500 g mass in it.

(cart alone) T = __________

(cart with mass) T = __________

Experimental Period

① Using the data in Table 3.2, calculate the average time for 5 oscillations with and without the 500 g mass in the cart.

② Calculate the period by dividing these times by 5 and record the periods in Table 3.2.

Table 3.2

Trial	Time for 5 Oscillations	Period
1		
2		Without additional mass= ______
3		
4		
5		
Average		
1		
2		With additional mass= ______
3		
4		
5		
Average		

Comparison

Calculate the percent difference between the measured and theoretical values:

(cart alone) % diff = ______________

(cart with mass) % diff = ______________

Questions

① Does the period of oscillation increase or decrease as the mass is increased? Does a more massive cart oscillate faster or slower?

② If the initial displacement from equilibrium (amplitude) is changed, does the period of oscillation change? Try it.

Experiment 5: Springs in Series and Parallel

EQUIPMENT NEEDED:

– Dynamics Cart with Mass (ME-9430)
– Dynamics Cart Track with End stop
– (2) Springs
– Base and Support rod (ME-9355)
– Mass balance
– Stopwatch

Purpose

The purpose is to measure the period of oscillation of springs in series and parallel and compare it to the period of oscillation of one spring.

Theory

For a mass attached to a spring, the theoretical period of oscillation is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where **T** is the time for one complete back-and-forth motion, **m** is the mass that is oscillating, and **k** is the spring constant. If the period of oscillation is measured, the spring constant can be determined:

$$k = \frac{4\pi^2 m}{T^2}$$

When two springs are combined in series or in parallel, the spring constants add in different ways. One possible way to add two spring constants is $k_{effective} = k + k = 2k$. Another way is

$$k_{effective} = \frac{1}{k} + \frac{1}{k} = \frac{2}{k}$$

which means that

$$k_{effective} = \frac{1}{2}k$$

Procedure

Measuring k For a Single Spring

① Use the balance to find the mass of the cart. Record this value at the top of Table 5.1.

② Set the cart on the track and attach a spring to one end of the cart by inserting the end of the spring in the hole provided in the cart. Then attach the other end of the spring to the end of the track (See Figure 5.1).

➤ **NOTE:** Remove the leveling feet for this experiment.

③ Incline the track by raising the end of the track that has the spring attached. As the end of the track is raised the spring will stretch. Keep the angle of inclination of the track small enough so the spring is not stretched more than half the length of the track.

④ Displace the cart from equilibrium a specific distance and let it go. Time 2 oscillations and record the time in Table 5.1. Repeat this measurement at least 5 times, using the same initial displacement (amplitude).

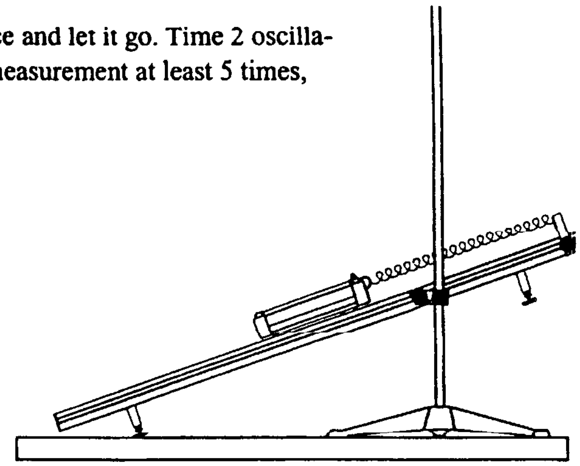


Figure 5.1 Equipment Setup

Measuring the Effective k For Pairs of Springs

⑤ Add a second spring in series as shown in Figure 5.2 and repeat Step ④.

⑥ Put the two springs in parallel as shown in Figure 5.3 and repeat Step ④.

⑦ Arrange the springs as shown in Figure 5.4 and repeat Step ④.

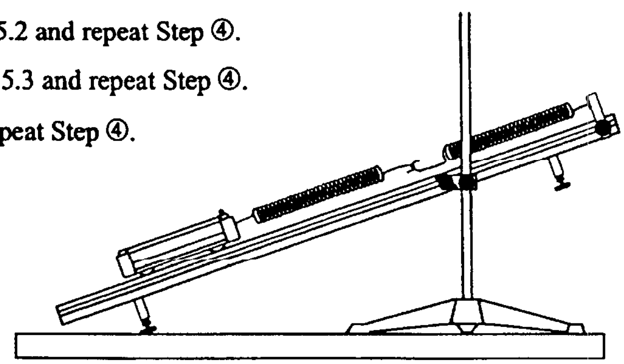


Figure 5.2 Springs in Series

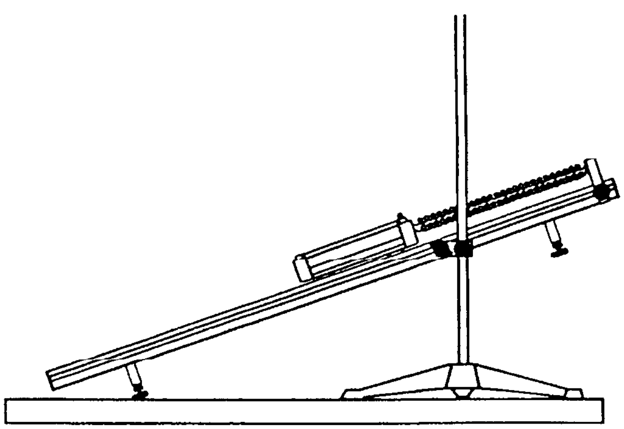


Figure 5.3 Springs in Parallel

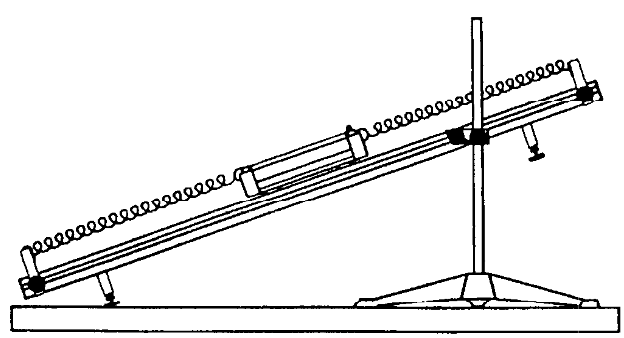


Figure 5.4 Final Spring Arrangement

Calculations

① Using the data in Table 5.1, calculate the average time for 2 oscillations.

② Calculate the period by dividing these times by 2 and record the periods in Table 5.1.

③ Using the periods and the mass of the cart, calculate the effective spring constants.

Table 5.1

Mass of Cart = ____________

Time for 2 oscillations

Springs	Trial 1	2	3	4	5	Avg	Period	k
One								
Series								
Parallel								
At Ends								

Questions

① Is $k_{effective} = 2k$ for springs in series or parallel?

② Is $k_{effective} = \frac{1}{2} k$ for springs in series or parallel?

③ Is the last spring arrangement series or parallel?

Experiment 6: Newton's Second Law

EQUIPMENT NEEDED:

- Dynamics Cart with Mass (ME-9430)
- Dynamics Cart Track
- Stopwatch

Purpose

The purpose is to show how the acceleration of an object is dependent on force and mass.

Procedure

① Level the track by setting the cart on the track to see which way it rolls. Adjust the leveling feet to raise or lower the ends until the cart placed at rest on the track will not move.

② To perform each of the following trials, cock the spring plunger on the cart and place the cart at rest at the end of the track with the plunger against the end stop. Then release the plunger by pressing the button on the cart with a ruler. Observe the resulting acceleration. This will be a qualitative measurement.

VARY THE FORCE: Perform the first trial with the spring plunger cocked to the first possible position (the least compression) and then do two more trials increasing the force applied to the cart by increasing the compression of the spring plunger.

VARY THE MASS: For these trials, always cock the spring plunger to the maximum. Observe the relative accelerations of the cart alone and the cart with one mass bar in it. If additional masses are available, use them to increase the mass for additional trials.

Data Analysis

① Does the acceleration increase or decrease as the force is increased?

② Does the acceleration increase or decrease as the mass is increased?

Question

From the results of this experiment, can you deduce the equation that relates acceleration to mass and force?

Experiment 7: Newton's Second Law II

EQUIPMENT NEEDED:

- Dynamics Cart (ME-9430)
- Super Pulley with Clamp
- String
- Stopwatch
- Mass balance
- Dynamics Cart Track
- Base and Support rod (ME-9355)
- Mass hanger and mass set
- Wooden or metal stopping block (See Procedure Step ③)

Purpose

The purpose is to verify Newton's Second Law, **F** = **ma**.

Theory

According to Newton's Second Law, **F** = **ma**. **F** is the net force acting on the object of mass **m** and **a** is the resulting acceleration of the object.

For a cart of mass m_1 on a horizontal track with a string attached over a pulley to a mass m_2 (see Figure 7.1), the net force **F** on the entire system (cart and hanging mass) is the weight of hanging mass, $F = m_2g$, assuming that friction is negligible.

According to Newton's Second Law, this net force should be equal to **ma**, where **m** is the total mass that is being accelerated, which in this case is $m_1 + m_2$. This experiment will check to see if m_2g is equal to $(m_1 + m_2)a$ when friction is ignored.

To obtain the acceleration, the cart will be started from rest and the time (**t**) it takes for it to travel a certain distance (**d**) will be measured. Then since $d = (½)at^2$, the acceleration can be calculated using

$$a = \frac{2d}{t^2} \qquad \text{(assuming a = constant)}$$

Procedure

① Level the track by setting the cart on the track to see which way it rolls. Adjust the leveling feet to raise or lower the ends until the cart placed at rest on the track will not move.

② Use the balance to find the mass of the cart and record in Table 7.1.

③ Attach the pulley to the end of the track as shown in Figure 7.1. Place the dynamics cart on the track and attach a string to the hole in the end of the cart and tie a mass hanger on the other end of the string. The string must be just long enough so the cart hits the stopping block before the mass hanger reaches the floor.

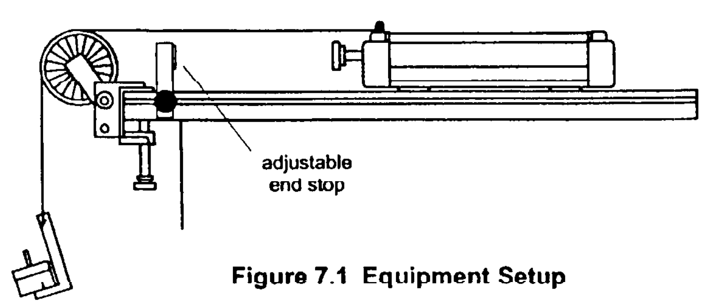


Figure 7.1 Equipment Setup

④ Pull the cart back until the mass hanger reaches the pulley. Record this position at the top of Table 7.1. This will be the release position for all the trials. Make a test run to determine how much mass is required on the mass hanger so that the cart takes about 2 seconds to complete the run. Because of reaction time, too short of a total time will cause too much error. However, if the cart moves too slowly, friction causes too much error. Record the hanging mass in Table 7.1.

⑤ Place the cart against the adjustable end stop on the pulley end of the track and record the final position of the cart in Table 7.1.

⑥ Measure the time at least 5 times and record these values in Table 7.1.

Table 7.1

Time

Cart Mass	Hanging Mass	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average Time

Initial release Position = __________

Final Position = __________

Total distance (d) = __________

⑦ Increase the mass of the cart and repeat the procedure.

Data Analysis

① Calculate the average times and record in Table 7.1.

② Calculate the total distance traveled by taking the difference between the initial and final positions of the cart as given in Table 7.1.

③ Calculate the accelerations and record in Table 7.2.

④ For each case, calculate the total mass multiplied by the acceleration and record in Table 7.2.

⑤ For each case, calculate the net force acting on the system and record in Table 7.2.

⑥ Calculate the percent difference between $\mathbf{F_{NET}}$ and $\mathbf{(m_1+m_2)a}$ and record in Table 7.2.

Table 7.2

Cart Mass	Acceleration	$(m_1+m_2)a$	$F_{NET} = m_2 g$	% Diff

Questions

① Did the results of this experiment verify that $\mathbf{F = ma}$?

② Considering frictional forces, which force would you expect to be greater: the hanging weight or the resulting total mass times acceleration? Did the results of this experiment consistently show that one was larger than the other?

③ Why is the mass in $\mathbf{F = ma}$ not just equal to the mass of the cart?

④ When calculating the force on the cart using mass times gravity, why isn't the mass of cart included?

Experiment 9: Conservation of Energy

EQUIPMENT NEEDED:

- Dynamics Cart with Mass (ME-9430)
- Super Pulley with Clamp
- Base and Support rod (ME-9355)
- String
- Mass balance
- Dynamics Cart Track
- Meter stick
- Mass hanger and mass set (several kilograms)
- Graph paper

Purpose

The purpose is to examine spring potential energy and gravitational potential energy and to show how energy is conserved.

Theory

The potential energy of a spring compressed a distance **x** from equilibrium is given by **PE = (½)kx²**, where **k** is the spring constant. According to Hooke's Law, the force exerted by the spring is proportional to the distance the spring is compressed or stretched, **F = kx**, where **k** is the proportionality constant. Thus the spring constant can be experimentally determined by applying different forces to stretch or compress the spring different distances. When the force is plotted versus distance, the slope of the resulting straight line is equal to **k**.

The gravitational potential energy gained by a cart as it climbs an incline is given by **potential energy = mgh**, where **m** is the mass of the cart, **g** is the acceleration due to gravity, and **h** is the vertical height the cart is raised. In terms of the distance, **d**, along the incline, the height is given by **h = d sinθ**.

If energy is conserved, the potential energy in the compressed spring will be completely converted into gravitational potential energy.

Procedure

① Level the track by setting the cart on the track to see which way it rolls. Adjust the leveling feet to raise or lower the ends until the cart placed at rest on the track will not move.

② Use the balance to find the mass of the cart. Record this value in Table 9.2.

Determining the Spring Constant

③ Set the cart on the track with the spring plunger against the stopping block as shown in Figure 9.1. Attach a string to the cart and attach the other end to a mass hanger, passing the string over the pulley.

④ Record the cart's position in Table 9.1.

⑤ Add mass to the mass hanger and record the new position. Repeat this for a total of 5 different masses.

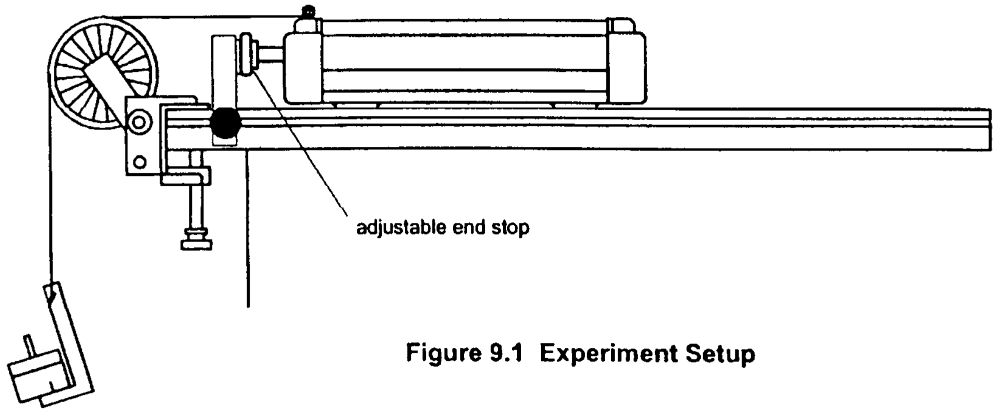


Figure 9.1 Experiment Setup

Table 9.1

Added Mass	Position	Displacement from Equilibrium	Force (mg)

Potential Energy

⑥ Remove the leveling feet.

⑦ Remove the string from the cart and cock the spring plunger to its maximum compression position. Place the cart against the end stop. Measure the distance the spring plunger is compressed and record this value in Table 9.2.

⑧ Incline the track and measure its height and hypotenuse (see Figure 9.2) to determine the angle of the track.

$$\text{angle} = \arc\sin\left(\frac{\text{height}}{\text{hypotenuse}}\right)$$

Record the angle in Table 9.2.

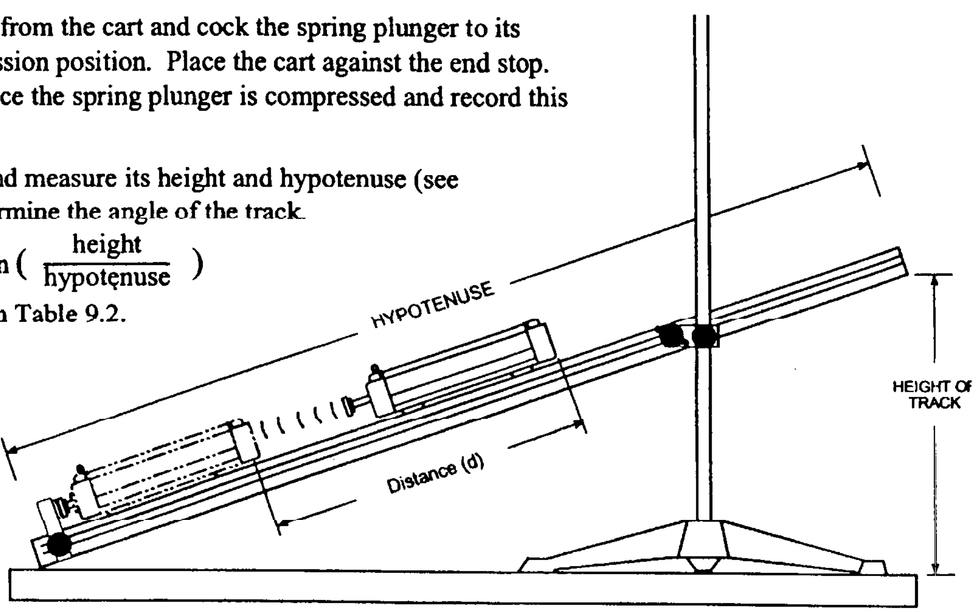


Figure 9.2

⑨ Record the initial position of the cart in Table 9.2.

⑩ Release the plunger by tapping it with a stick and record the distance the cart goes up the track. Repeat this five times. Record the maximum distance the cart went in Table 9.2.

⑪ Change the angle of inclination and repeat the measurements.

⑫ Add mass to the cart and repeat the measurements.

Table 9.2

Distance traveled by the cart (d)

Angle	Mass	Trial 1	2	3	4	5	Max	$h = d \sin\theta$

Distance spring is compressed (x) = ____________

Initial position of cart = ____________

Data Analysis

① Using the data in Table 9.1, plot force versus displacement. Draw the best-fit straight line through the data points and determine the slope of the line. The slope is equal to the effective spring constant, k.

k = ____________

② Calculate the spring potential energy and record in Table 9.3.

③ Calculate the gravitational potential energy for each case and record in Table 9.3.

④ Calculate the percent difference between the spring potential energy and the gravitational potential energy.

Table 9.3

Angle/Mass	Spring PE ($\frac{1}{2}kx^2$)	Gravitational PE (mgh)	% Difference

Questions

① Which of the potential energies was larger? Where did this "lost" energy go?

② When the mass of the cart was doubled, why did the gravitational potential energy remain about the same?

Teacher's Guide

Experiment 1: Conservation of Energy in Explosions

Notes on Data Analysis

M1	M2	Position	X1	X2	X1/X2	M2/M1
497.5	500.7	181.0	42.0	41.5	1.01	1.01
497.5	996.4	195.0	56.0	27.5	2.04	2.00
497.5	1494.9	201.5	62.5	21.0	2.98	3.00
995.7	1494.9	189.0	50.0	33.5	1.49	1.50

Answers to Questions

① Momentum is conserved in each case.

② As shown in this lab, the momentum of each cart is the same.

③ $KE_2 = \frac{m_1}{m_2} KE_1$

The lighter cart will have a higher kinetic energy.

④ The starting position does not depend on which cart has the plunger cocked. During the "explosion", the momentum of the carts will be affected by the fact that the plunger is moving at a different velocity than either cart. However, since each plunger eventually ends up moving at the same speed as the cart it is on, there is no difference once the carts are separated.

Experiment 2: Conservation of Momentum in Collisions

> ➤ **NOTE:** Without some method of actually measuring the velocities of the carts, this lab should be used for qualitative analysis only.

Part I

a. Since the carts have the same mass, they will exchange velocity in each case.

b. The momentum transfer will be proportional to the ratio of the cart masses.

Questions

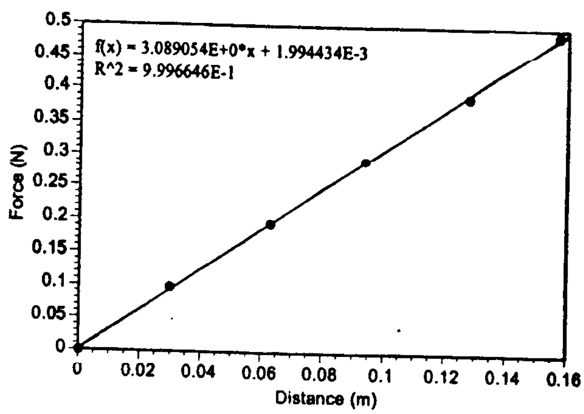
① Each cart loses its momentum. The *total* momentum is unchanged, because the total momentum is zero both before and after the collision.

② The total momentum in this case is still zero both before and after the collision.

Experiment 3: Simple Harmonic Oscillator

Notes on Procedure

⑥ For best results, make sure that the springs are neither over-stretched nor hanging loose. For these tests, we used 10-50g masses only.



Notes on Calculations

① The spring constant k = 3.089 N/m for the springs used here. This value will vary from spring to spring.

② Theoretical values will vary, depending on the value for k and for m. For best results, measure the carts rather than assume their weight to be the stated 500g.

Notes on Comparison

The percent difference between experimental and theoretical values should be less than 2%, and it is not unusual to obtain errors of less than 0.5%.

Notes on Questions

① The period of oscillation increases with mass. The more massive cart oscillates slower.

② The period is not changed, as long as the initial displacement does not exceed the linear region of the spring. You will notice a slight difference if the displacement is enough to permanently deform the spring.

Experiment 5: Springs in Series and Parallel

Notes

Keep the angle of the track low, especially if you are using a short (1.2m) track. Otherwise, the carts will go off the end of the track when the springs are in series.

Notes on Calculations

The two springs used for this experiment had spring constants of 1.53 and 1.60.

- In series, the spring constant was 0.76. (k/2)
- In parallel, the spring constant was 3.12 (2k)
- The spring constant was 3.06 (2k) when the springs were attached to the ends of the cart.

Notes on Questions

① The effective spring constant is 2k for springs in parallel.

② The effective spring constant is k/2 for springs in series.

③ The springs are effectively in parallel when they are attached to opposite ends of the cart.

Experiment 6: Newton's Second Law

This lab is intended to be a qualitative lab only. For a quantitative analysis of Newton's second law, see experiment 7.

Notes on Data Analysis

① Acceleration increases with force

② Acceleration decreases with mass.

Notes on Questions

$F = ma$

Experiment 7: Newton's Second Law II

If the mass used to accelerate the cart is too low, friction will be a very significant source of error. If it is too high, then the time will be short and accurate measurement will be difficult. It would be best for this lab to use a photogate timing system, such as the PASCO ME-9215.

Notes on Questions

① The results of this experiment generally show that F = ma. Errors can be high, due to friction and timing inaccuracy.

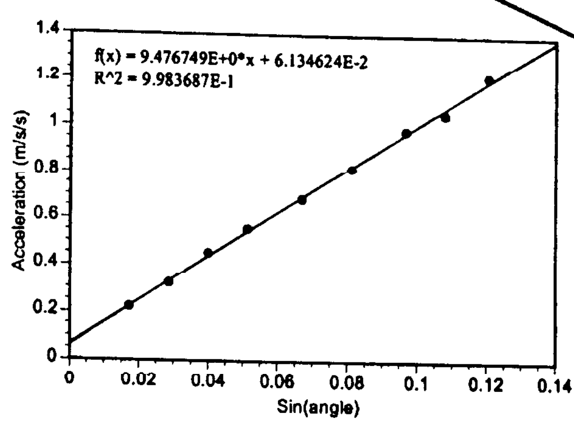
② The force of the hanging weight is larger than the total mass times acceleration. The difference between the two is the force of friction.

③ The hanging mass is accelerating at the same rate as the cart, so its mass must be considered as well as that of the cart.

④ The cart is on a level track, so it is not accelerated by gravity.

Experiment 8: Acceleration Down an Incline

Data Analysis



The value of the slope will be slightly lower than 9.8, due to friction. (Our value 3.3% low.)

Notes on Questions

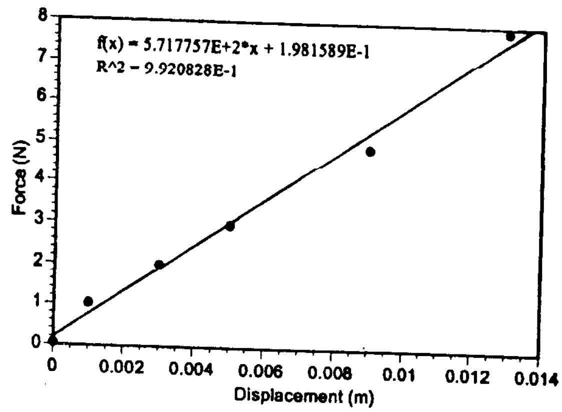
① Assuming that reaction time relatively constant, the percent error due to reaction time would be greater for shorter times and higher angles.

② Changing the mass of the cart will affect the results slightly due to changing frictional characteristics.

Experiment 9: Conservation of Energy

Analysis

①



②-④

k = 572 Spring PE = 0.193336

Angle	Mass	dmax (cm)	h (m)	mgh	%diff
14.57	0.4971	15.1	0.0380	0.1851	-4.28%
11.07	0.4971	19.5	0.0374	0.1824	-5.66%
11.07	0.9926	10.1	0.0194	0.1886	-2.43%
3.026	0.9926	39.0	0.0206	0.2003	3.58%
3.026	0.4971	75.1	0.0396	0.1931	-0.11%

Notes on Questions

① The initial spring potential energy is larger. (Generally. There are experimental errors, which can make the gravitational energy appear larger than the initial spring potential.) The "lost" energy goes into friction.

② Why not? The increased mass will mean that the cart does not travel as high, but the final gravitational potential energy will be the same.

Technical Support

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- If your problem is with the PASCO apparatus, note:

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  Approximate age of apparatus.

  A detailed description of the problem/sequence of events. (In case you can't call PASCO right away, you won't lose valuable data.)

  If possible, have the apparatus within reach when calling. This makes descriptions of individual parts much easier.

- If your problem relates to the instruction manual, note:

  Part number and Revision (listed by month and year on the front cover).

Have the manual at hand to discuss your questions.

Appendix 3:

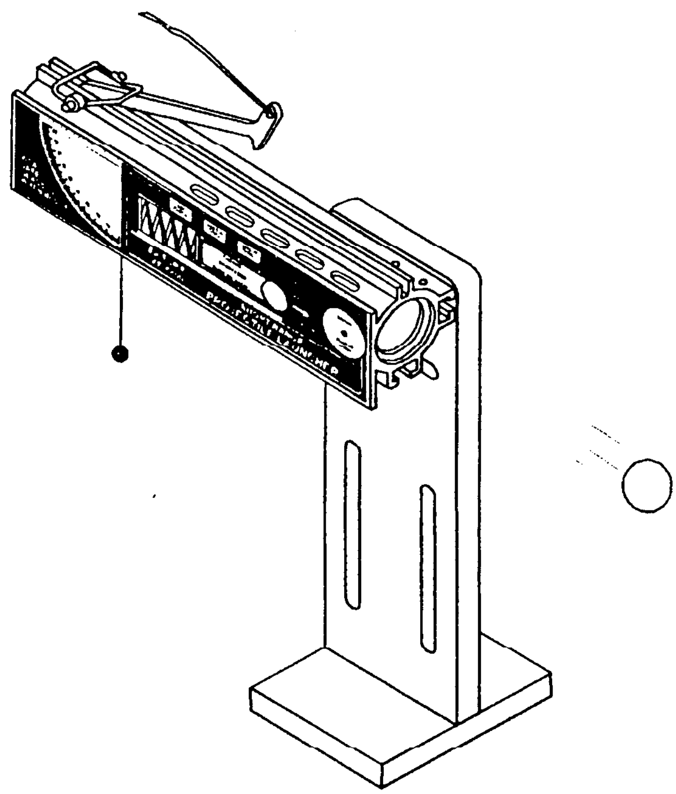
Projectile Launcher Short/Long Version

Includes Teacher's Notes and Typical Experiment Results

Instruction Manual and Experiment Guide for the PASCO scientific Model ME-6800, 6801

012-05043E
3/95

Projectile Launcher Short / Long Version



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This manual authored by: Ann Hanks

This manual edited by: Jon Hanks

Teacher's guide written by: Eric Ayars

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① The packing carton must be strong enough for the item shipped.

② Make certain there are at least two inches of packing material between any point on the apparatus and the inside walls of the carton.

③ Make certain that the packing material cannot shift in the box or become compressed, allowing the instrument come in contact with the packing carton.

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Introduction

The PASCO Projectile Launcher has been designed for projectile experiments and demonstrations. The only additional equipment required is a C-clamp for clamping the Launcher to a table. The features of the Projectile Launcher include:

- LAUNCH AT ANY ANGLE:

  Balls can be launched at any angle from zero to 90 degrees measured from the horizontal. The angle is easily adjusted using thumb screws. The built-in protractor and plumb-bob on the side of the launcher give a convenient and accurate way of determining the angle of inclination.

- THREE RANGE SETTINGS:

  There are three ranges from which to choose. For the Short Range Projectile Launcher these three ranges are approximately 1.2 meters, 3 meters, and 5 meters, when the angle is 45 degrees. For the Long Range Demonstration Projectile Launcher, the three ranges are approximately 2.5 meters, 5 meters, and 9 meters. The difference between these two versions of the Projectile Launcher is the strength of the spring. The Long Range version is intended for large classroom demonstrations.

- FIXED ELEVATION INDEPENDENT OF ANGLE:

  The Projectile Launcher pivots at the muzzle end so the elevation of the ball as it leaves the barrel does not change as the angle is varied. The base has two sets of slots: The top curved slot is used when it is desired to change the angle and the bottom two slots are used when it is desired to shoot horizontally only, such as into a pendulum or a Dynamics Cart.

- REPEATABLE RESULTS:

  There is no spin on the ball since the piston keeps the ball from rubbing on the walls as it travels up the barrel. The sturdy base can be secured to a table with a C- clamp (not included) so there is very little recoil. The trigger is pulled with a string to minimize the jerking.

- BARREL SIGHTS AND SAFETY PRECAUTIONS:

  There are sights for aiming the Projectile Launcher. These sights can be viewed from the back of the Projectile Launcher by looking through the back end of the barrel.

> ➤ **WARNING:** Never look down the front of the barrel because it may be loaded. To check to see if the ball is in the barrel and whether the Projectile Launcher is cocked, look at the slots in the side of the barrel. Safety goggles are provided. The yellow indicator seen through the side slot indicates the position of the piston and the ball can also be seen through these slots when it is in the piston.

- COMPUTER COMPATIBLE: Photogates may be attached with the accessory bracket (ME-6821) to connect the Projectile Launcher to a computer to measure the muzzle speed. Also a photogate at the muzzle and the Time of Flight accessory (ME-6810) can be used to time the flight of the ball.

- COMPACT STORAGE: The Projectile Launcher stores away in a small space. The ramrod attaches to the Projectile Launcher with Velcro® and the Projectile Launcher can be aligned with the base so it takes up the minimum amount of space on the shelf.

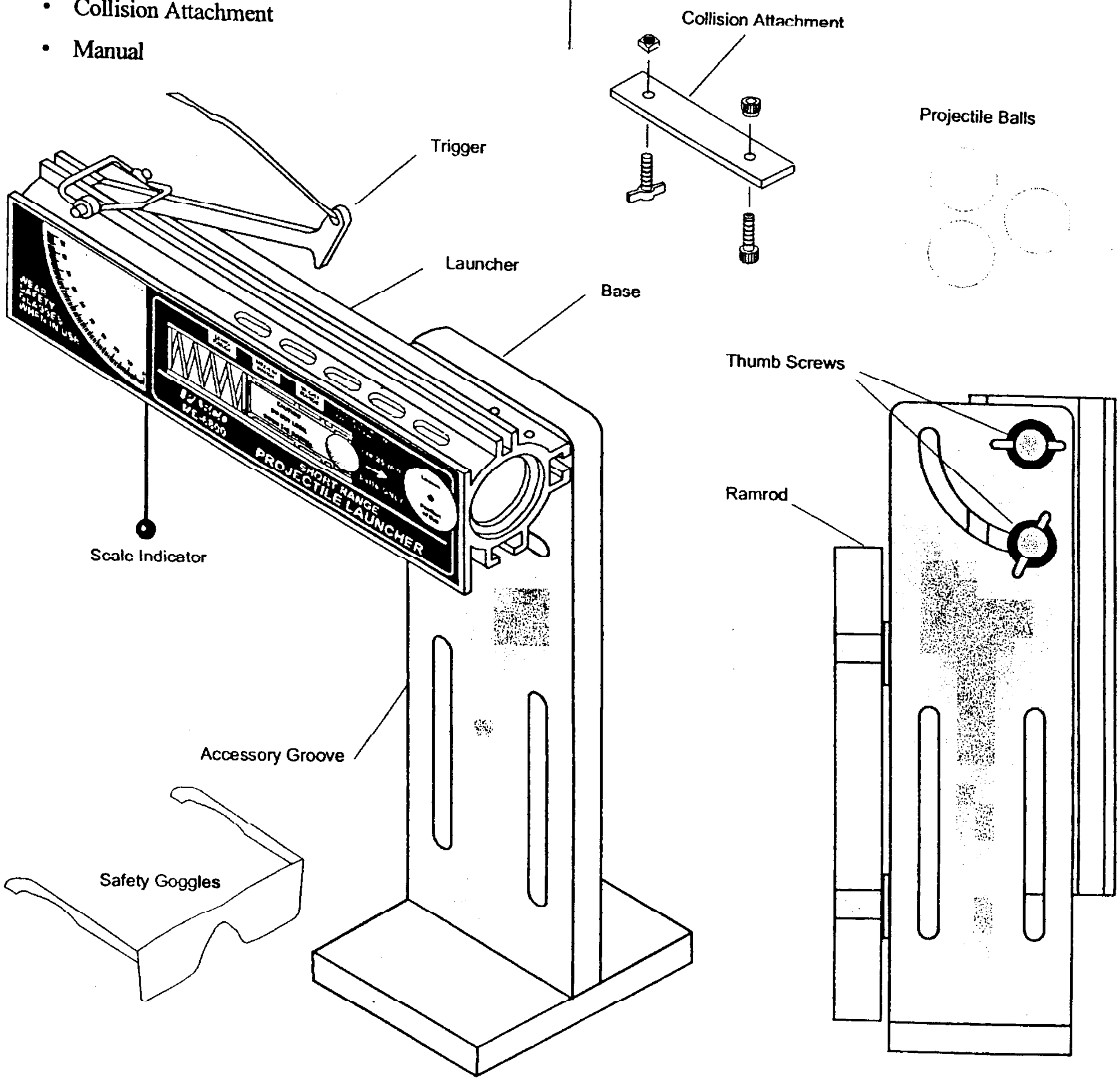
Equipment

The following is a description of the equipment that is included with various models of the Projectile Launcher.

The ME-6800 (Short Range) Projectile Launcher Student/Demo Version includes the following:

- Launcher and Base (Assembled)
- (3) Plastic Balls
- Ramrod (Attached with Velcro® to stand)
- (2) Safety Goggles
- Collision Attachment
- Manual

The ME-6801 (Long Range) Projectile Launcher Student/ Demo Version includes the same items as the ME-6800 but is capable of significantly greater projectile range.



General Operation of the Projectile Launcher

① Ready

- Always wear safety goggles when you are in a room where the Projectile Launcher is being used.
- The base of the Projectile Launcher must be clamped to a sturdy table using the clamp of your choice. When clamping to the table, it is desirable to have the label side of the Launcher even with one edge of the table so a plumb bob can be used to locate the position of the muzzle with respect to the floor.
- The Projectile Launcher can be mounted to the bracket using the curved slot when it is desired to change the launch angle. It can also be mounted to the lower two slots in the base if you are only going to shoot horizontally, such as into a pendulum or a Dynamics Cart.

② Aim

- The angle of inclination above the horizontal is adjusted by loosening both thumb screws and rotating the Launcher to the desired angle as indicated by the plumb bob and protractor on the side of the Launcher. When the angle has been selected, both thumb screws are tightened.
- You can bore-sight at a target (such as in the Monkey-Hunter demonstration) by looking through the Launcher from the back end when the Launcher is not loaded. There are two sights inside the barrel. Align the centers of both sights with the target by adjusting the angle and position of the Launcher.

③ Load

- Always cock the piston with the ball in the piston. Damage to the piston may occur if the ramrod is used without the ball.
- Place the ball in the piston. Remove the ramrod from its Velcro® storage place on the base. While viewing the range-setting slots in the side of the Launcher, push the ball down the barrel with the ramrod until the trigger catches the piston at the desired range setting.
- Remove the ramrod and place it back in its storage place on the base.
- When the Projectile Launcher is loaded, the yellow indicator is visible in one of the range slots in the side of the barrel and the ball is visible in another one of the slots in the side of the barrel. To check to see if the Launcher is loaded, always check the side of the barrel. Never look down the barrel!

④ Shoot

- Before shooting the ball, make certain that no person is in the way.
- To shoot the ball, pull straight up on the lanyard (string) that is attached to the trigger. It is only necessary to pull it about a centimeter.
- The spring on the trigger will automatically return the trigger to its initial position when you release it.

⑤ Maintenance and Storage

- No special maintenance of the Projectile Launcher is required.
- Do not oil the Launcher!!
- To store the Launcher in the least amount of space, align the barrel with the base by adjusting the angle to 90 degrees. If the photogate bracket and photogates are attached to the Launcher, the bracket can be slid back along the barrel with the photogates still attached.

Installing the Optional Photogate Bracket (ME-6821)

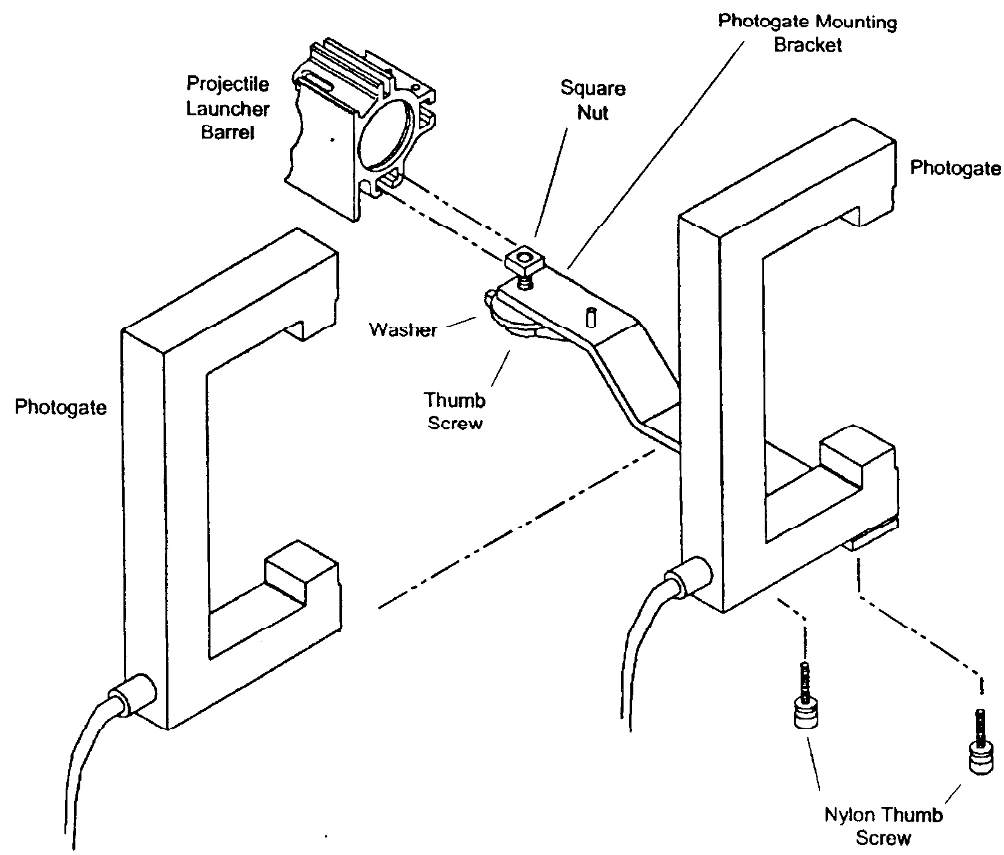
The Photogate Bracket is an optional accessory for mounting one or two photogates on the Projectile Launcher to measure the muzzle velocity of the ball.

Installation is as follows:

① Prepare the bracket by inserting the thumb screw through the hole in the bracket near the end that has the post (see diagram for orientation) and start the square nut onto the end of the thumb screw. Attach the photogates to the bracket using the remaining holes in the bracket and the screws provided with the photogates.

② To mount the bracket to the Launcher, align the square nut in the slot on the bottom of the barrel and slide the nut and the post into the slot. Slide the bracket back until the photogate nearest to the barrel is as close to the barrel as possible without blocking the beam. Tighten the thumb screw to secure the bracket in place.

③ When storing the Projectile Launcher, the photogate bracket need not be removed. It can be slid back along the barrel with or without the photogates in place, making as compact a package as possible.



Installing the 2-Dimensional Collision Attachment

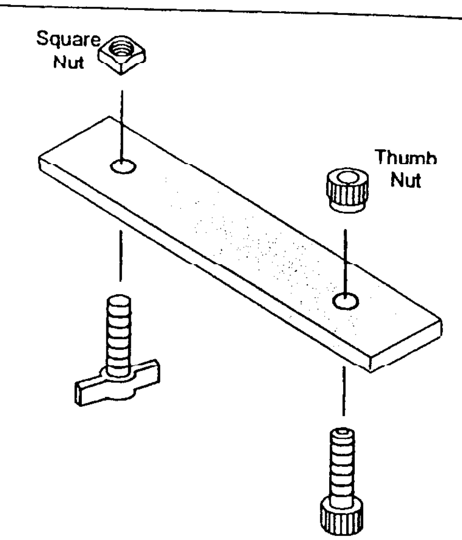
Introduction

The two dimensional Collision Attachment consists of 2 screws, 2 nuts, and a flat plastic bar. It is used with the Projectile Launcher to hold a second ball in front of the muzzle so the launched ball will collide with the second ball, creating a 2-dimensional collision.

Assembly

To assemble the collision attachment, insert the screws through the holes and secure with the nuts as shown below.

To mount the collision attachment to the Launcher the square nut slides into the T-shaped channel on the bottom of the barrel. (See Figure 6.2 on page 28)



Expectations for the Projectile Launcher

The following are helpful hints and approximate values you may find useful:

① The muzzle speed will vary slightly with angle. The difference between muzzle speed when shot horizontally versus vertically can be anywhere from zero to 8%, depending on the range setting and the particular launcher.

② Although the muzzle end of the Projectile Launcher doesn't change height with angle, it is about 30 cm (12 inches) above table level, so if it is desired to use the simple range formula, it is necessary to shoot to a table that is at the same height as the muzzle.

③ The scatter pattern is minimized when the Projectile Launcher base is securely clamped to a sturdy table. Any wobble in the table will show up in the data.

④ The angle of inclination can be determined to within one- half of a degree.

Experiment 1: Projectile Motion

EQUIPMENT NEEDED:

-Projectile Launcher and plastic ball
-Plumb bob
-Meter stick
-Carbon paper
-White paper

Purpose

The purpose of this experiment is to predict and verify the range of a ball launched at an angle. The initial velocity of the ball is determined by shooting it horizontally and measuring the range and the height of the Launcher.

Theory

To predict where a ball will land on the floor when it is shot off a table at some angle above the horizontal, it is necessary to first determine the initial speed (muzzle velocity) of the ball. This can be determined by shooting the ball horizontally off the table and measuring the vertical and horizontal distances through which the ball travels. Then the initial velocity can be used to calculate where the ball will land when the ball is shot at an angle.

HORIZONTAL INITIAL VELOCITY:

For a ball shot horizontally off a table with an initial speed, v_o, the horizontal distance travelled by the ball is given by $x = v_o t$, where t is the time the ball is in the air. Air friction is assumed to be negligible.

The vertical distance the ball drops in time t is given by $y = \frac{1}{2}gt^2$.

The initial velocity of the ball can be determined by measuring x and y. The time of flight of the ball can be found using:

$$t = \sqrt{\frac{2y}{g}}$$

and then the initial velocity can be found using $v_0 = \frac{x}{t}$.

INITIAL VELOCITY AT AN ANGLE:

To predict the range, **x**, of a ball shot off with an initial velocity at an angle, θ, above the horizontal, first predict the time of flight using the equation for the vertical motion:

$$y = y_0 + (v_0 \sin\theta)\, t - \frac{1}{2}gt^2$$

where y_o is the initial height of the ball and **y** is the position of the ball when it hits the floor. Then use $x = (v_0 \cos\theta)\, t$ to find the range.

Setup

① Clamp the Projectile Launcher to a sturdy table near one end of the table.

② Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

Procedure

Part A: Determining the Initial Velocity of the Ball

① Put the plastic ball into the Projectile Launcher and cock it to the long range position. Fire one shot to locate where the ball hits the floor. At this position, tape a piece of white paper to the floor. Place a piece of carbon paper (carbon-side down) on top of this paper and tape it down. When the ball hits the floor, it will leave a mark on the white paper.

② Fire about ten shots.

③ Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance in Table 1.1.

④ Use a plumb bob to find the point on the floor that is directly beneath the release point on the barrel. Measure the horizontal distance along the floor from the release point to the leading edge of the paper. Record in Table 1.1.

⑤ Measure from the leading edge of the paper to each of the ten dots and record these distances in Table 1.1.

⑥ Find the average of the ten distances and record in Table 1.1.

⑦ Using the vertical distance and the average horizontal distance, calculate the time of flight and the initial velocity of the ball. Record in Table 1.1.

Part B: Predicting the Range of the Ball Shot at an Angle

① Adjust the angle of the Projectile Launcher to an angle between 30 and 60 degrees and record this angle in Table 1.2.

② Using the initial velocity and vertical distance found in the first part of this experiment, assume the ball is shot off at the new angle you have just selected and calculate the new time of flight and the new horizontal distance. Record in Table 1.2.

③ Draw a line across the middle of a white piece of paper and tape the paper on the floor so the line is at the predicted horizontal distance from the Projectile Launcher. Cover the paper with carbon paper.

④ Shoot the ball ten times.

⑤ Measure the ten distances and take the average. Record in Table 1.2.

Analysis

① Calculate the percent difference between the predicted value and the resulting average distance when shot at an angle.

② Estimate the precision of the predicted range. How many of the final 10 shots landed within this range?

Table 1.1 Determining the Initial Velocity

Vertical distance = ____________ Horizontal distance to paper edge = ____________

Calculated time of flight = ________ Initial velocity = ______________

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Table 1.2 Confirming the Predicted Range

Angle above horizontal = ____________ Horizontal distance to paper edge = ____________

Calculated time of flight = ____________ Predicted Range = ____________

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

Experiment 4: Projectile Path

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-carbon paper
-movable vertical target board (Must reach from floor to muzzle)
- graph paper
-measuring tape or meter stick
-white paper

Purpose

The purpose of this experiment is to find how the vertical distance the ball drops is related to the horizontal distance the ball travels when the ball is launched horizontally from a table.

Theory

The range is the horizontal distance, x, between the muzzle of the Launcher and the place where the ball hits, given by $x = v_0 t$, where vo is the initial speed of the ball as it leaves the muzzle and t is the time of flight.

If the ball is shot horizontally, the time of flight of the ball will be

$$t = \frac{x}{v_0}$$

The vertical distance, y, that the ball falls in time t is given by

$$y = \frac{1}{2} g t^2$$

where g is the acceleration due to gravity.

Substituting for t into the equation for y gives

$$y = \left(\frac{g}{2{v_0}^2}\right) x^2$$

A plot of y versus x^2 will give a straight line with a slope equal to $\frac{g}{2{v_0}^2}$.

Setup

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed away from the table.

② Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

③ Fire a test shot on medium range to determine the initial position of the vertical target. Place the target so the ball hits it near the bottom. See Figure 4.1.

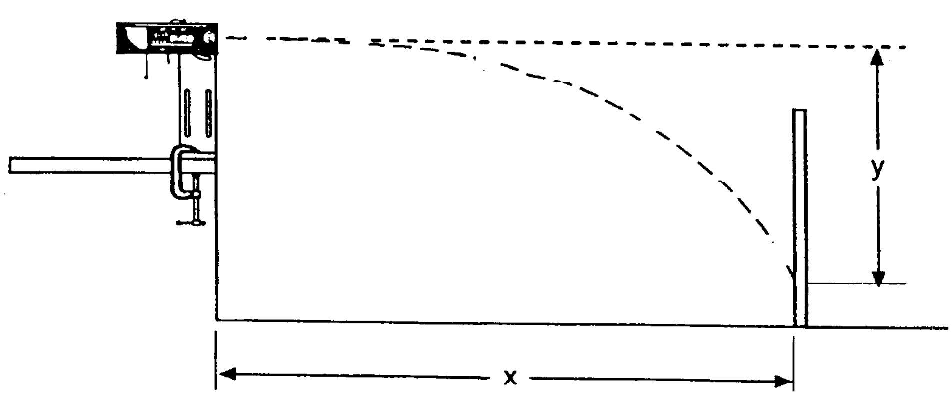


Figure 4.1 Set Up

④ Cover the target board with white paper. Tape carbon paper over the white paper.

Procedure

① Measure the vertical height from the floor to the muzzle and record in Table 4.1. Mark this height on the target.

② Measure the horizontal distance from the muzzle of the Projectile Launcher to the target and record in Table 4.1.

③ Shoot the ball.

④ Move the target about 10 to 20 cm closer to the Launcher.

⑤ Repeat Steps 2 through 4 until the height of the ball when it strikes the target is about 10 to
20 cm below the height of the muzzle.

Table 4.1 Data

Height of Muzzle = ___________

Horizontal (x)	Height (y)	x^2

Analysis

① On the target, measure the vertical distances from the muzzle level mark down to the ball marks and record in Table 4.1.

② Calculate x^2 for all the data points and record in Table 4.1.

③ Plot y vs. x^2 and draw the best-fit straight line.

④ Calculate the slope of the graph and record in Table 4.2.

⑤ From the slope of the graph, calculate the initial speed of the ball as it leaves the muzzle and record in Table 4.2.

⑥ Using any data point for x and y, calculate the time using y and then calculate the initial speed using this time and x. Record the results in Table 4.2.

⑦ Calculate the percent difference between the initial speeds found using these two methods. Record in Table 4.2.

Table 4.2 Initial Speed

Slope of graph	
Initial speed from slope	
Time of flight	
Initial speed from x, y	
Percent Difference	

Questions

① Was the line straight? What does this tell you about the relationship between y and x?

② If you plotted y vs. x, how would the graph differ from the y vs. x^2 graph?

③ What shape is the path of a projectile?

Experiment 5: Conservation of Energy

EQUIPMENT NEEDED

-Projectile Launcher and plastic ball
-measuring tape or meter stick
-(optional) 2 Photogates and Photogate Bracket
-plumb bob
-white paper
-carbon paper

Purpose

The purpose of this experiment is to show that the kinetic energy of a ball shot straight up is transformed into potential energy.

Theory

The total mechanical energy of a ball is the sum of its potential energy (PE) and its kinetic energy (KE). In the absence of friction, total energy is conserved. When a ball is shot straight up, the initial PE is defined to be zero and the KE = $(1/2)mv_0^2$, where m is the mass of the ball and vo is the muzzle speed of the ball. See Figure 5.1. When the ball reaches its maximum height, h, the final KE is zero and the PE = mgh, where g is the acceleration due to gravity. Conservation of energy gives that the initial KE is equal to the final PE.

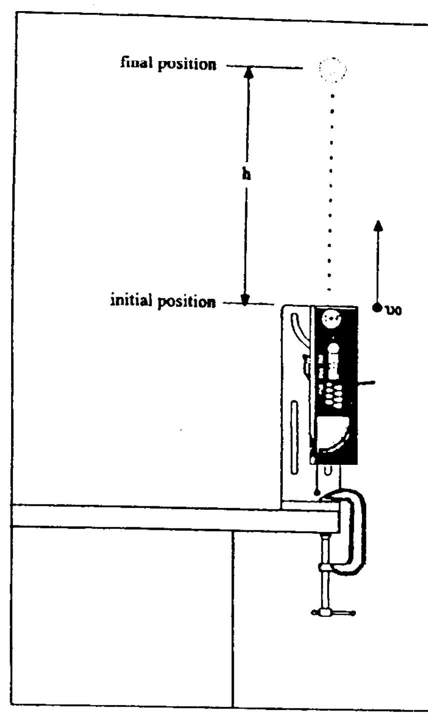


Figure 5.1 Conservation of Energy

To calculate the kinetic energy, the initial velocity must be determined. To calculate the initial velocity, vo, for a ball shot horizontally off a table, the horizontal distance travelled by the ball is given by $x = v_0 t$, where t is the time the ball is in the air. Air friction is assumed to be negligible. See Figure 5.2.

The vertical distance the ball drops in time t is given by $y = (1/2)gt^2$.

The initial velocity of the ball can be determined by measuring x and y. The time of flight of the ball can be found using

$$t = \sqrt{\frac{2y}{g}}$$

and then the initial velocity can be found using $v_0 = x/t$.

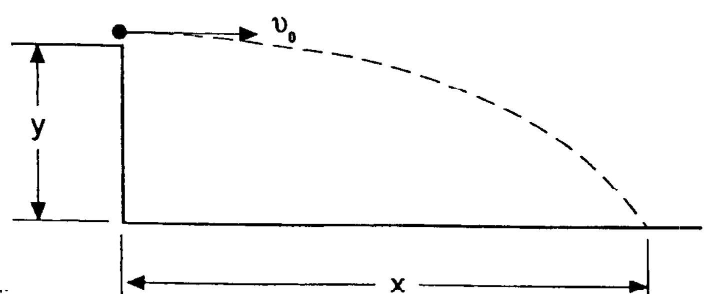


Figure 5.2 Finding the Initial Velocity

Set up

① Clamp the Projectile Launcher to a sturdy table near one end of the table with the Launcher aimed away from the table. See Figure 5.1.

② Point the Launcher straight up and fire a test shot on medium range to make sure the ball doesn't hit the ceiling. If it does, use the short range throughout this experiment or put the Launcher closer to the floor.

③ Adjust the angle of the Projectile Launcher to zero degrees so the ball will be shot off horizontally.

Procedure

PART I: Determining the Initial Velocity of the Ball (without photogates)

① Put the plastic ball into the Projectile Launcher and cock it to the medium range position. Fire one shot to locate where the ball hits the floor. At this position, tape a piece of white paper to the floor. Place a piece of carbon paper (carbon-side down) on top of this paper and tape it down. When the ball hits the floor, it will leave a mark on the white paper.

② Fire about ten shots.

③ Measure the vertical distance from the bottom of the ball as it leaves the barrel (this position is marked on the side of the barrel) to the floor. Record this distance in Table 5.1.

④ Use a plumb bob to find the point on the floor that is directly beneath the release point on the barrel. Measure the horizontal distance along the floor from the release point to the leading edge of the paper. Record in Table 5.1.

⑤ Measure from the leading edge of the paper to each of the ten dots and record these distances in Table 5.1.

⑥ Find the average of the ten distances and record in Table 5.1.

⑦ Using the vertical distance and the average horizontal distance, calculate the time of flight and the initial velocity of the ball. Record in Table 5.1.

Table 5.1 Determining the Initial Velocity without Photogates

Vertical distance = ____________ Calculated time of flight= ____________

Horizontal distance to paper edge = ____________ Initial velocity = ____________

Trial Number	Distance
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
Average	
Total Distance	

ALTERNATE METHOD FOR DETERMINING THE INITIAL VELOCITY OF THE BALL (USING PHOTOGATES)

① Attach the photogate bracket to the Launcher and attach two photogates to the bracket. Plug the photogates into a computer or other timer.

② Adjust the angle of the Projectile Launcher to 90 degrees (straight up).

③ Put the plastic ball into the Projectile Launcher and cock it to the long range position.

④ Run the timing program and set it to measure the time between the ball blocking the two photogates.

⑤ Shoot the ball three times and take the average of these times. Record in Table 5.2.

Table 5.2 Initial Speed Using Photogates

TRIAL NUMBER	TIME
1	
2	
3	
AVERAGE TIME	
INITIAL SPEED	

⑥ Using that the distance between the photogates is 10 cm, calculate the initial speed and record it in Table 5.2.

MEASURING THE HEIGHT

① Adjust the angle of the Launcher to 90 degrees (straight up).

② Shoot the ball on the medium range setting several times and measure the maximum height attained by the ball. Record in Table 5.3.

③ Determine the mass of the ball and record in Table 5.3.

Analysis

① Calculate the initial kinetic energy and record in Table 5.3.

② Calculate the final potential energy and record in Table 5.3.

③ Calculate the percent difference between the initial and final energies and record in Table 5.3.

Table 5.3 Results

Maximuim Height of Ball	
Mass of Ball	
Initial Kinetic Energy	
Final Potential Energy	
Percent Difference	

Questions

① How does friction affect the result for the kinetic energy?

② How does friction affect the result for the potential energy?

Teacher's Guide

Experiment 1: Projectile Motion

> ➤ **NOTE:** For best results, make sure that the projectile launcher is clamped securely to a firm table. Any movement of the gun will result in inconsistent data.

A) The muzzle velocity of the gun tested for this manual was 6.5 m/s (Short range launcher at maximum setting, nylon ball)

B) To find the range at the chosen angle, it is necessary to solve the quadratic equation given in the theory section. You may wish for the students to do this, or you may provide them with the solution:

$$t = \frac{v_0 \sin\theta + \sqrt{(v_0 \sin\theta)^2 + 2g(y_0 - y)}}{g}$$

① The difference depended on the angle at which the gun was fired. The following table gives typical results:

Angle	Predicted Range	Actual Range	Percent Error
30	5.22	5.19	0.57%
45	5.30	5.16	2.64%
60	4.35	4.23	2.87%
39	5.39	5.31	1.48%

> ➤ **NOTE:** The maximum angle is not 45° in this case, nor is the range at 60° equal to that at 30°. This is because the initial height of the ball is not the same as that of the impact point. The maximum range for this setup (with the launcher 1.15 m above ground level) was calculated to be 39°, and this was experimentally verified as well.

② Answers will vary depending on the method of estimating the precision. The primary source of error is in ignoring the effect of air resistance.

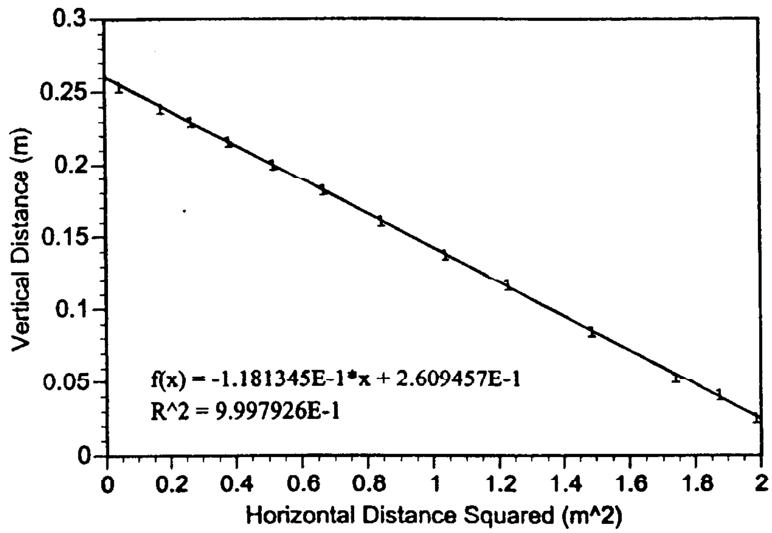
Experiment 2: Projectile Motion Using Photogates

> ➤ **NOTE:** Other than the method of determining initial velocity, this experiment and experiment 1 are equivalent.

Experiment 4: Projectile Path

① Alternately, measure your distances from the ground up.

③ Vertical distances measured from the ground up for this graph. The intercept is the height of the launcher above ground when done this way.



④ The slope (measuring from the ground) is -0.118 for this test. (Measuring down from the initial height will give the same value, only positive.) In either case, the slope is

$$\frac{g}{2v_0^2}$$

⑤ The slope calculated here gives us an initial velocity of 6.44 m/s. This compares favorably with the velocity calculated in experiments 1 and 2.

① Yes. This tells us that y is a function of x^2.

② A plot of y versus x would be parabolic instead of linear.

③ The projectile moves in a parabolic curve. (neglecting air friction)

Experiment 5: Conservation of Energy

① Using the photogate method, we found that the initial speed of the ball was 4.93 m/s. (Nylon ball, short range launcher at medium setting) The ball mass was 9.6 g, so our total kinetic energy was 0.117 J.

② The ball reached an average height of 1.14 m. Potential energy was then 0.107 J.

③ Energy lost was 8.5% of original energy.

Experiment 6: Conservation of Momentum in Two Dimensions

② If possible use medium-range rather than short. The medium-range setting gives more predictable results than the short-range setting.

④⑥ Results for the x component of momentum should be within 5% of initial values. The total y component should be small compared to the x component.

①② Momentum is conserved on both axes.

③ Kinetic energy is nearly conserved in the elastic collision. There is some loss due the fact that the collision is not completely elastic.

④ Energy is conserved for the inelastic collision; but kinetic energy is not.

⑤ The angle should be nearly 90°. (Our tests had angles of about 84°)

⑥ In the inelastic case, the angle will be less than in the elastic case. The exact angle will depend on the degree of inelasticity, which will depend on the type and amount of tape used.

Experiment 7: Varying Angle to Maximize Height on a Wall

① You should be able to measure the angle of maximum height to within ±2%.

④ Measure the distance to the front edge of the ball.

⑤ Measure the initial height to the center of the ball.

Technical Support

If you have any comments about this product or this manual please let us know. If you have any suggestions on alternate experiments or find a problem in the manual please tell us. PASCO appreciates any customer feed-back. Your input helps us evaluate and improve our product.

For Technical Support call us at 1-800-772-8700 (toll-free within the U.S.) or (916) 786-3800.

email: techsupp@PASCO.com

Before you call the PASCO Technical Support staff it would be helpful to prepare the following information:

- If your problem is computer/software related, note:

  Title and Revision Date of software.

  Type of Computer (Make, Model, Speed).

  Type of external Cables/Peripherals.

- If your problem is with the PASCO apparatus, note:

  Title and Model number (usually listed on the label).

  Approximate age of apparatus.

  A detailed description of the problem/sequence of events. (In case you can't call PASCO right away, you won't lose valuable data.)

  If possible, have the apparatus within reach when calling. This makes descriptions of individual parts much easier.

- If your problem relates to the instruction manual, note:

  Part number and Revision (listed by month and year on the front cover).

  Have the manual at hand to discuss your questions.