

Engineering Mechanics Studio Classroom
Statics Modules

Gateway Funded Research
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Executive Summary

The Albert Nerken School of Engineering intends to develop an interactive method of instruction for Engineering Mechanics (ESC 100). This new approach to undergraduate education adapts and furthers the "studio" concept of instruction. In this system of learning, student workstations, tabletop experiments, computer software, videotapes and traditional textbooks will be combined to create a dynamic teaching environment, which integrates the traditional separate activities of instruction, lecture, recitation and laboratory. The current research effort focused on the development of teaching modules for the statics portion of the course. Dynamics teaching modules will be developed in a future research effort.

Introduction

This research effort was performed by Professors Grossman, Guido and Li, all of whom have taught Engineering Mechanism (ESC 100) numerous times. There are currently three sections of ESC 100, which is required to be taken by the Civil Engineering, Electrical Engineering and Mechanical Engineering students in the fall semester of their sophomore year. All three investigators concurred that the course is currently taught as two-thirds statics and one-third dynamics. It was determined that to develop teaching modules for both statics and dynamics would not be possible in the time frame of the project (June, July, August 2000). Therefore, only teaching modules in statics were developed.

It was decided that ten teaching modules would be developed for the statics portion of Engineering Mechanics (ESC 100). These modules can be found in Table 1. In the development of each module: Prof. Grossman researched the lecture material; Prof. Guido the table top experiments; and Prof. Li the available CD Roms, software, videos, etc.

Table 1
ESC 100 - Engineering Mechanics
Teaching Modules for Statics

Topics	Statics Modules	Chapters*
Force Systems & Equilibrium (4 modules)	Basic Concepts – Dot Product, Cross Product, Moment of a force, Couples	1,2,3
	Resultant of Concurrent and Nonconcurrent Force system	2,3
	Equilibrium 1: Free Body Diagram, and 2-D Equilibrium	4
	Equilibrium 2: 3-D	4
Centroids (1 module)	Centroids and Distributed Loads	5
Applications (2 modules)	Structures: Trusses, Frames & Machines	6
	Beams and Cables	8
Friction (1 module)	Static & Kinetic Friction	
Moment of Inertia (2 modules)	Areas	9
	Masses	9

Note: Each module in 3 hours or 1 week and 2 2-hour exams are included in this frame, one after the first 4 modules, and one after moment of inertia.

* The textbook currently used for ESC 100 is "Vector Mechanics for Engineers: Statics and Dynamics," by F. P. Beer and E.R. Johnston, Jr., Sixth Edition. McGraw-Hill, 1997.

Teaching Modules

Professor Grossman has suggested that the Review and summary sections at the end of each chapter of the textbook be used as the lecture material for each of the ten modules. See Table 2.

Table 2
Lecture Material

Topics	Modules	Lecture Material*
I. Force Systems	1. Basic Concepts	pp. 64-66 pp. 144-147
	2. Resultants	pp.148
	3.&4. Equilibrium 1 and 2	pp. 202-204
II. Centroid	5. Distributed Systems	pp. 266-269
III. Applications	6. Structures, Trusses, Frames & Machines	pp. 333-325
	7. Beams and Cables	pp. 390-392
IV. Friction	8. Static & Kinetic Friction	pp. 447-448
V. Moment of Inertia	9. Areas	pp. 529-530
	10. Masses	pp. 532-533

* Text: "Vector Mechanics for Engineers: Statics and Dynamics," by F. P. Beer and E.R. Johnston, Jr., Sixth Edition. McGraw-Hill, 1997. See Appendix 1 for actual pages.

Professor Guido contacted PASCO Scientific, the company that manufactures the table top equipment needed for the table top experiments, and obtained a web site (www2.PASCO.com/TECHSUPP) from which he obtained experiments for the statics equipment needed. This equipment was specified in the summer 1999 report entitled "Engineering Mechanics Studio Classroom." The main table top equipment needed for statics is the "Super Force Pulley Table" (ME-9447) and the "Introductory Mechanics System" (ME-9299). Professor Guido has specified which table top experiments should be performed by the instructor as class demonstrations and which experiments should be performed by the students as homework assignments. See Table 3.

Professor Li has searched the web and has obtained websites containing information pertaining to each of the ten modules. These websites can be gone to during regular class time or visited by the students at their leisure. In addition, Professor Li has reviewed the 2 CD-ROMs now available with the textbook. They are "Working Model @ 3D Simulations CD-Rom," and "Student Resources CD-Rom." She has reviewed both sample problems and homework problems. See page 4.

Table 3
Table Top Experiments

Module 1	<p>Basic Concepts – Dot Product, Moment of a Force, Couples</p> <p>(Class demo) Using "Introductory Mechanics System" from PASCO: (Class demo) Experiment No. 1 – Hooke's Law Measuring Forces. (HW) Experiment No. 4 – Torque-Parallel Forces Experiment No. 5 – Torque-Non-Parallel Forces</p>
Module 2	<p>Resultants of Concurrent and Non-Concurrent Force System</p> <p>(Class demo) Using "Force Table" from PASCO: Experiment No. 1 – Vector Addition (Class demo) Using "Introductory Mechanics System" from PASCO: (HW) Experiment No. 2 – Adding Force Resultants and Equilibrants Experiment No. 3 – Resolving Forces-Components</p>
Module 3	<p>Equilibrium 1: Free Body Diagram, and 2D Equilibrium</p> <p>(Class demo) Using "Introductory Mechanics System" from PASCO: (Class demo + HW) Experiment No. 7 – Equilibrium of Physical Bodies Experiment No. 8 – The Inclined Plane</p>
Module 4	<p>Equilibrium 2-3D</p> <p>Using "Introductory Mechanics System" from PASCO: (HW) Experiment No. 15 – Designing a Beam Balance</p>
Module 5	<p>Centroids and Distributed Loads</p> <p>Using "Introductory Mechanics System" from PASCO: (Class demo) Experiment No. 6 – Center of Mass</p>
Module 6	<p>Structures: Trusses, Frames, and Machines</p> <p>Using "Introductory Mechanics System" from PASCO: (Class demo) Experiment No. 12 – Lever (mentions work principle) (HW) Experiment No. 13 The Inclined Plane (mentions work principle)</p>
Module 7	<p>Beams and Cables</p> <p>Using "Introductory Mechanics System" from PASCO: (Class demo) Experiment No. 15 – The Pulley</p>
Module 8	<p>Static & Kinetic Friction</p> <p>Using "Introductory Mechanics Systems" from PASCO: (Class demo) Experiment No. 9 – Sliding Friction</p>
Module 9	<p>Moment of Inertia (Areas)</p> <p>NONE</p>
Module 10	<p>Moment of Inertia (Masses)</p> <p>NONE</p>

*Note: See Appendix 2 for the experiments associated with the "Force Table," and Appendix 3 for the experiments associated with the "Introductory Mechanics System."

CD-Rom Accompanying "Vector Mechanics for Engineers: Statics and Dynamics"

Pertinent Websites

Module 1: Basic Concepts - Dot Product, Cross Product, Moment of a force, Couples

CD-ROM: Sample Problem 2.1 (p.22) - Visualize vector addition
 Sample Problem 2.2 (p.23) - See the effect of angle on tension in tugboat Ropes
 Homework Problem 2.45 (p.41) - See the effect of force direction on tension in a rope and cable
 Sample Problem 2.9 (p.58) - View the effect of cylinder mass and its distance from a wall on the tension in its supporting cables

Student Resources CD-ROM:

Statics Tutorial → Analysis Tools:
 3-D Vectors, Cross Products, Moments and Couples

Resource from Web:

http://claymore.engineer.gvsu.edu/~jackh/eod_new/mechanic/statics/statics.html
 Basic Concepts: Statics, Force, Scalar, Vector
<http://www.encyclopedia.com/articles/12285.html>
<http://www.eng.iastate.edu/efmd/statics.htm#statfor>
<http://www.aa.washington.edu/courses/engr210/handouts.shtml>
http://web.mit.edu/4.441/1_lectures/1_lecture4/1_lecture4.html
http://web.mit.edu/4.441/1_lectures/1_lecture5/1_lecture5.html

Cross Product:

<http://www.phy.syr.edu/courses/java-suite/crosspro.html>

Module 2: Resultant of Concurrent and Nonconcurrent Force system

CD-ROM: Sample Problem 3.6 (p.111)- Gain insights into force and its relationship to torque (couples)
 Sample Problem 3.11 (p.130) - Interact with the resultant force's magnitude and point of application and see the effects on the stability of the foundation mat
 Homework Problem 3.147 (p. 149) -View the relationship between force and moment

Student Resources CD-ROM:

Statics Tutorial → Analysis Tools:
 3-D Vectors: Addition of Concurrent Forces
 Couples: Resultant Moment Examples and Quizzes

Resource from Web:

http://web.mit.edu/4.441/1_lectures/1_lecture7/1_lecture7.html
http://web.mit.edu/4.441/1_lectures/1_lecture9/1_lecture9.html
http://claymore.engineer.gvsu.edu/~jackh/eod_new/mechanic/statics/statics.html
<http://www.eng.iastate.edu/efmd/statics.htm#scalvec>

Module 3: Equilibrium 1: Free Body Diagram, and 2-D Equilibrium

CD-ROM: Sample Problem 4.2 (p.163) - Visualize the effect of applied forces on reaction forces
 Sample Problem 4.5 (p.165) - Determine the effects of mass and spring constant on the equilibrium configuration of a scale
 Sample Problem 4.6 (p.179) - Experiment with the relative ease of lifting a joist as a function of joist angle

Student Resources CD-ROM:

Statics Tutorial→Equilibrium:
 Particle Equilibrium (2.9&2.15)
 2-D Rigid Body Equilibrium (4.1-4.4)

Resource from Web:

What is Equilibrium:

http://web.mit.edu/4.441/1_lectures/1_lecture5/1_lecture5.html

Free Body Diagram:

http://www.eng.iastate.edu/efmd/statics.htm#free_bod

2-D Equilibrium:

<http://www.eng.iastate.edu/efmd/statics.htm#equilibrium>

<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 4: Equilibrium 2: 3-DCD-ROM:Student Resources CD-ROM:

Statics Tutorial→Equilibrium:
 3-D Rigid Body Equilibrium (4.8-4.9)

Resource from Web:

http://claymore.engineer.gvsu.edu/~jackh/eod_new/mechanic/statics/statics.html

<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 5: Centroids and Distributed Loads

CD-ROM: Sample Problem 5.9 (p.242) - Modify distributed loads on a beam and observe the effect on reaction forces at the beam's supports

Student Resources CD-ROM:

Statics Tutorial→ Distributed Forces:
 Centroids (5.2-5.5)
 Distributed Loads (5.8)
 Fluid Statics (5.9)

Resource from Web:

<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 6: Structures: Trusses, Frames & Machines

CD-ROM: Sample Problem 6.1 (p.284) - View the effect of applied forces on internal forces in a truss
 Sample Problem 6.144 (p.328) - Observe the relationship between barrel weight and reaction forces

Student Resources CD-ROM:

Statics Tutorial > Analysis of Structures:
 Trusses (6.1-6.8)
 Frames & Machines (6.9-6.12)

Resource from Web:

<http://www.ce.ufl.edu/activities/trusslab/trussndx.html>
<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 7: Beams and Cables

CD-ROM: Sample Problem 7.1 (p.344) - Gain engineering experience with axial forces, shear forces, and bending moments

Student Resources CD-ROM:

Statics Tutorial→Beams and Friction
 Shear & Moment Diagrams (7.3-7.6)

Resource from Web:

<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 8: Static & Kinetic Friction

CD-ROM: Sample Problem 8.1 (p.403) - Learn about coefficient of static friction and slipping.
 Sample Problem 8.3 (p.405) - View a complex relationship between force, friction, and sliding.

Student Resources CD-ROM:

Statics Tutorial→Beams and Friction:
 Friction (8.1-8.5)

Resource from Web:

<http://www.aa.washington.edu/courses/engr210/handouts.shtml>

Module 9: Moment of Inertia of Areas

CD-ROM: Student Resources CD-ROM:
 Statics Tutorial→ Distributed Forces:
 Moments of Inertia: _Area Moments (9.2-9.6)

Resource from Web: Not found.

Module 10: Moment of Inertia of Masses

CD-ROM: Student Resources CD-ROM:
 Statics Tutorial→ Distributed Forces:
 Moments of Inertia: _Mass Moments (9.11-9.15)

Resource from Web: Not found.

Conclusions and Recommendations

The next phase of course development would be to address the Dynamics portion of Engineering Mechanics. Professors Grossman, Guido and Li would first agree on the number of topics and modules that would be necessary to thoroughly cover the dynamics material – it must be kept in mind that only one-third of the semester is to be devoted to dynamics, approximately 5 weeks. Next, each professor would take on a particular task as was done with the Statics Project. Professor Grossman – lecture material, Professor Guido – table top experiments, and Professor Li – web sites and CD-Roms. It is recommended that this project be completed by January 2001. Therefore, new funding is required for the Fall 2000.

The integration of table top experiments for the demonstration of relevant ideas, and the use of the internet and other available multimedia resources into the traditional lecture mode of teaching Engineering Mechanics (ESC 100) will take a concerted effort on the part of the instructors. When the Engineering Mechanics Studio/Classroom is made available and the necessary equipment purchased (see Gateway report Summer 1999), a familiarity and expertise with the equipment will be essential by the instructors for the development of the new Engineering Mechanics course. Professors Grossman, Guido and Li should be the development team. Additional funding should be made available for this very important phase of the project.

APPENDIX 1:

Lecture Material

MODULE No. 1

REVIEW AND SUMMARY FOR CHAPTER 2

Resultant of two forces

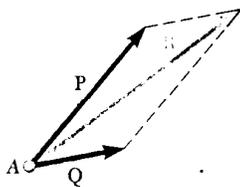


Fig. 2.35

Component of a force

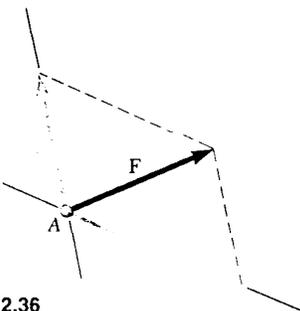


Fig. 2.36

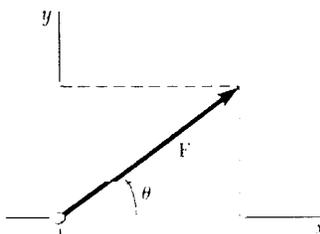


Fig. 2.37

In this chapter we have studied the effect of forces on particles, i.e., on bodies of such shape and size that all forces acting on them may be assumed applied at the same point.

Forces are *vector quantities*; they are characterized by a *point of application*, a *magnitude*, and a *direction*, and they add according to the *parallelogram law* (Fig. 2.35). The magnitude and direction of the resultant **R** of two forces **P** and **Q** can be determined either graphically or by trigonometry, using successively the law of cosines and the law of sines [Sample Prob. 2.1].

Any given force acting on a particle can be resolved into two or more *components*, i.e., it can be replaced by two or more forces which have the same effect on the particle. A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram which has **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram (Fig. 2.36) and can be determined either graphically or by trigonometry [Sec. 2.6].

A force **F** is said to have been resolved into two *rectangular components* if its components **F_x** and **F_y** are perpendicular to each other and are directed along the coordinate axes (Fig. 2.37). Introducing the *unit vectors* **i** and **j** along the *x* and *y* axes, respectively, we write [Sec. 2.7]

$$\mathbf{F}_x = F_x \mathbf{i} \quad \mathbf{F}_y = F_y \mathbf{j} \quad (2.6)$$

and

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} \quad (2.7)$$

where F_x and F_y are the *scalar components* of **F**. These components, which can be positive or negative, are defined by the relations

$$F_x = F \cos \theta \quad F_y = F \sin \theta \quad (2.8)$$

When the rectangular components F_x and F_y of a force **F** are given, the angle θ defining the direction of the force can be obtained by writing

$$\tan \theta = \frac{F_y}{F_x} \quad (2.9)$$

The magnitude F of the force can then be obtained by solving one of the equations (2.8) for F or by applying the Pythagorean theorem and writing

$$F = \sqrt{F_x^2 + F_y^2} \quad (2.10)$$

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Fig. 2.3

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When three or more coplanar forces act on a particle, the rectangular components of their resultant \mathbf{R} can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.8]. We have

$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad (2.13)$$

The magnitude and direction of \mathbf{R} can then be determined from relations similar to Eqs. (2.9) and (2.10) [Sample Prob. 2.3].

A force \mathbf{F} in three-dimensional space can be resolved into rectangular components F_x , F_y , and F_z [Sec. 2.12]. Denoting by θ_x , θ_y , and θ_z , respectively, the angles that \mathbf{F} forms with the x , y , and z axes (Fig. 2.38), we have

$$F_x = F \cos \theta_x \quad F_y = F \cos \theta_y \quad F_z = F \cos \theta_z \quad (2.19)$$

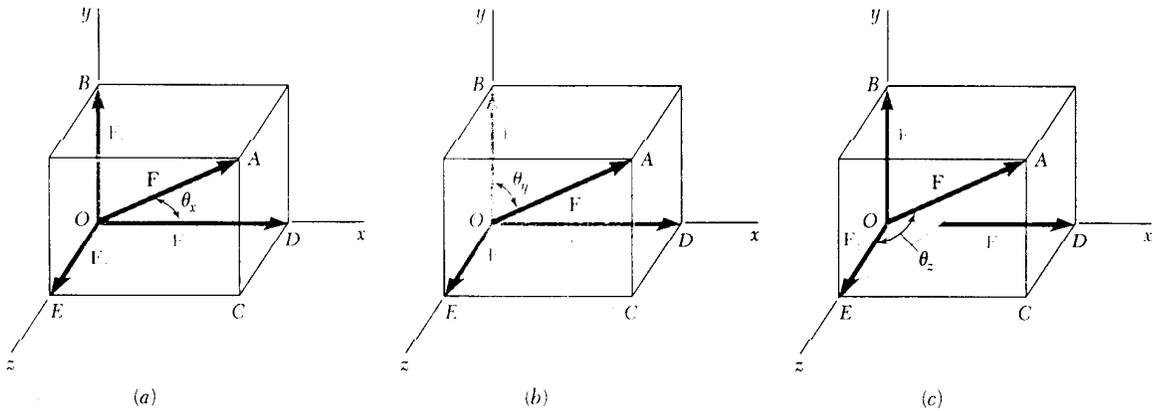


Fig. 2.38

The cosines of θ_x , θ_y , θ_z are known as the *direction cosines* of the force \mathbf{F} . Introducing the unit vectors \mathbf{i} , \mathbf{j} , \mathbf{k} along the coordinate axes, we write

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k} \quad (2.20)$$

or

$$\mathbf{F} = F(\cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}) \quad (2.21)$$

which shows (Fig. 2.39) that \mathbf{F} is the product of its magnitude F and the unit vector

$$\boldsymbol{\lambda} = \cos \theta_x \mathbf{i} + \cos \theta_y \mathbf{j} + \cos \theta_z \mathbf{k}$$

Since the magnitude of $\boldsymbol{\lambda}$ is equal to unity, we must have

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1 \quad (2.24)$$

When the rectangular components F_x , F_y , F_z of a force \mathbf{F} are given, the magnitude F of the force is found by writing

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} \quad (2.18)$$

and the direction cosines of \mathbf{F} are obtained from Eqs. (2.19). We have

$$\cos \theta_x = \frac{F_x}{F} \quad \cos \theta_y = \frac{F_y}{F} \quad \cos \theta_z = \frac{F_z}{F} \quad (2.25)$$

Direction cosines

Direction cosines

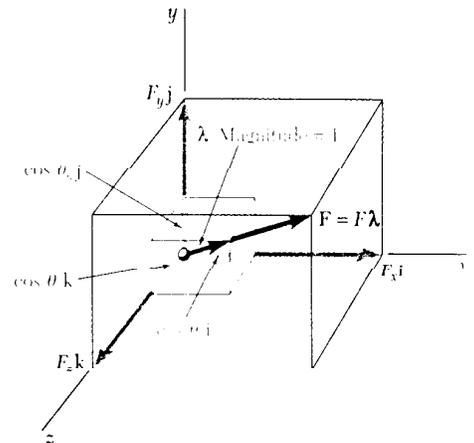


Fig. 2.39

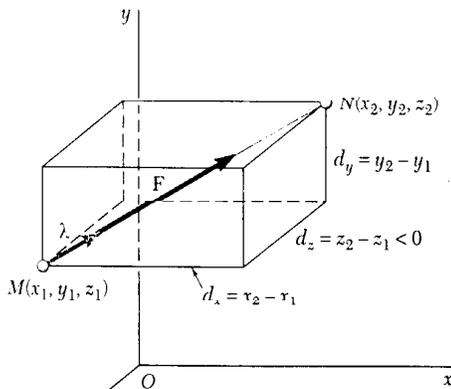


Fig. 2.40

When a force \mathbf{F} is defined in three-dimensional space by its magnitude F and two points M and N on its line of action [Sec. 2.13], its rectangular components can be obtained as follows. We first express the vector \overrightarrow{MN} joining points M and N in terms of its components d_x , d_y , and d_z (Fig. 2.40); we write

$$\overrightarrow{MN} = d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k} \quad (2.26)$$

We next determine the unit vector $\boldsymbol{\lambda}$ along the line of action of \mathbf{F} by dividing \overrightarrow{MN} by its magnitude $MN = d$:

$$\boldsymbol{\lambda} = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \quad (2.27)$$

Recalling that \mathbf{F} is equal to the product of F and $\boldsymbol{\lambda}$, we have

$$\mathbf{F} = F\boldsymbol{\lambda} = \frac{F}{d}(d_x\mathbf{i} + d_y\mathbf{j} + d_z\mathbf{k}) \quad (2.28)$$

from which it follows [Sample Probs. 2.7 and 2.8] that the scalar components of \mathbf{F} are, respectively,

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d} \quad (2.29)$$

Resultant of forces in space

When *two or more forces* act on a particle in *three-dimensional space*, the rectangular components of their resultant \mathbf{R} can be obtained by adding algebraically the corresponding components of the given forces [Sec. 2.14]. We have

$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R_z = \Sigma F_z \quad (2.31)$$

The magnitude and direction of \mathbf{R} can then be determined from relations similar to Eqs. (2.18) and (2.25) [Sample Prob. 2.8].

Equilibrium of a particle

A particle is said to be in *equilibrium* when the resultant of all the forces acting on it is zero [Sec. 2.9]. The particle will then remain at rest (if originally at rest) or move with constant speed in a straight line (if originally in motion) [Sec. 2.10].

Free-body diagram

To solve a problem involving a particle in equilibrium, one first should draw a *free-body diagram* of the particle showing all the forces acting on it [Sec. 2.11]. If *only three coplanar forces* act on the particle, a *force triangle* may be drawn to express that the particle is in equilibrium. Using graphical methods of trigonometry, this triangle can be solved for no more than two unknowns [Sample Prob. 2.4]. If *more than three coplanar forces* are involved, the equations of equilibrium

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad (2.15)$$

should be used. These equations can be solved for no more than two unknowns [Sample Prob. 2.6].

Equilibrium in space

When a particle is in *equilibrium in three-dimensional space* [Sec. 2.15], the three equations of equilibrium

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0 \quad (2.31)$$

should be used. These equations can be solved for no more than three unknowns [Sample Prob. 2.9].

MODULE NO. 1 (CONT'D)

REVIEW AND SUMMARY FOR CHAPTER 3

Principle of transmissibility

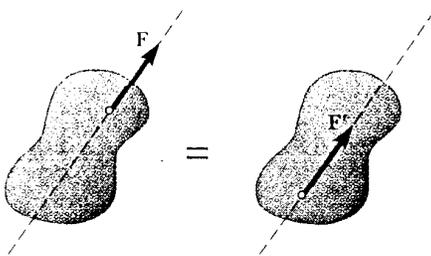


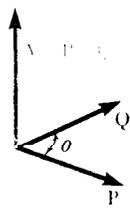
Fig. 3.48

In this chapter we studied the effect of forces exerted on a rigid body. We first learned to distinguish between *external* and *internal* forces [Sec. 3.2] and saw that, according to the *principle of transmissibility*, the effect of an external force on a rigid body remains unchanged if that force is moved along its line of action [Sec. 3.3]. In other words, two forces \mathbf{F} and \mathbf{F}' acting on a rigid body at two different points have the same effect on that body if they have the same magnitude, same direction, and same line of action (Fig. 3.48). Two such forces are said to be *equivalent*.

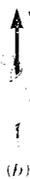
Before proceeding with the discussion of *equivalent systems of forces*, we introduced the concept of the *vector product of two vectors* [Sec. 3.4]. The vector product

Vector product of two vectors

$$\mathbf{V} = \mathbf{P} \times \mathbf{Q}$$



(a)



(b)

Fig. 3.49

of the vectors \mathbf{P} and \mathbf{Q} was defined as a vector perpendicular to the plane containing \mathbf{P} and \mathbf{Q} (Fig. 3.49), of magnitude

$$V = PQ \sin \theta \quad (3.1)$$

and directed in such a way that a person located at the tip of \mathbf{V} will observe as counterclockwise the rotation through θ which brings the vector \mathbf{P} in line with the vector \mathbf{Q} . The three vectors \mathbf{P} , \mathbf{Q} , and \mathbf{V} —taken in that order—are said to form a *right-handed triad*. It follows that the vector products $\mathbf{Q} \times \mathbf{P}$ and $\mathbf{P} \times \mathbf{Q}$ are represented by equal and opposite vectors. We have

$$\mathbf{Q} \times \mathbf{P} = -(\mathbf{P} \times \mathbf{Q}) \quad (3.4)$$

It also follows from the definition of the vector product of two vectors that the vector products of the unit vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} are

$$\mathbf{i} \times \mathbf{i} = 0 \quad \mathbf{i} \times \mathbf{j} = \mathbf{k} \quad \mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

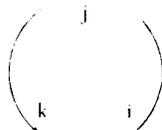


Fig. 3.50

and so on. The sign of the vector product of two unit vectors can be obtained by arranging in a circle and in counterclockwise order the three letters representing the unit vectors (Fig. 3.50): The vector product of two unit vectors will be positive if they follow each other in counterclockwise order and negative if they follow each other in clockwise order.

The rectangular components of the vector product \mathbf{V} of two vectors \mathbf{P} and \mathbf{Q} were expressed [Sec. 3.5] as

$$\begin{aligned} V_x &= P_y Q_z - P_z Q_y \\ V_y &= P_z Q_x - P_x Q_z \\ V_z &= P_x Q_y - P_y Q_x \end{aligned} \quad (3.9)$$

Using a determinant, we also wrote

$$\mathbf{V} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \quad (3.10)$$

The moment of a force \mathbf{F} about a point O was defined [Sec. 3.6] as the vector product

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} \quad (3.11)$$

where \mathbf{r} is the position vector drawn from O to the point of application A of the force \mathbf{F} (Fig. 3.51). Denoting by θ the angle between the lines of action of \mathbf{r} and \mathbf{F} , we found that the magnitude of the moment of \mathbf{F} about O can be expressed as

$$M_O = rF \sin \theta = Fd \quad (3.12)$$

where d represents the perpendicular distance from O to the line of action of \mathbf{F} .

The rectangular components of the moment \mathbf{M}_O of a force \mathbf{F} were expressed [Sec. 3.8] as

$$\begin{aligned} M_x &= yF_z - zF_y \\ M_y &= zF_x - xF_z \\ M_z &= xF_y - yF_x \end{aligned} \quad (3.18)$$

where x, y, z are the components of the position vector \mathbf{r} (Fig. 3.52). Using a determinant form, we also wrote

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \quad (3.19)$$

In the more general case of the moment about an arbitrary point B of a force \mathbf{F} applied at A , we had

$$\mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_{A/B} & y_{A/B} & z_{A/B} \\ F_x & F_y & F_z \end{vmatrix} \quad (3.21)$$

where $x_{A/B}, y_{A/B}$, and $z_{A/B}$ denote the components of the vector $\mathbf{r}_{A/B}$:

$$x_{A/B} = x_A - x_B \quad y_{A/B} = y_A - y_B \quad z_{A/B} = z_A - z_B$$

Moment about a point

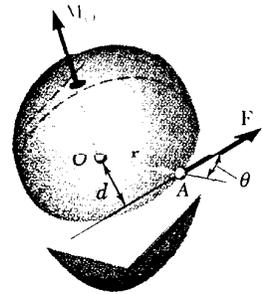


Fig. 3.51

Rectangular components of moment

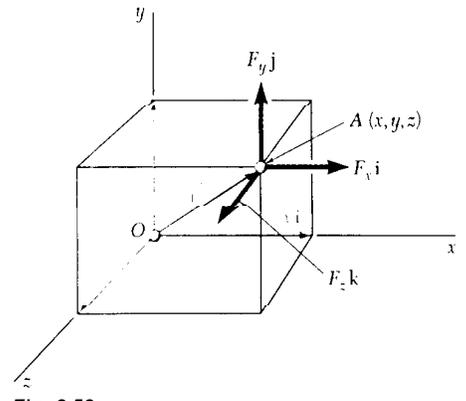


Fig. 3.52

In the case of *problems involving only two dimensions*, the force \mathbf{F} can be assumed to lie in the xy plane. Its moment \mathbf{M}_B about a point B in the same plane is perpendicular to that plane (Fig. 3.53) and is completely defined by the scalar

$$M_B = (x_A - x_B)F_y - (y_A - y_B)F_x \quad (3.23)$$

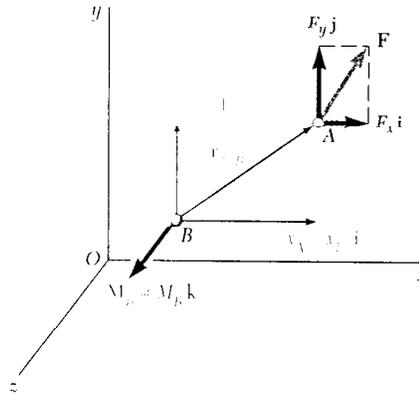


Fig. 3.53

Various methods for the computation of the moment of a force about a point were illustrated in Sample Probs. 3.1 through 3.4.

The *scalar product* of two vectors \mathbf{P} and \mathbf{Q} [Sec. 3.9] was denoted by $\mathbf{P} \cdot \mathbf{Q}$ and was defined as the scalar quantity

$$\mathbf{P} \cdot \mathbf{Q} = PQ \cos \theta \quad (3.24)$$

where θ is the angle between \mathbf{P} and \mathbf{Q} (Fig. 3.54). By expressing the scalar product of \mathbf{P} and \mathbf{Q} in terms of the rectangular components of the two vectors, we determined that

$$\mathbf{P} \cdot \mathbf{Q} = P_x Q_x + P_y Q_y + P_z Q_z \quad (3.30)$$

The *projection of a vector \mathbf{P} on an axis OL* (Fig. 3.55) can be obtained by forming the scalar product of \mathbf{P} and the unit vector $\boldsymbol{\lambda}$ along OL . We have

$$P_{OL} = \mathbf{P} \cdot \boldsymbol{\lambda} \quad (3.36)$$

or, using rectangular components,

$$P_{OL} = P_x \cos \theta_x + P_y \cos \theta_y + P_z \cos \theta_z \quad (3.37)$$

where θ_x , θ_y , and θ_z denote the angles that the axis OL forms with the coordinate axes.

The *mixed triple product* of the three vectors \mathbf{S} , \mathbf{P} , and \mathbf{Q} was defined as the scalar expression

$$\mathbf{S} \cdot \mathbf{P} \times \mathbf{Q} \quad (3.38)$$

obtained by forming the scalar product of \mathbf{S} with the vector product

Scalar product of two vectors

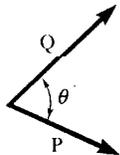


Fig. 3.54

Projection of a vector on an axis

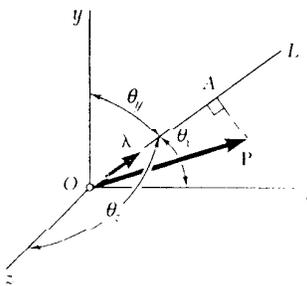


Fig. 3.55

Mixed triple product of three vectors

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of \mathbf{P} and \mathbf{Q} [Sec. 3.10]. It was shown that

$$\mathbf{S} \cdot (\mathbf{P} \times \mathbf{Q}) = \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix} \quad (3.41)$$

where the elements of the determinant are the rectangular components of the three vectors.

The *moment of a force \mathbf{F} about an axis OL* [Sec. 3.11] was defined as the projection OC on OL of the moment \mathbf{M}_O of the force \mathbf{F} (Fig. 3.56), i.e., as the mixed triple product of the unit vector $\boldsymbol{\lambda}$, the position vector \mathbf{r} , and the force \mathbf{F} :

$$M_{OL} = \boldsymbol{\lambda} \cdot \mathbf{M}_O = \boldsymbol{\lambda} \cdot (\mathbf{r} \times \mathbf{F}) \quad (3.42)$$

Using the determinant form for the mixed triple product, we have

$$M_{OL} = \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix} \quad (3.43)$$

where $\lambda_x, \lambda_y, \lambda_z =$ direction cosines of axis OL
 $x, y, z =$ components of \mathbf{r}
 $F_x, F_y, F_z =$ components of \mathbf{F}

An example of the determination of the moment of a force about a skew axis was given in Sample Prob. 3.5.

Two forces \mathbf{F} and $-\mathbf{F}$ having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple* [Sec. 3.12]. It was shown that the moment of a couple is independent of the point about which it is computed; it is a vector \mathbf{M} perpendicular to the plane of the couple and equal in magnitude to the product of the common magnitude F of the forces and the perpendicular distance d between their lines of action (Fig. 3.57).

Two couples having the same moment \mathbf{M} are *equivalent*, i.e., they have the same effect on a given rigid body [Sec. 3.13]. The sum of two couples is itself a couple [Sec. 3.14], and the moment \mathbf{M} of the resultant couple can be obtained by adding vectorially the moments \mathbf{M}_1 and \mathbf{M}_2 of the original couples [Sample Prob. 3.6]. It follows that a couple can be represented by a vector, called a *couple vector*, equal in magnitude and direction to the moment \mathbf{M} of the couple [Sec. 3.15]. A couple vector is a *free vector* which can be attached to the origin O if so desired and resolved into components (Fig. 3.58).

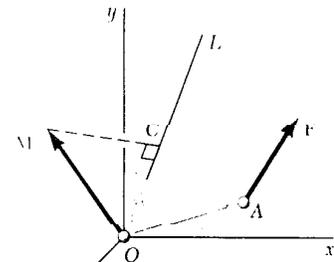


Fig. 3.56

Couples

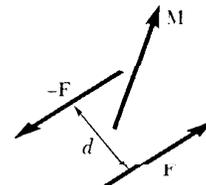


Fig. 3.57

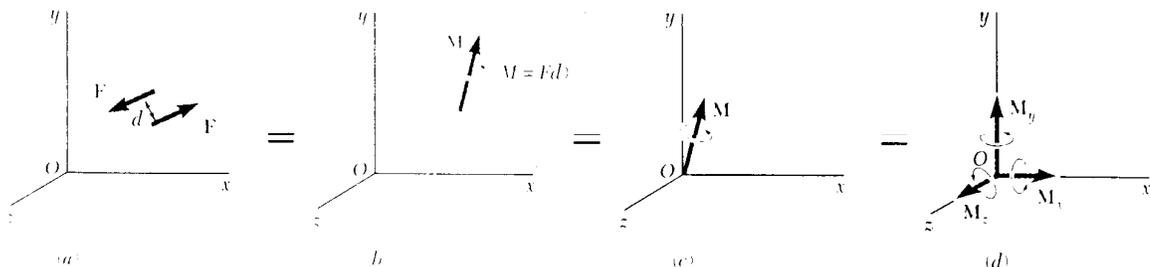


Fig. 3.58

MODULE NO. 2



Fig. 3.59

Any force \mathbf{F} acting at a point A of a rigid body can be replaced by a *force-couple system* at an arbitrary point O , consisting of the force \mathbf{F} applied at O and a couple of moment \mathbf{M}_O equal to the moment about O of the force \mathbf{F} in its original position [Sec. 3.16]; it should be noted that the force \mathbf{F} and the couple vector \mathbf{M}_O are always perpendicular to each other (Fig. 3.59).

It follows [Sec. 3.17] that *any system of forces can be reduced to a force-couple system at a given point O* by first replacing each of the forces of the system by an equivalent force-couple system at O (Fig. 3.60) and then adding all the forces and all the couples determined in this manner to obtain a resultant force \mathbf{R} and a resultant couple vector \mathbf{M}_O^R [Sample Probs. 3.8 through 3.11]. Note that, in general, the resultant \mathbf{R} and the couple vector \mathbf{M}_O^R will not be perpendicular to each other.

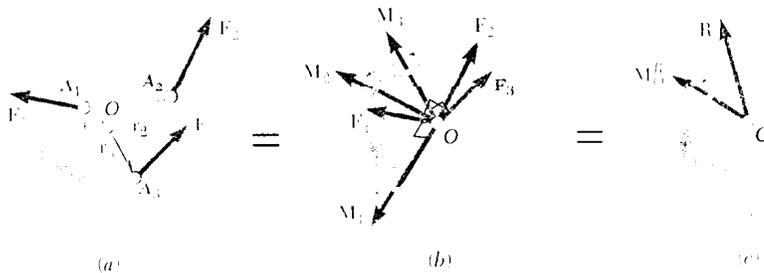


Fig. 3.60

We concluded from the above [Sec. 3.18] that, as far as rigid bodies are concerned, *two systems of forces, $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \dots$ and $\mathbf{F}'_1, \mathbf{F}'_2, \mathbf{F}'_3, \dots$, are equivalent if, and only if,*

$$\Sigma \mathbf{F} = \Sigma \mathbf{F}' \quad \text{and} \quad \Sigma \mathbf{M}_O = \Sigma \mathbf{M}'_O \quad (3.57)$$

If the resultant force \mathbf{R} and the resultant couple vector \mathbf{M}_O^R are perpendicular to each other, the force-couple system at O can be further reduced to a single resultant force [Sec. 3.20]. This will be the case for systems consisting either of (a) concurrent forces (cf. Chap. 2), (b) coplanar forces [Sample Probs. 3.5 and 3.9], or (c) parallel forces [Sample Prob. 3.11]. If the resultant \mathbf{R} and the couple vector \mathbf{M}_O^R are *not* perpendicular to each other, the system *cannot* be reduced to a single force. It can, however, be reduced to a special type of force-couple system called a *wrench*, consisting of the resultant \mathbf{R} and a couple vector \mathbf{M}_1 directed along \mathbf{R} [Sec. 3.21 and Sample Prob. 3.12].

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MODULES 3 AND 4

REVIEW AND SUMMARY

FOR CHAPTER 4

Equilibrium equations

This chapter was devoted to the study of the *equilibrium of rigid bodies*, i.e., to the situation when the external forces acting on a rigid body *form a system equivalent to zero* [Sec. 4.1]. We then have

$$\Sigma \mathbf{F} = 0 \quad \Sigma \mathbf{M}_O = \Sigma (\mathbf{r} \times \mathbf{F}) = 0 \quad (4.1)$$

Resolving each force and each moment into its rectangular components, we can express the necessary and sufficient conditions for the equilibrium of a rigid body with the following six scalar equations:

$$\begin{aligned} \Sigma F_x = 0 & \quad \Sigma F_y = 0 & \quad \Sigma F_z = 0 & \quad (4.2) \\ \Sigma M_x = 0 & \quad \Sigma M_y = 0 & \quad \Sigma M_z = 0 & \quad (4.3) \end{aligned}$$

These equations can be used to determine unknown forces applied to the rigid body or unknown reactions exerted by its supports.

Free-body diagram

When solving a problem involving the equilibrium of a rigid body, it is essential to consider *all* of the forces acting on the body. Therefore, the first step in the solution of the problem should be to draw a *free-body diagram* showing the body under consideration and all of the unknown as well as known forces acting on it [Sec. 4.2].

Equilibrium of a two-dimensional structure

In the first part of the chapter, we considered the *equilibrium of a two-dimensional structure*; i.e., we assumed that the structure considered and the forces applied to it were contained in the same plane. We saw that each of the reactions exerted on the structure by its supports could involve one, two, or three unknowns, depending upon the type of support [Sec. 4.3].

In the case of a two-dimensional structure, Eqs. (4.1), or Eqs. (4.2) and (4.3), reduce to *three equilibrium equations*, namely:

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma M_A = 0 \quad (4.5)$$

where A is an arbitrary point in the plane of the structure [Sec. 4.4]. These equations can be used to solve for three unknowns. While the three equilibrium equations (4.5) cannot be *augmented* with additional equations, any of them can be *replaced* by another equation. Therefore, we can write alternative sets of equilibrium equations such as

$$\Sigma F_x = 0 \quad \Sigma M_A = 0 \quad \Sigma M_B = 0 \quad (4.6)$$

where point B is chosen in such a way that the line AB is not parallel to the y axis, or

$$\Sigma M_A = 0 \quad \Sigma M_B = 0 \quad \Sigma M_C = 0 \quad (4.7)$$

where the points A , B , and C do not lie in a straight line.

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Since any set of equilibrium equations can be solved for only three unknowns, the reactions at the supports of a rigid two-dimensional structure cannot be completely determined if they involve more than three unknowns; they are said to be *statically indeterminate* [Sec. 4.5]. On the other hand, if the reactions involve fewer than three unknowns, equilibrium will not be maintained under general loading conditions; the structure is said to be *partially constrained*. The fact that the reactions involve exactly three unknowns is no guarantee that the equilibrium equations can be solved for all three unknowns. If the supports are arranged in such a way that the reactions are either concurrent or parallel, the reactions are statically indeterminate, and the structure is said to be *improperly constrained*.

Two particular cases of equilibrium of a rigid body were given special attention. In Sec. 4.6, a *two-force body* was defined as a rigid body subjected to forces at only two points, and it was shown that the resultants \mathbf{F}_1 and \mathbf{F}_2 of these forces must have the same magnitude, the same line of action, and opposite sense (Fig. 4.11), a property which will simplify the solution of certain problems in later chapters. In Sec. 4.7, a *three-force body* was defined as a rigid body subjected to forces at only three points, and it was shown that the resultants \mathbf{F}_1 , \mathbf{F}_2 , and \mathbf{F}_3 of these forces must be either concurrent (Fig. 4.12) or parallel. This property provides us with an alternative approach to the solution of problems involving a three-force body [Sample Prob. 4.6].

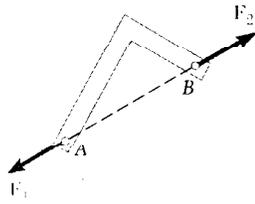


Fig. 4.11

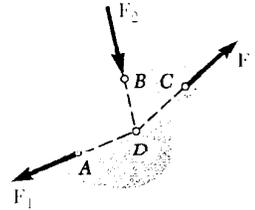


Fig. 4.12

In the second part of the chapter, we considered the equilibrium of a three-dimensional body and saw that each of the reactions exerted on the body by its supports could involve between one and six unknowns, depending upon the type of support [Sec. 4.8].

In the general case of the equilibrium of a three-dimensional body all of the six scalar equilibrium equations (4.2) and (4.3) listed at the beginning of this review should be used and solved for six unknowns [Sec. 4.9]. In most problems, however, these equations will be more conveniently obtained if we first write

$$\Sigma \mathbf{F} = 0 \quad \Sigma \mathbf{M}_O = \Sigma (\mathbf{r} \times \mathbf{F}) = 0 \quad (4.1)$$

and express the forces \mathbf{F} and position vectors \mathbf{r} in terms of scalar components and unit vectors. The vector products can then be computed

Equilibrium of a three-dimensional body

either directly or by means of determinants, and the desired scalar equations obtained by equating to zero the coefficients of the unit vectors [Sample Probs. 4.7 through 4.9].

We noted that as many as three unknown reaction components may be eliminated from the computation of $\Sigma \mathbf{M}_O$ in the second of the relations (4.1) through a judicious choice of point O . Also, the reactions at two points A and B can be eliminated from the solution of some problems by writing the equation $\Sigma M_{AB} = 0$, which involves the computation of the moments of the forces about an axis AB joining points A and B [Sample Prob. 4.10].

If the reactions involve more than six unknowns, some of the reactions are *statically indeterminate*; if they involve fewer than six unknowns, the rigid body is only *partially constrained*. Even with six or more unknowns, the rigid body will be *improperly constrained* if the reactions associated with the given supports either are parallel or intersect the same line.

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Review Problems

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4.142 The semicircular rod $ABCD$ is maintained in equilibrium by the small wheel at D and the rollers at B and C . Knowing that $\alpha = 45^\circ$, determine the reactions at B , C , and D .

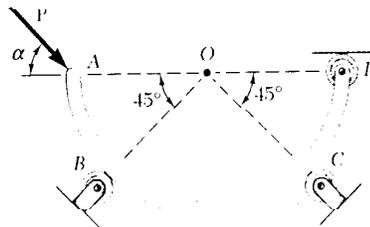


Fig. P4.142 and P4.143

4.143 Determine the range of values of α for which the semicircular rod shown can be maintained in equilibrium by the small wheel at D and the rollers at B and C .

Fig

4.144 A force \mathbf{P} of magnitude 280 lb is applied to member ABC which is supported by a frictionless pin at A and by the cable CED . Since the cable passes over a small pulley at E , the tension may be assumed to be the same in portions CE and ED of the cable. For the case when $a = 3$ ft, determine (a) the tension in the cable, (b) the reaction at A .

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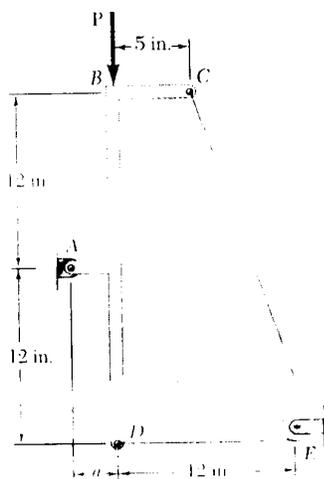


Fig. P4.144

MODULE No. 5

REVIEW AND SUMMARY FOR CHAPTER 5

This chapter was devoted chiefly to the determination of the *center of gravity* of a rigid body, i.e., to the determination of the point G where a single force \mathbf{W} , called the *weight* of the body, can be applied to represent the effect of the earth's attraction on the body.

In the first part of the chapter, we considered *two-dimensional bodies*, such as flat plates and wires contained in the xy plane. By adding force components in the vertical z direction and moments about the horizontal y and x axes [Sec. 5.2], we derived the relations

$$W = \int dW \quad \bar{x}W = \int x dW \quad \bar{y}W = \int y dW \quad (5.2)$$

which define the weight of the body and the coordinates \bar{x} and \bar{y} of its center of gravity.

In the case of a *homogeneous flat plate of uniform thickness* [Sec. 5.3], the center of gravity G of the plate coincides with the *centroid* C of the area A of the plate, the coordinates of which are defined by the relations

$$\bar{x}A = \int x dA \quad \bar{y}A = \int y dA \quad (5.3)$$

Similarly, the determination of the center of gravity of a *homogeneous wire of uniform cross section* contained in a plane reduces to the determination of the *centroid* C of the *line* L representing the wire; we have

$$\bar{x}L = \int x dL \quad \bar{y}L = \int y dL \quad (5.4)$$

The integrals in Eqs. (5.3) are referred to as the *first moments* of the area A with respect to the y and x axes and are denoted by Q_y and Q_x , respectively [Sec. 5.4]. We have

$$Q_y = \bar{x}A \quad Q_x = \bar{y}A \quad (5.5)$$

The first moments of a line can be defined in a similar way.

The determination of the centroid C of an area or line is simplified when the area or line possesses certain *properties of symmetry*. If the area or line is symmetric with respect to an axis, its centroid C lies

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Fig. 5.24

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on the axes; if it is symmetric with respect to two axes, C is located at the intersection of the two axes; if it is symmetric with respect to a center O , C coincides with O .

The areas and the centroids of various common shapes are tabulated in Fig. 5.8. When a flat plate can be divided into several of these shapes, the coordinates \bar{X} and \bar{Y} of its center of gravity G can be determined from the coordinates $\bar{x}_1, \bar{x}_2, \dots$ and $\bar{y}_1, \bar{y}_2, \dots$ of the centers of gravity G_1, G_2, \dots of the various parts [Sec. 5.5]. Equating moments about the y and x axes, respectively (Fig. 5.24), we have

$$\bar{X}\Sigma W = \Sigma \bar{x}W \quad \bar{Y}\Sigma W = \Sigma \bar{y}W \quad (5.7)$$

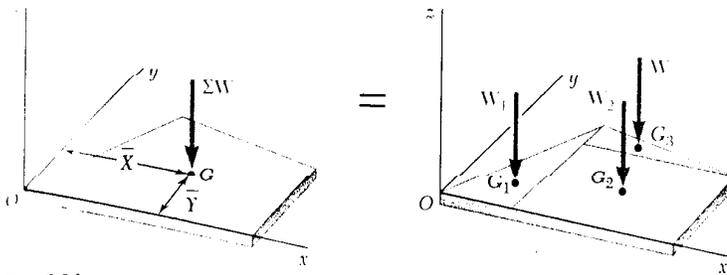


Fig. 5.24

If the plate is homogeneous and of uniform thickness, its center of gravity coincides with the centroid C of the area of the plate, and Eqs. (5.7) reduce to

$$Q_y = \bar{X}\Sigma A = \Sigma \bar{x}A \quad Q_x = \bar{Y}\Sigma A = \Sigma \bar{y}A \quad (5.8)$$

These equations yield the first moments of the composite area, or they can be solved for the coordinates \bar{X} and \bar{Y} of its centroid [Sample Prob. 5.1]. The determination of the center of gravity of a composite wire is carried out in a similar fashion [Sample Prob. 5.2].

When an area is bounded by analytical curves, the coordinates of its centroid can be determined by *integration* [Sec. 5.6]. This can be done by evaluating either the double integrals in Eqs. (5.3) or a *single integral* which uses one of the thin rectangular or pie-shaped elements of area shown in Fig. 5.12. Denoting by \bar{x}_{el} and \bar{y}_{el} the coordinates of the centroid of the element dA , we have

$$Q_y = \bar{x}A = \int \bar{x}_{el} dA \quad Q_x = \bar{y}A = \int \bar{y}_{el} dA \quad (5.9)$$

It is advantageous to use the same element of area to compute both of the first moments Q_y and Q_x ; the same element can also be used to determine the area A [Sample Prob. 5.4].

Determination of centroid by integration

Theorems of Pappus-Guldinus

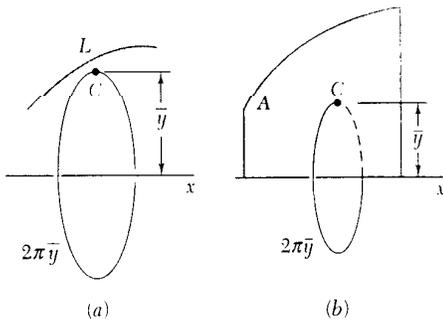


Fig. 5.25

Distributed loads

The *theorems of Pappus-Guldinus* relate the determination of the area of a surface of revolution or the volume of a body of revolution to the determination of the centroid of the generating curve or area [Sec. 5.7]. The area A of the surface generated by rotating a curve of length L about a fixed axis (Fig. 5.25a) is

$$A = 2\pi\bar{y}L \quad (5.10)$$

where \bar{y} represents the distance from the centroid C of the curve to the fixed axis. Similarly, the volume V of the body generated by rotating an area A about a fixed axis (Fig. 5.25b) is

$$V = 2\pi\bar{y}A \quad (5.11)$$

where \bar{y} represents the distance from the centroid C of the area to the fixed axis.

The concept of centroid of an area can also be used to solve problems other than those dealing with the weight of flat plates. For example, to determine the reactions at the supports of a beam [Sec. 5.8], we can replace a *distributed load* w by a concentrated load W equal in magnitude to the area A under the load curve and passing through the centroid C of that area (Fig. 5.26). The same approach can be used to determine the resultant of the hydrostatic forces exerted on a *rectangular plate submerged in a liquid* [Sec. 5.9].

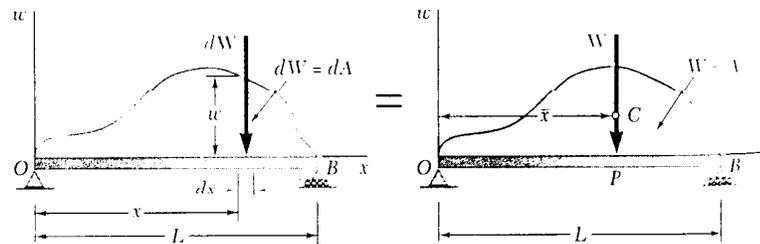


Fig. 5.26

The last part of the chapter was devoted to the determination of the *center of gravity* G of a *three-dimensional body*. The coordinates \bar{x} , \bar{y} , \bar{z} of G were defined by the relations

$$\bar{x}W = \int x dW \quad \bar{y}W = \int y dW \quad \bar{z}W = \int z dW \quad (5.16)$$

In the case of a *homogeneous body*, the center of gravity G coincides with the *centroid* C of the volume V of the body; the coordinates of C are defined by the relations

$$\bar{x}V = \int x dV \quad \bar{y}V = \int y dV \quad \bar{z}V = \int z dV \quad (5.17)$$

If the volume possesses a *plane of symmetry*, its centroid C will lie in that plane; if it possesses two planes of symmetry, C will be located on the line of intersection of the two planes; if it possesses three planes of symmetry which intersect at only one point, C will coincide with that point [Sec. 5.10].

Center of gravity of a three-dimensional body

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elements 5.27. De the cen

which plane tion Ch into thin relation

with a a tin

The volumes and centroids of various common three-dimensional shapes are tabulated in Fig. 5.21. When a body can be divided into several of these shapes, the coordinates \bar{X} , \bar{Y} , \bar{Z} of its center of gravity \bar{C} can be determined from the corresponding coordinates of the centers of gravity of its various parts [Sec. 5.11]. We have

$$\bar{X}\Sigma W = \Sigma \bar{x}W \quad \bar{Y}\Sigma W = \Sigma \bar{y}W \quad \bar{Z}\Sigma W = \Sigma \bar{z}W \quad (5.19)$$

If the body is made of a homogeneous material, its center of gravity coincides with the centroid C of its volume, and we write [Sample Probs. 5.11 and 5.12]

$$\bar{X}\Sigma V = \Sigma \bar{x}V \quad \bar{Y}\Sigma V = \Sigma \bar{y}V \quad \bar{Z}\Sigma V = \Sigma \bar{z}V \quad (5.20)$$

When a volume is bounded by analytical surfaces, the coordinates of its centroid can be determined by *integration* [Sec. 5.12]. To avoid the computation of the triple integrals in Eqs. (5.18), we can use

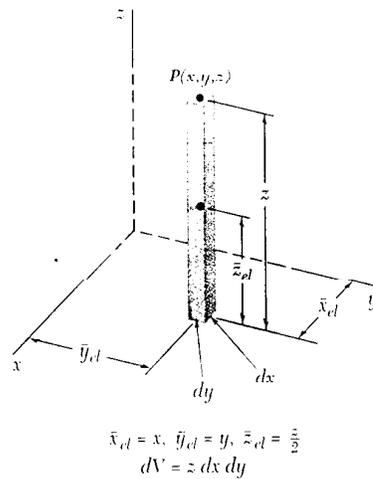


Fig. 5.27

elements of volume in the shape of thin filaments, as shown in Fig. 5.27. Denoting by \bar{x}_{el} , \bar{y}_{el} , and \bar{z}_{el} the coordinates of the centroid of the element dV , we rewrite Eqs. (5.18) as

$$\bar{x}V = \int \bar{x}_{el} dV \quad \bar{y}V = \int \bar{y}_{el} dV \quad \bar{z}V = \int \bar{z}_{el} dV \quad (5.22)$$

which involve only double integrals. If the volume possesses *two planes of symmetry*, its centroid C is located on their line of intersection. Choosing the x axis to lie along that line and dividing the volume into thin slabs parallel to the yz plane, we can determine C from the relation

$$\bar{x}V = \int \bar{x}_{el} dV \quad (5.23)$$

with a *single integration* [Sample Prob. 5.13]. For a body of revolution, these slabs are circular and their volume is given in Fig. 5.28.

Fig. 5.28. Determination of centroid of a body of revolution.

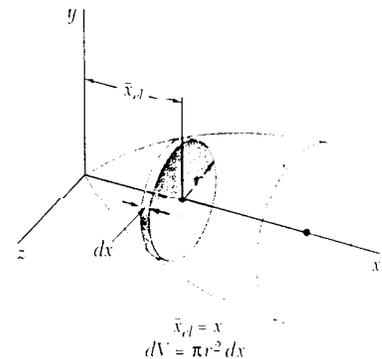


Fig. 5.28

MODULE NO. 6

REVIEW AND SUMMARY FOR CHAPTER 6

In this chapter you learned to determine the *internal forces* holding together the various parts of a structure.

The first half of the chapter was devoted to the analysis of *trusses*, i.e., to the analysis of structures consisting of *straight members connected at their extremities only*. The members being slender and unable to support lateral loads, all the loads must be applied at the joints; a truss may thus be assumed to consist of *pins and two-force members* [Sec. 6.2].

A truss is said to be *rigid* if it is designed in such a way that it will not greatly deform or collapse under a small load. A triangular truss consisting of three members connected at three joints is clearly a rigid truss (Fig. 6.25a) and so will be the truss obtained by adding two new members to the first one and connecting them at a new joint (Fig. 6.25b). Trusses obtained by repeating this procedure are called *simple trusses*. We may check that in a simple truss the total number of members is $m = 2n - 3$, where n is the total number of joints [Sec. 6.3].

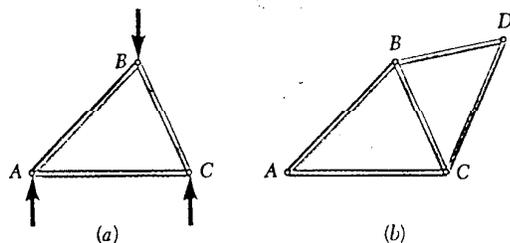


Fig. 6.25

The forces in the various members of a simple truss can be determined by the *method of joints* [Sec. 6.4]. First, the reactions at the supports can be obtained by considering the entire truss as a free body. The free-body diagram of each pin is then drawn, showing the forces exerted on the pin by the members or supports it connects. Since the members are straight two-force members, the force exerted by a member on the pin is directed along that member, and only the magnitude of the force is unknown. It is always possible in the case of a simple truss to draw the free-body diagrams of the pins in such an order that only two unknown forces are included in each diagram. These forces can be obtained from the corresponding two equilibrium equations or—if only three forces are involved—from the cor-

Analysis of trusses

Simple trusses

Method of joints

responding force triangle. If the force exerted by a member on a pin is directed toward that pin, the member is in *compression*; if it is directed away from the pin, the member is in *tension* [Sample Prob. 6.1]. The analysis of a truss is sometimes expedited by first recognizing *joints under special loading conditions* [Sec. 6.5]. The method of joints can also be extended to the analysis of *three-dimensional or space trusses* [Sec. 6.6].

Method of sections

The *method of sections* is usually preferred to the method of joints when the force in only one member—or very few members—of a truss is desired [Sec. 6.7]. To determine the force in member BD of the truss of Fig. 6.26a, for example, we pass a section through members BD , BE , and CE , remove these members, and use the portion ABC of the truss as a free body (Fig. 6.26b). Writing $\sum M_E = 0$, we determine the magnitude of the force F_{BD} , which represents the force in member BD . A positive sign indicates that the member is in *tension*; a negative sign indicates that it is in *compression* [Sample Probs. 6.2 and 6.3].

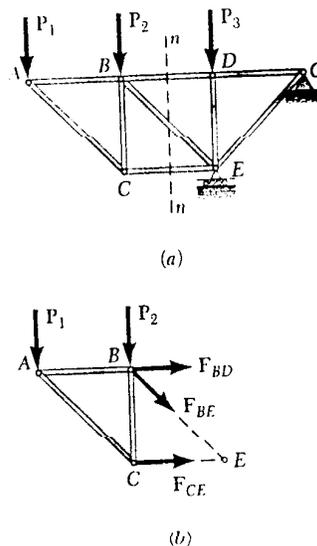


Fig. 6.26

Compound trusses

The method of sections is particularly useful in the analysis of *compound trusses*, i.e., trusses which cannot be constructed from a basic triangular truss of Fig. 6.25a but which can be obtained by rigidly connecting several simple trusses [Sec. 6.8]. If the compound trusses have been properly connected (e.g., one pin and one roller), three nonconcurrent and nonparallel links) and if the resulting structure is properly supported (e.g., one pin and one roller), the compound truss is *statically determinate, rigid, and completely strained*. The following necessary—but not sufficient—condition is then satisfied: $m + r = 2n$, where m is the number of members, r is the number of unknowns representing the reactions at the supports, and n is the number of joints.

FRAMES AND MACHINES

The second part of the chapter was devoted to the analysis of *frames and machines*. Frames and machines are structures which contain *multiforce members*, i.e., members acted upon by three or more forces. Frames are designed to support loads and are usually stationary, fully constrained structures. Machines are designed to transmit or modify forces and always contain moving parts [Sec. 6.9].

Analysis of a frame

To analyze a frame, we first consider the *entire frame as a free body* and write three equilibrium equations [Sec. 6.10]. If the frame remains rigid when detached from its supports, the reactions involve only three unknowns and may be determined from these equations [Sample Probs. 6.4 and 6.5]. On the other hand, if the frame ceases to be rigid when detached from its supports, the reactions involve more than three unknowns and cannot be completely determined from the equilibrium equations of the frame [Sec. 6.11; Sample Prob. 6.6].

Multiforce members

We then *dismember the frame* and identify the various members as either *two-force members* or *multiforce members*; pins are assumed to form an integral part of one of the members they connect. We draw the free-body diagram of each of the multiforce members, noting that when two multiforce members are connected to the same two-force member, they are acted upon by that member with *equal and opposite forces of unknown magnitude but known direction*. When two multiforce members are connected by a pin, they exert on each other *equal and opposite forces of unknown direction*, which should be represented by *two unknown components*. The equilibrium equations obtained from the free-body diagrams of the multiforce members can then be solved for the various internal forces [Sample Probs. 6.4 and 6.5]. The equilibrium equations can also be used to complete the determination of the reactions at the supports [Sample Prob. 6.6]. Actually, if the frame is *statically determinate and rigid*, the free-body diagrams of the multiforce members could provide as many equations as there are unknown forces (including the reactions) [Sec. 6.11]. However, as suggested above, it is advisable to first consider the free-body diagram of the entire frame to minimize the number of equations that must be solved simultaneously.

Analysis of a machine

To analyze a machine, we dismember it and, following the same procedure as for a frame, draw the free-body diagram of each of the multiforce members. The corresponding equilibrium equations yield the *output forces* exerted by the machine in terms of the *input forces* applied to it, as well as the *internal forces* at the various connections [Sec. 6.12; Sample Prob. 6.7].

shear and bending in the beam. The loads may be either concentrated at specific points, or distributed along the entire length or a portion of the beam. The beam itself may be supported in various ways, since only statically determinate beams are considered in this text; we limited our analysis to that of simply supported beams, overhanging beams, and cantilever beams [Sec. 7.3].

To obtain the shear V and bending moment M at a given point C of a beam, we first determine the reactions at the supports by considering the entire beam as a free body. We then cut the beam at C and draw the free-body diagram of one of the two portions obtained in this fashion to determine V and M . In order to avoid any confusion regarding the sense of the shearing force V and couple M (which act in opposite directions on the two portions of the beam), the sign convention illustrated in Fig. 7.21 was adopted [Sec. 7.4]. Once the values of the shear and bending moment have been determined at a few selected points of the beam, it is usually possible to draw a shear diagram and a bending-moment diagram representing, respectively, the shear and bending moment at any point of the beam [Sec. 7.5]. When a beam is subjected to concentrated loads only, the shear is of constant value between loads and the bending moment varies linearly between loads [Sample Prob. 7.2]. On the other hand, when a beam is subjected to distributed loads, the shear and bending moment vary quite differently [Sample Prob. 7.3].

The construction of the shear and bending-moment diagrams is facilitated if the following relations are taken into account. Denoting by w the distributed load per unit length (assumed positive if directed downward), we have [Sec. 7.5]:

$$\frac{dV}{dx} = -w \quad (7.1)$$

$$\frac{dM}{dx} = V \quad (7.3)$$

or, in integrated form,

$$V_D - V_C = -(\text{area under load curve between } C \text{ and } D) \quad (7.2')$$

$$M_D - M_C = \text{area under shear curve between } C \text{ and } D \quad (7.4')$$

Equation (7.2') makes it possible to draw the shear diagram of a beam from the curve representing the distributed load on that beam and the value of V at one end of the beam. Similarly, Eq. (7.4') makes it possible to draw the bending-moment diagram from the shear diagram and the value of M at one end of the beam. However, concentrated loads introduce discontinuities in the shear diagram and concentrated couples in the bending-moment diagram, none of which are accounted for in these equations [Sample Probs. 7.4 and 7.7]. Finally, we note from Eq. (7.3) that the points of the beam where the bending moment is maximum or minimum are also the points where the shear is zero [Sample Prob. 7.5].

The second half of the chapter was devoted to the analysis of flexible cables. We first considered a cable of negligible weight supporting concentrated loads [Sec. 7.7]. Using the entire cable AB as a

Fig. 7.21 Internal forces at section in a beam



Internal forces at section (positive shear and positive bending moment)
Fig. 7.21

Relations among load, shear, and bending moment

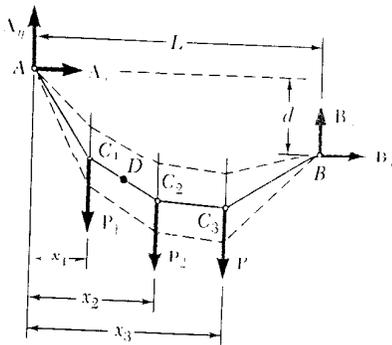


Fig. 7.22

Cables with distributed loads

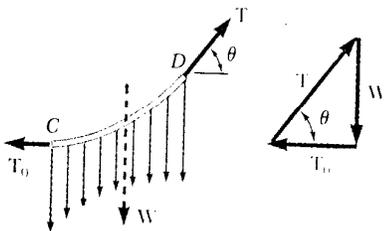


Fig. 7.23

Parabolic cable

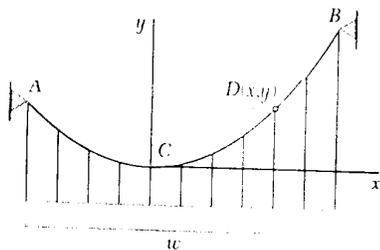


Fig. 7.24

Catenary

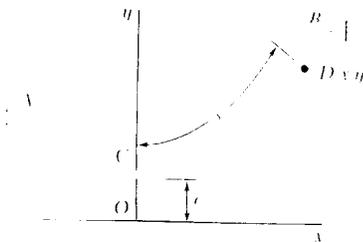


Fig. 7.25

free body (Fig. 7.22), we noted that the three available equilibrium equations were not sufficient to determine the four unknowns representing the reactions at the supports A and B. However, if the coordinates of a point D of the cable are known, an additional equation can be obtained by considering the free-body diagram of the portion AD or DB of the cable. Once the reactions at the supports have been determined, the elevation of any point of the cable and the tension in any portion of the cable can be found from the appropriate free-body diagram [Sample Prob. 7.8]. It was noted that the horizontal component of the force **T** representing the tension is the same at any point of the cable.

We next considered cables carrying *distributed loads* [Sec. 7.8]. Using as a free body a portion of cable CD extending from the lowest point C to an arbitrary point D of the cable (Fig. 7.23), we observed that the horizontal component of the tension force **T** at D is constant and equal to the tension T_0 at C, while its vertical component is equal to the weight W of the portion of cable CD. The magnitude and direction of **T** were obtained from the force triangle:

$$T = \sqrt{T_0^2 + W^2} \quad \tan \theta = \frac{W}{T_0} \quad (7.6)$$

In the case of a load *uniformly distributed along the horizontal* as in a suspension bridge (Fig. 7.24)—the load supported by portion CD is $W = wx$, where w is the constant load per unit horizontal length [Sec. 7.9]. We also found that the curve formed by the cable is a *parabola* of equation

$$y = \frac{wx^2}{2T_0} \quad (7.6)$$

and that the length of the cable can be found by using the expansion in series given in Eq. (7.10) [Sample Prob. 7.9]

In the case of a load *uniformly distributed along the cable itself*—e.g., a cable hanging under its own weight (Fig. 7.25)—the load supported by portion CD is $W = ws$, where s is the length measured along the cable and w is the constant load per unit length [Sec. 7.10]. Choosing the origin O of the coordinate axes at a distance $c = T_0/w$ below C , we derived the relations

$$s = c \sinh \frac{x}{c} \quad (7.15)$$

$$y = c \cosh \frac{x}{c} \quad (7.16)$$

$$y^2 - s^2 = c^2 \quad (7.17)$$

$$T_0 = wc \quad W = ws \quad T = wy \quad (7.18)$$

which can be used to solve problems involving cables hanging under their own weight [Sample Prob. 7.10]. Equation (7.16), which defines the shape of the cable, is the equation of a *catenary*.

INDEX AND SUMMARY

CHAPTER 8

This chapter was devoted to the study of *dry friction*, i.e., to problems involving rigid bodies which are in contact along *nonlubricated surfaces*.

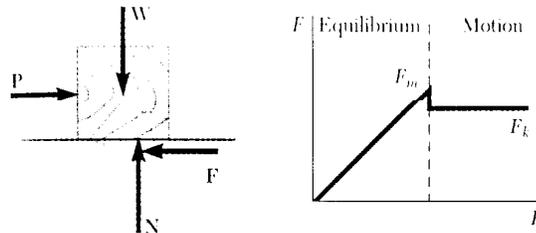


Fig. 8.16

Applying a horizontal force \mathbf{P} to a block resting on a horizontal surface [Sec. 8.2], we note that the block at first does not move. This shows that a *friction force* \mathbf{F} must have developed to balance \mathbf{P} (Fig. 8.16). As the magnitude of \mathbf{P} is increased, the magnitude of \mathbf{F} also increases until it reaches a maximum value F_m . If \mathbf{P} is further increased, the block starts sliding and the magnitude of \mathbf{F} drops from F_m to a lower value F_k . Experimental evidence shows that F_m and F_k are proportional to the normal component N of the reaction of the surface. We have

$$F_m = \mu_s N \quad F_k = \mu_k N \quad (8.1, 8.2)$$

where μ_s and μ_k are called, respectively, the *coefficient of static friction* and the *coefficient of kinetic friction*. These coefficients depend on the nature and the condition of the surfaces in contact. Approximate values of the coefficients of static friction were given in Table 5.1.

It is sometimes convenient to replace the normal force \mathbf{N} and the friction force \mathbf{F} by their resultant \mathbf{R} (Fig. 8.17). As the friction force increases and reaches its maximum value $F_m = \mu_s N$, the angle ϕ that \mathbf{R} forms with the normal to the surface increases and reaches a maximum value ϕ_s , called the *angle of static friction*. If motion actually takes place, the magnitude of \mathbf{F} drops to F_k ; similarly the angle ϕ drops to a lower value ϕ_k , called the *angle of kinetic friction*. As shown in Sec. 8.3, we have

$$\tan \phi_s = \mu_s \quad \tan \phi_k = \mu_k \quad (8.3, 8.4)$$

Static and kinetic friction

Angles of friction

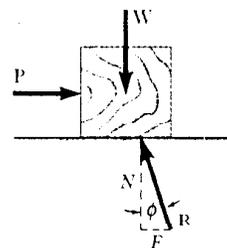


Fig. 8.17

Problems involving friction

When solving equilibrium problems involving friction, we should keep in mind that the magnitude F of the friction force is equal to $F_m = \mu_s N$ only if the body is about to slide [Sec. 8.4]. If motion is not impending, F and N should be considered as independent unknowns to be determined from the equilibrium equations (Fig. 8.18a). We

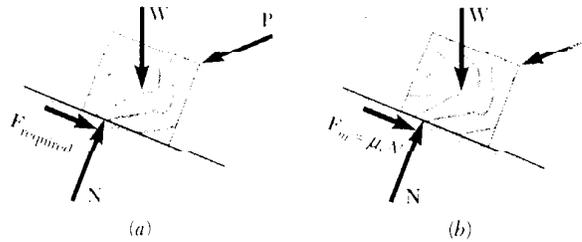


Fig. 8.18

should also check that the value of F required to maintain equilibrium is not larger than F_m ; if it were, the body would move and the magnitude of the friction force would be $F_k = \mu_k N$ [Sample Prob. 8.1]. On the other hand, if motion is known to be impending, F has reached its maximum value $F_m = \mu_s N$ (Fig. 8.18b), and this expression may be substituted for F in the equilibrium equations [Sample Prob. 8.3]. When only three forces are involved in a free-body diagram, including the reaction \mathbf{R} of the surface in contact with the body, it is usually more convenient to solve the problem by drawing a force triangle [Sample Prob. 8.2].

When a problem involves the analysis of the forces exerted on each other by two bodies A and B , it is important to show the friction forces with their correct sense. The correct sense for the friction force exerted by B on A , for instance, is opposite to that of the relative motion (or impending motion) of A with respect to B [Fig. 8.6].

Wedges and screws

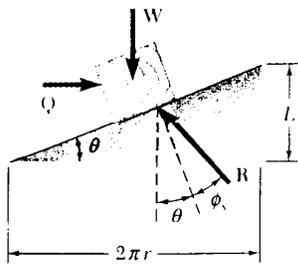


Fig. 8.19

In the second part of the chapter we considered a number of specific engineering applications where dry friction plays an important role. In the case of wedges, which are simple machines used to raise heavy loads [Sec. 8.5], two or more free-body diagrams were drawn and care was taken to show each friction force with its correct sense [Sample Prob. 8.4]. The analysis of square-threaded screws which are frequently used in jacks, presses, and other mechanisms was reduced to the analysis of a block sliding on an incline by unwrapping the thread of the screw and showing it as a straight line [Sec. 8.6]. This is done again in Fig. 8.19, where r denotes the mean radius of the thread, L is the lead of the screw (i.e., the distance through which the screw advances in one turn), \mathbf{W} is the load, and Q is equal to the torque exerted on the screw. It was noted that in the case of multiple-threaded screws the lead L of the screw is not equal to its pitch, which is the distance measured between two consecutive threads.

Other journal bearings friction. See belt friction.

In some cylinder, it belt slips or impending rotating direction.

slip to the drum on the larger in the direction. Determine coefficient of friction.

which we actually should be replaced by formulas.

MODULE NO. 9

REVIEW AND SUMMARY FOR CHAPTER 9

In the first half of this chapter, we discussed the determination of the resultant \mathbf{R} of forces ΔF distributed over a plane area A when the magnitudes of these forces are proportional to both the areas ΔA of the elements on which they act and the distances y from these elements to a given x axis; we thus had $\Delta F = ky \Delta A$. We found that the magnitude of the resultant \mathbf{R} is proportional to the first moment $Q_x = \int y dA$ of the area A , while the moment of \mathbf{R} about the x axis is proportional to the *second moment*, or *moment of inertia*, $I_x = \int y^2 dA$ of A with respect to the same axis [Sec. 9.2].

The *rectangular moments of inertia* I_x and I_y of an area [Sec. 9.3] were obtained by evaluating the integrals

$$I_x = \int y^2 dA \quad I_y = \int x^2 dA \quad (9.1)$$

These computations can be reduced to single integrations by choosing dA to be a thin strip parallel to one of the coordinate axes. We also recall that it is possible to compute I_x and I_y from the same elemental strip (Fig. 9.35) using the formula for the moment of inertia of a rectangular area [Sample Prob. 9.3].

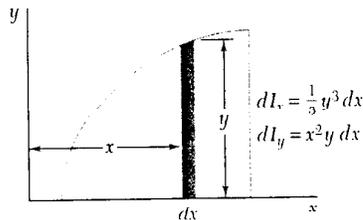


Fig. 9.35

The *polar moment of inertia of an area* A with respect to the pole O [Sec. 9.4] was defined as

$$J_O = \int r^2 dA \quad (9.3)$$

where r is the distance from O to the element of area dA (Fig. 9.36). Observing that $r^2 = x^2 + y^2$, we established the relation

$$J_O = I_x + I_y \quad (9.4)$$

Rectangular moments of inertia

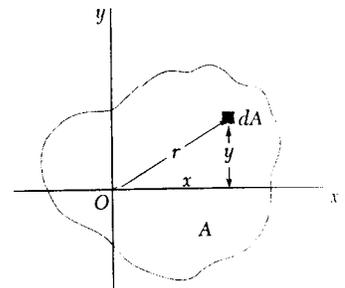


Fig. 9.36

Polar moment of inertia

The *radius of gyration of an area A* with respect to the *x* axis [Sec. 9.5] was defined as the distance k_x , where $I_x = k_x^2 A$. With similar definitions for the radii of gyration of *A* with respect to the *y* axis and with respect to *O*, we had

$$k_x = \sqrt{\frac{I_x}{A}} \quad k_y = \sqrt{\frac{I_y}{A}} \quad k_O = \sqrt{\frac{J_O}{A}} \quad (9.5-9.7)$$

The *parallel-axis theorem* was presented in Sec. 9.6. It states that the moment of inertia I of an area with respect to any given axis AA' (Fig. 9.37) is equal to the moment of inertia \bar{I} of the area with respect to the centroidal axis BB' that is parallel to AA' plus the product of the area A and the square of the distance d between the two axes

$$I = \bar{I} + Ad^2 \quad (9.9)$$

This formula can also be used to determine the moment of inertia I of an area with respect to a centroidal axis BB' when its moment of inertia I with respect to a parallel axis AA' is known. In this case, however, the product Ad^2 should be *subtracted* from the known moment of inertia I .

A similar relation holds between the polar moment of inertia J_O of an area about a point O and the polar moment of inertia \bar{J}_C of the same area about its centroid C . Letting d be the distance between O and C , we have

$$J_O = \bar{J}_C + Ad^2 \quad (9.11)$$

The parallel-axis theorem can be used very effectively to compute the *moment of inertia of a composite area* with respect to a given axis [Sec. 9.7]. Considering each component area separately, we first compute the moment of inertia of each area with respect to its centroidal axis, using the data provided in Figs. 9.12 and 9.13 whenever possible. The parallel-axis theorem is then applied to determine the moment of inertia of each component area with respect to the desired axis, and the various values obtained are added [Sample Probs. 9.4 and 9.5].

Sections 9.8 through 9.10 were devoted to the transformation of the moments of inertia of an area *under a rotation of the coordinate axes*. First, we defined the *product of inertia of an area A* as

$$I_{xy} = \int xy \, dA \quad (9.12)$$

and showed that $I_{xy} = 0$ if the area A is symmetrical with respect to either or both of the coordinate axes. We also derived the *parallel-axis theorem for products of inertia*. We had

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A \quad (9.13)$$

where $\bar{I}_{x'y'}$ is the product of inertia of the area with respect to the centroidal axes x' and y' which are parallel to the x and y axis and \bar{x} and \bar{y} are the coordinates of the centroid of the area (Sec. 9.8).

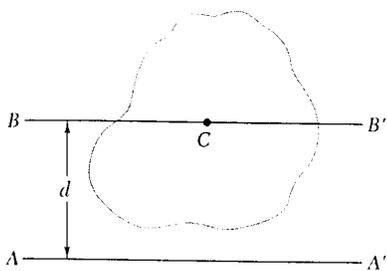


Fig. 9.37

In Sec. 9.6, we defined the moments of inertia I_x and I_y of an area with respect to the x and y axes, respectively, as the sum of the squares of the distances from the x and y axes to the centroid of the area. In this case, however, the product Ad^2 should be subtracted from the known moment of inertia I .

The parallel-axis theorem can be used very effectively to compute the moment of inertia of a composite area with respect to a given axis [Sec. 9.7].

The product of inertia of an area A is defined as $I_{xy} = \int xy \, dA$.

We also showed that $I_{xy} = 0$ if the area A is symmetrical with respect to either or both of the coordinate axes.

The parallel-axis theorem for products of inertia states that $I_{xy} = \bar{I}_{x'y'} + \bar{x}\bar{y}A$.

MODULE NO. 10

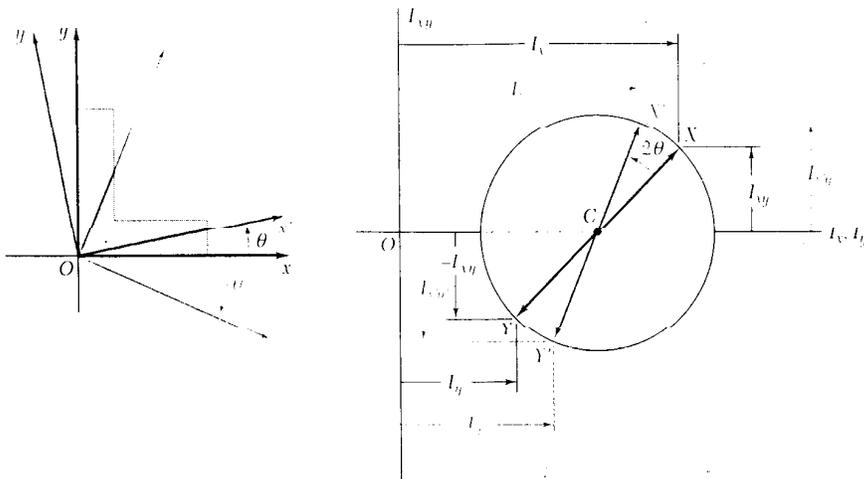


Fig. 9.39

axes, we plot points $X (I_x, I_{xy})$ and $Y (I_y, -I_{xy})$ and draw the line joining these two points (Fig. 9.39). This line is a diameter of Mohr's circle and thus defines this circle. As the coordinate axes are rotated through θ , the diameter rotates through *twice that angle*, and the coordinates of X' and Y' yield the new values $I_{x'}$, $I_{y'}$, and $I_{x'y'}$ of the moments and product of inertia of the area. Also, the angle θ_m and the coordinates of points A and B define the principal axes a and b and the principal moments of inertia of the area [Sample Prob. 9.8]

Moments of Inertia of Masses

The second half of the chapter was devoted to the determination of *moments of inertia of masses*, which are encountered in dynamics in problems involving the rotation of a rigid body about an axis. The mass moment of inertia of a body with respect to an axis AA' (Fig. 9.40) was defined as

$$I = \int r^2 dm \quad (9.28)$$

where r is the distance from AA' to the element of mass [Sec. 9.11]. The *radius of gyration* of the body was defined as

$$k = \sqrt{\frac{I}{m}} \quad (9.29)$$

The moments of inertia of a body with respect to the coordinate axes were expressed as

$$\begin{aligned} I_x &= \int (y^2 + z^2) dm \\ I_y &= \int (z^2 + x^2) dm \\ I_z &= \int (x^2 + y^2) dm \end{aligned} \quad (9.30)$$

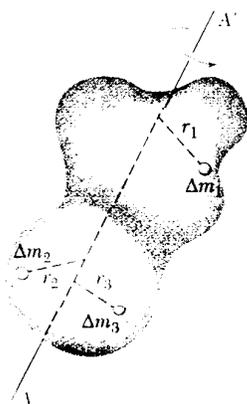


Fig. 9.40

We saw that moments of inertia with respect to

where \bar{I} is the centroidal axis of the body, and

The moment of inertia from the mass for a rectangular body shown (Fig.

while for a

When possible to use with respect to a thin plate body moment of inertia formulas (see Sample Probs.

In the moment of inertia when

We saw that the *parallel-axis theorem* also applies to mass moments of inertia [Sec. 9.12]. Thus, the moment of inertia I of a body with respect to an arbitrary axis AA' (Fig. 9.41) can be expressed as

$$I = \bar{I} + md^2 \tag{9.33}$$

where \bar{I} is the moment of inertia of the body with respect to the centroidal axis BB' which is parallel to the axis AA' , m is the mass of the body, and d is the distance between the two axes.

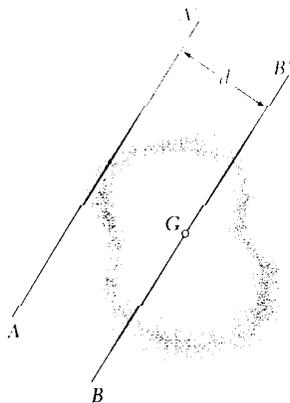


Fig. 9.41

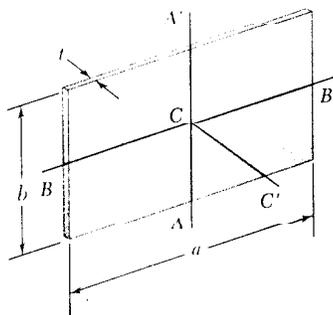


Fig. 9.42

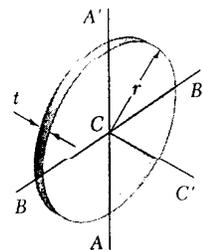


Fig. 9.43

The moments of inertia of *thin plates* can be readily obtained from the moments of inertia of their areas [Sec. 9.13]. We found that for a *rectangular plate* the moments of inertia with respect to the axes shown (Fig. 9.42) are

$$I_{AA'} = \frac{1}{12}ma^2 \quad I_{BB'} = \frac{1}{12}mb^2 \tag{9.39}$$

$$I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{12}m(a^2 + b^2) \tag{9.40}$$

while for a *circular plate* (Fig. 9.43) they are

$$I_{AA'} = I_{BB'} = \frac{1}{4}mr^2 \tag{9.41}$$

$$I_{CC'} = I_{AA'} + I_{BB'} = \frac{1}{2}mr^2 \tag{9.42}$$

When a body possesses *two planes of symmetry*, it is usually possible to use a single integration to determine its moment of inertia with respect to a given axis by selecting the element of mass dm to be a thin plate [Sample Probs. 9.10 and 9.11]. On the other hand, when a body consists of *several common geometric shapes*, its moment of inertia with respect to a given axis can be obtained by using the formulas given in Fig. 9.28 together with the parallel-axis theorem [Sample Probs. 9.12 and 9.13].

In the last portion of the chapter, we learned to determine the moment of inertia of a body *with respect to an arbitrary axis* OL which is drawn through the origin O [Sec. 9.16]. Denoting by λ_x , λ_y ,

Moments of inertia of thin plates

Composite bodies

Moment of inertia with respect to an arbitrary axis

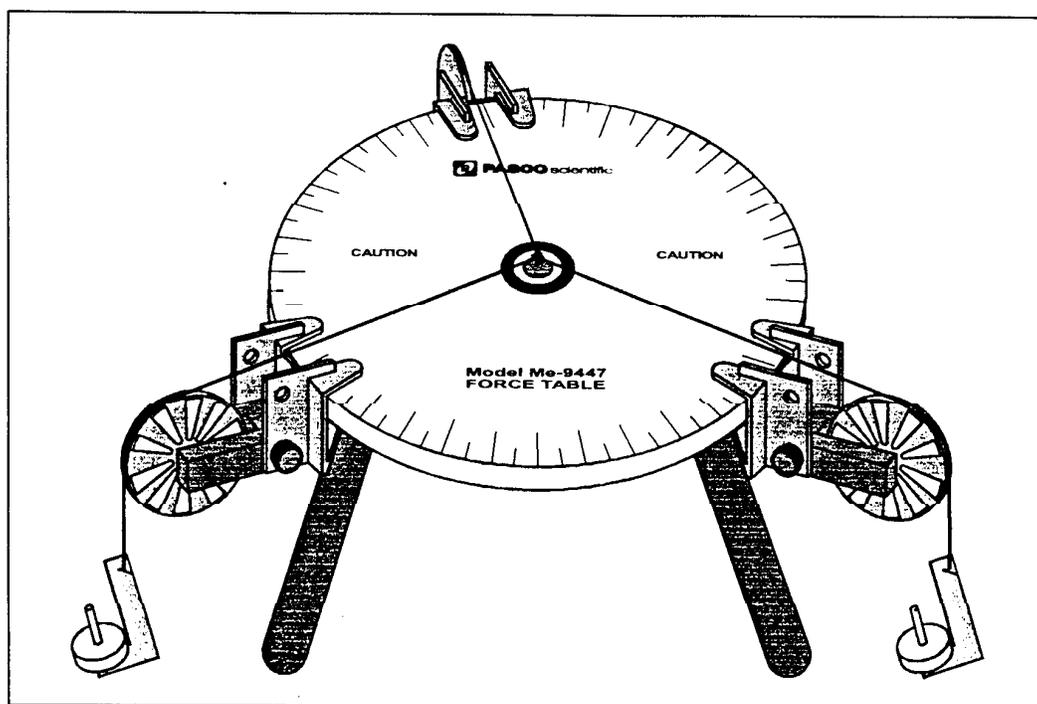
APPENDIX 2:

Force Table

**Instruction Manual
and Experiment Guide
for the PASCO scientific
Model ME-9447**

012-04715
4/96
Revision B

Force Table



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Introduction

The PASCO Model ME-9447 Force Table is used to physically demonstrate the addition of vectors using the concept of equilibrium. The vectors are forces supplied by the weight of the masses that hang over the pulleys. Masses hanging over pulleys placed at given angles are balanced by another mass over a pulley at another angle.

The PASCO force table has several excellent features:

- It is lightweight.
- **NOTE:** Do not exceed 200 g on each pulley.
- It can be stored in a small space.
- The pulleys have very little friction.
- The pulley clamps allow the string to be lowered close to the angle markings on the table, reducing the parallax in reading the angle.
- Equilibrium can be obtained by centering a knot (which gives greater precision) or by centering the conventional ring.

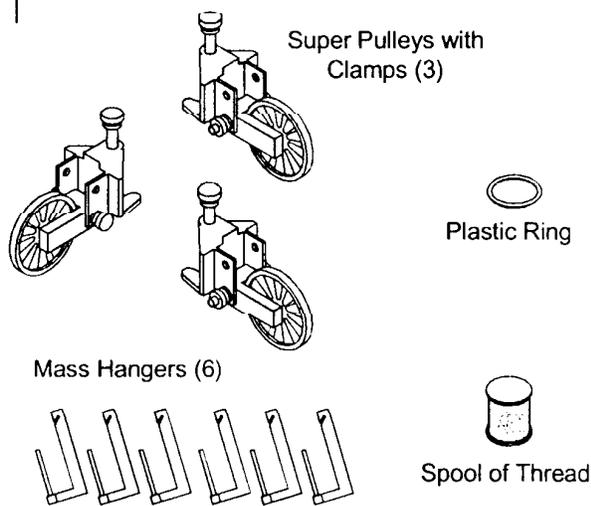
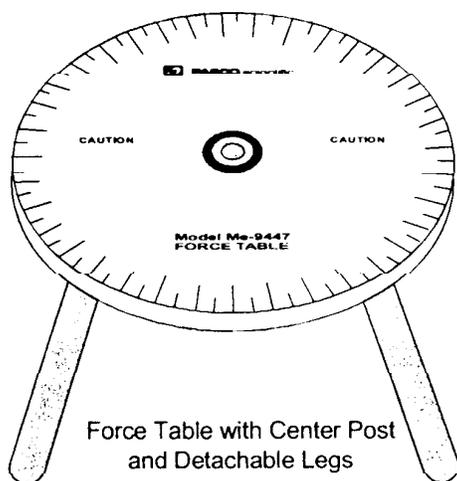
Equipment

The ME-9447 Force Table includes the following:

- Force Table Assembly with Center Post and three Detachable Legs
- Three Super Pulley Clamps
- Three Mass Hangers
- Plastic Ring
- Spool of Thread
- Instruction Manual/Experiments Guide which includes protractors that may be reproduced

Additional Equipment Recommended

- Masses such as PASCO model ME-9348 Mass and Hanger Set or PASCO model SE-8704 Slotted Mass Set
- Metric Ruler such as PASCO model SE-8731
- Protractor such as PASCO model SE-8732



Assembly

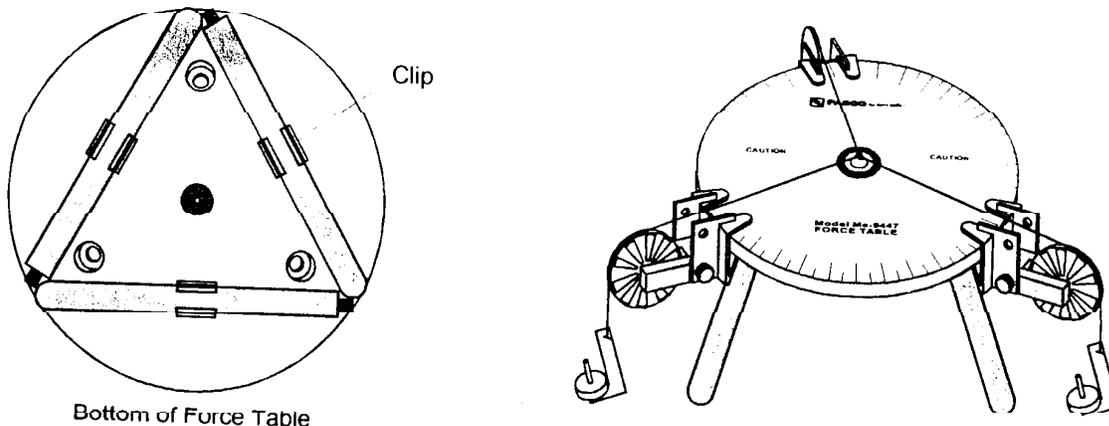


Figure 1 Force Table: Before and After Assembly

Assembly

- ① Remove the three legs from the clips on the bottom of the plastic force table disk.
- ② Screw the legs into the holes on the bottom of the disk (see Figure 1.)
- ③ Attach three pulleys and clamps to the rim of the disk. If more than two forces are to be added, use the desired number plus one pulley and clamp for the equilibrant force.

There are two ways to attach the strings to the table: The first way uses the conventional ring in the center of the table and the second way uses an anchor string

through the hole in the center of the table. The advantage of the anchor string is that a higher precision can be achieved because a single knot is being centered instead of the massive ring. The anchor string keeps the masses from falling to one side when the system is not in equilibrium.

► **NOTE:** In both methods it is important to adjust the pulleys so that the strings are parallel to the top surface of the Force Table, and as close to the top surface as possible. When adjusting the pulleys, don't let the ring rest on the top surface.

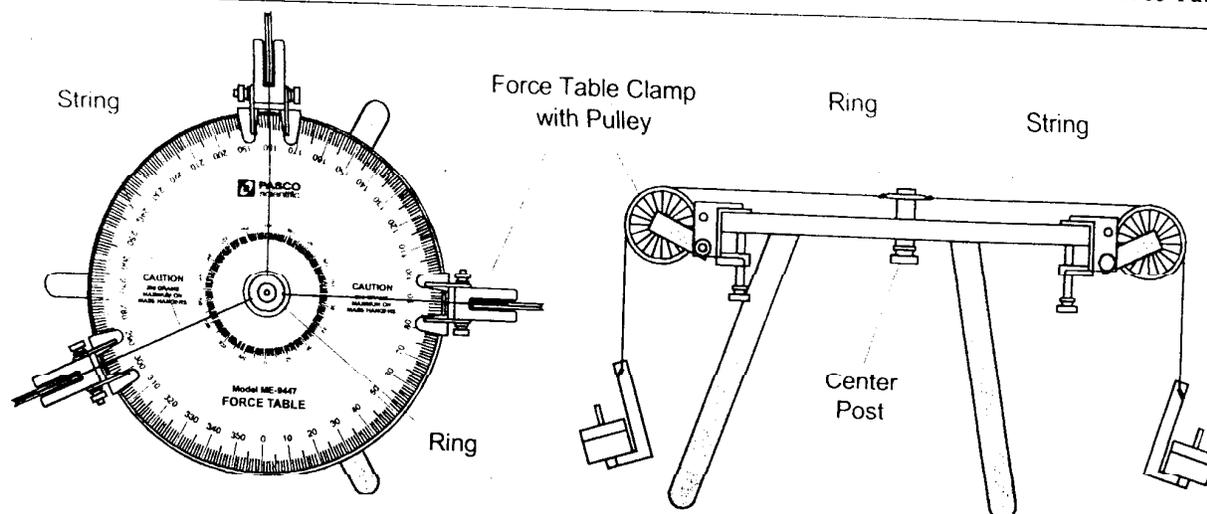


Figure 2 Ring Method of Stringing Force Table

Ring Method

See Figure 2. To use this method, screw the center post up until it stops so that it sticks up above the table. Place the ring over the post and tie one 30 cm long string to the ring for each pulley. The strings must be long enough to reach over the pulleys. Place each string over a pulley and tie a mass hanger to it.

► **NOTE:** A string can be attached to the PASCO mass hanger by wrapping the string several times (4 or 5) around the notch at the top of each mass hanger.

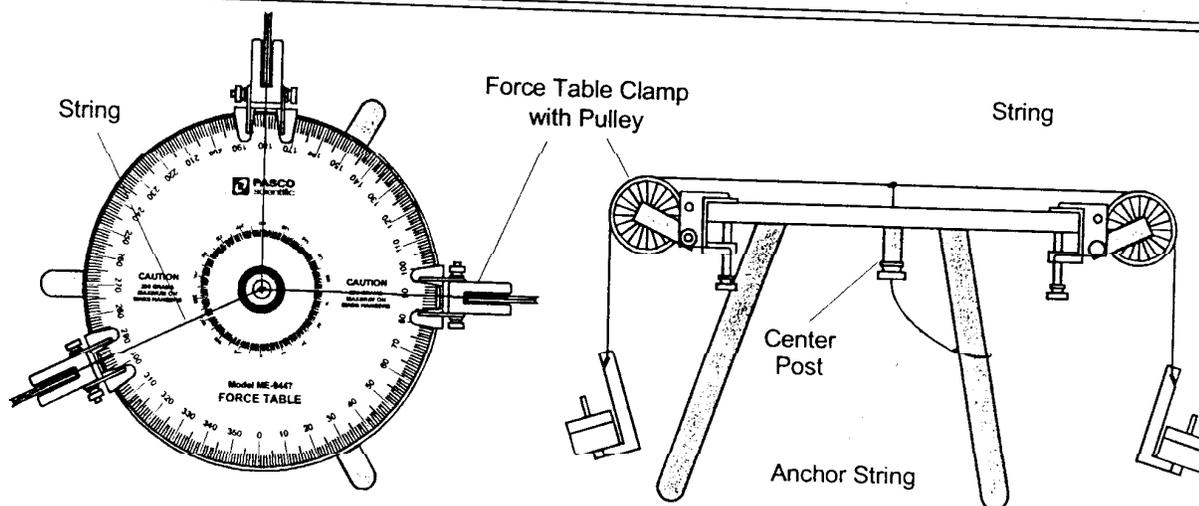


Figure 3 Anchor Method of Stringing Force Table

Anchor String Method

See Figure 3. Cut two 60cm lengths of string and tie them together at their centers (to form an "X"). Three of the ends will reach from the center of the table over a pulley; the fourth will be threaded down through the hole in the center post to act as the anchor string. Screw the center post down so it is flush with the top surface of the table. Thread the anchor string down through the hole in the center post and tie that end to

one of the legs. Put each of the other strings over a pulley and tie a mass hanger on the end of each string.

► **NOTE:** A string can be attached to the PASCO mass hanger by wrapping the string several times (4 or 5) around the notch at the top of each mass hanger.

Experiment 1: Vector Addition

EQUIPMENT NEEDED:

ME-9447 Force Table, -3 pulleys and pulley clamps, -3 mass hangers, -mass set, -string, -metric ruler, -protractor, -2 sheets of paper

Purpose

The purpose of this experiment is to use the force table to experimentally determine the force which balances two other forces. This result is checked by adding the two forces by using their components and by graphically adding the forces.

Theory

This experiment finds the resultant of adding two vectors by three methods: experimentally, by components, and graphically

- **NOTE:** In all cases, the force caused by the mass hanging over the pulley is found by multiplying the mass by the acceleration due to gravity.

Experimental Method

Two forces are applied on the force table by hanging masses over pulleys positioned at certain angles. Then the angle and mass hung over a third pulley are adjusted until it balances the other two forces. This third force is called the equilibrant (\vec{F}_E) since it is the force which establishes equilibrium. The equilibrant is not the same as the resultant (\vec{F}_R). The resultant is the addition of the two forces. While the equilibrant is equal in magnitude to the resultant, it is in the opposite direction because it balances the resultant (see Figure 4). So the equilibrant is the negative of the resultant:

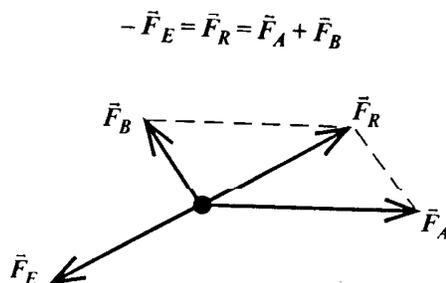


Figure 4 The Equilibrant Balances the Resultant

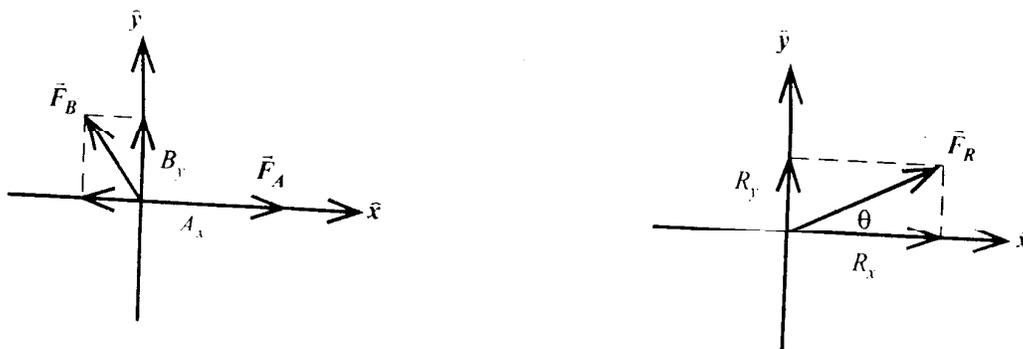


Figure 5 Components

Component Method

Two forces are added together by adding the x- and y-components of the forces. First the two forces are broken into their x- and y-components using trigonometry:

$$\vec{F}_A = A_x \hat{x} + A_y \hat{y} \text{ and } B_x \hat{x} + B_y \hat{y}$$

where A_x is the x-component of vector \vec{F}_A and \hat{x} is the unit vector in the x-direction. See Figure 5. To determine the sum of \vec{F}_A and \vec{F}_B , the components are added to get the components of the resultant \vec{F}_R :

$$\vec{F}_R = (A_x + B_x) \hat{x} + (A_y + B_y) \hat{y} = R_x \hat{x} + R_y \hat{y}$$

To complete the analysis, the resultant force must be in the form of a magnitude and a direction (angle). So the components of the resultant (R_x and R_y) must be combined using the Pythagorean Theorem since the components are at right angles to each other:

$$F_R = \sqrt{R_x^2 + R_y^2}$$

$$\tan(\theta) = \frac{R_x}{R_y}$$

And using trigonometry gives the angle:

Graphical Method

Two forces are added together by drawing them to scale using a ruler and protractor. The second force (\vec{F}_B) is drawn with its tail to the head of the first force (\vec{F}_A). The resultant (\vec{F}_R) is drawn from the tail of \vec{F}_A to the head of \vec{F}_B . See Figure 6. Then the magnitude of the resultant can be measured directly from the diagram and converted to the proper force using the chosen scale. The angle can also be measured using the protractor.

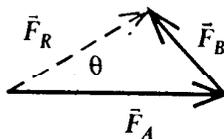


Figure 6 Adding Vectors Head to Tail

Setup

- ① Assemble the force table as shown in the Assembly section. Use three pulleys (two for the forces that will be added and one for the force that balances the sum of the two forces).
- ② If you are using the Ring Method, screw the center post up so that it will hold the ring in place when the masses are suspended from the two pulleys. If you are using the Anchor String Method, leave the center post so that it is flush with the top surface of the force table. Make sure the anchor string is tied to one of the legs of the force table so the anchor string will hold the strings that are attached to the masses that will be suspended from the two pulleys.
- ③ Hang the following masses on two of the pulleys and clamp the pulleys at the given angles:

Force A = 50 g at 0°

Force B = 100 g at 120°

Procedure (Experimental Method)

By trial and error, find the angle for the third pulley and the mass which must be suspended from it that will balance the forces exerted on the strings by the other two masses. The third force is called the equilibrant (\vec{F}_E) since it is the force which establishes equilibrium. The equilibrant is the negative of the resultant:

$$-\vec{F}_E = \vec{F}_R = \vec{F}_A + \vec{F}_B$$

Record the mass and angle required for the third pulley to put the system into equilibrium in Table 1.

To determine whether the system is in equilibrium, use the following criteria.

Ring Method of Finding Equilibrium

The ring should be centered over the post when the system is in equilibrium. Screw the center post down so that it is flush with the top surface of the force table and no longer able to hold the ring in position. Pull the ring slightly to one side and let it go. Check to see that the ring returns to the center. If not, adjust the mass and/or angle of the pulley until the ring always returns to the center when pulled slightly to one side.

Anchor String Method of Finding Equilibrium

The knot should be centered over the hole in the middle of the center post when the system is in equilibrium. The anchor string should be slack. Adjust the pulleys downward until the strings are close to the top surface of the force table. Pull the knot slightly to one side and let it go. Check to see that the knot returns to the center. If not, adjust the mass and/or angle of the third pulley until the knot always returns to the center when pulled slightly to one side.

Analysis

To determine theoretically what mass should be suspended from the third pulley, and at what angle, calculate the magnitude and direction of the equilibrant (\vec{F}_E) by the component method and the graphical method.

Component Method

On a separate piece of paper, add the vector components of Force A and Force B to determine the magnitude of the equilibrant. Use trigonometry to find the direction (remember, the equilibrant is exactly opposite in direction to the resultant). Record the results in Table 1.

Graphical Method

On a separate piece of paper, construct a tail-to-head diagram of the vectors of Force A and Force B. Use a metric rule and protractor to measure the magnitude and direction of the resultant. Record the results in Table 1. Remember to record the direction of the equilibrant, which is opposite in direction to the resultant.

- ① How do the theoretical values for the magnitude and direction of the equilibrant compare to the actual magnitude and direction?

Table 1 Results of the Three Methods of Vector Addition

Method	Equilibrant (F_E)	
	Magnitude	Direction (θ)
Experiment:		
Component: $R_x =$ _____ $R_y =$ _____		
Graphical:		

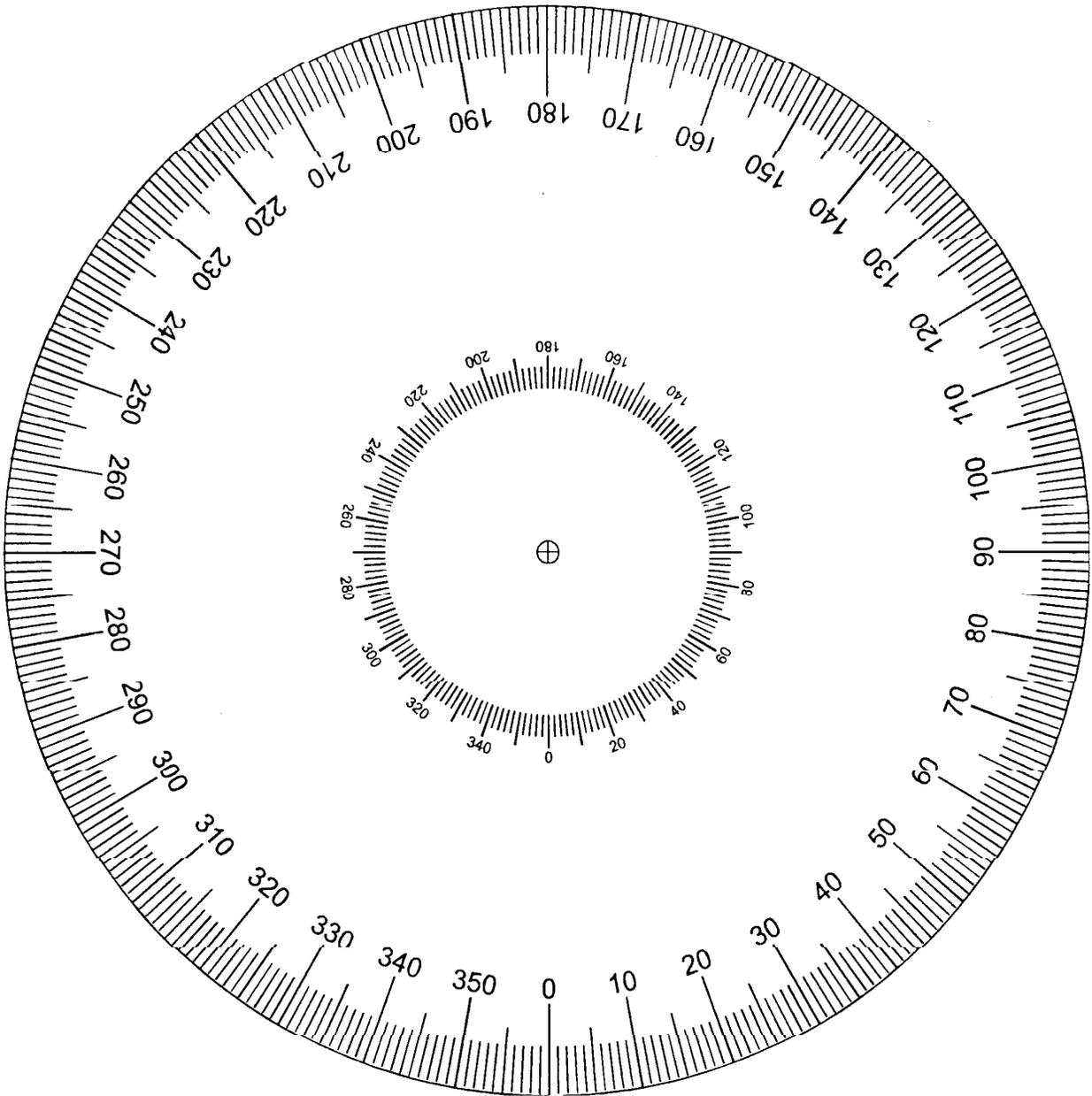
Appendix

Storage

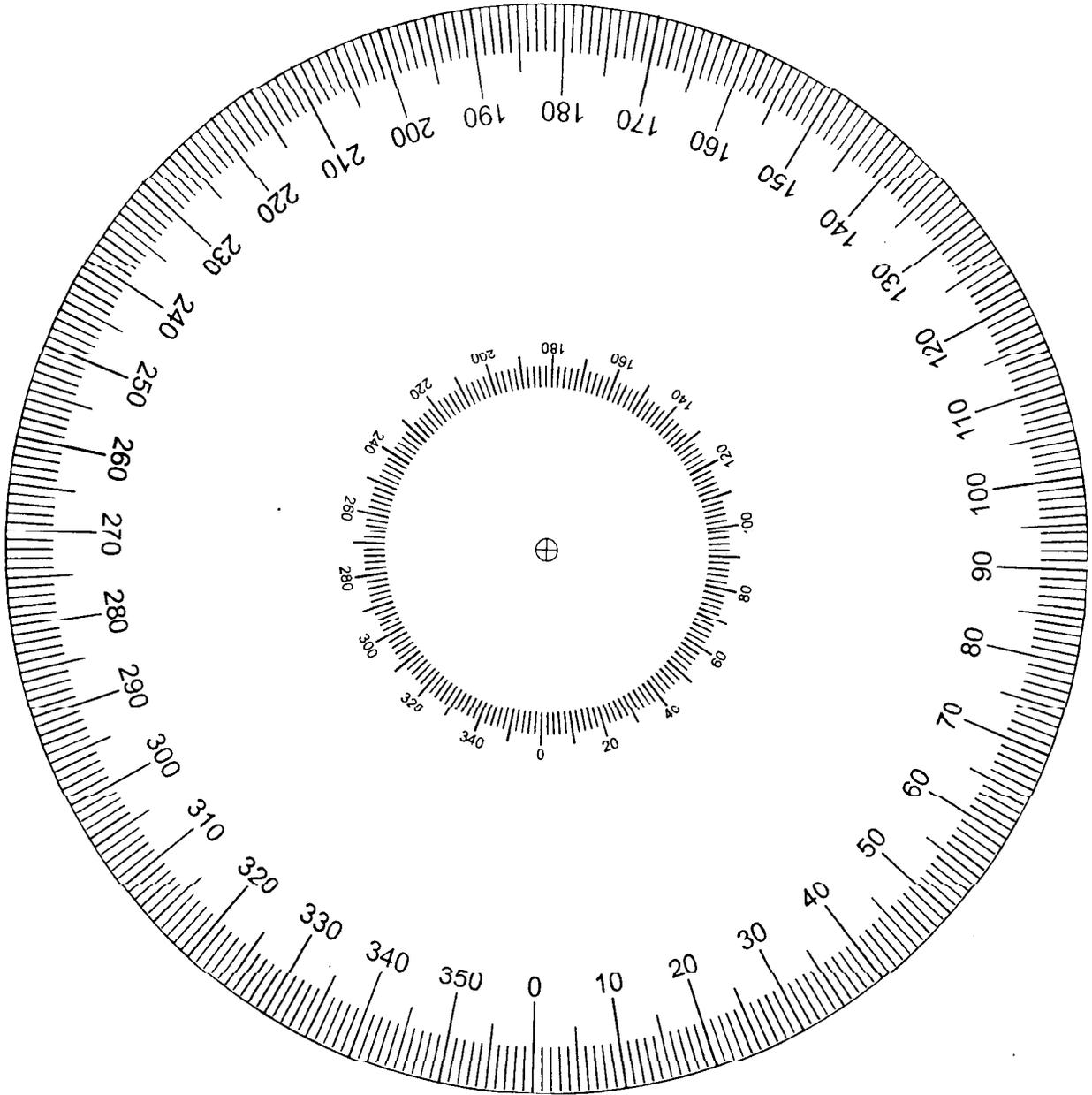
The force table may be stored with or without the pulleys and pulley clips attached. To minimize the storage space needed for the force table, remove the legs by unscrewing them from the table. Then put the legs in the clips that are on the underside of the force table. Then the force tables are ready to be stacked on a shelf.

Protractor

The protractors on the following pages are smaller versions of the top surface of the Force Table. These can be duplicated, trimmed and used as overlays on the Force Table for drawing and tracing of the string positions.



Protractor Template



Protractor Template

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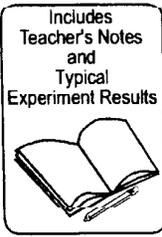
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APPENDIX 3:

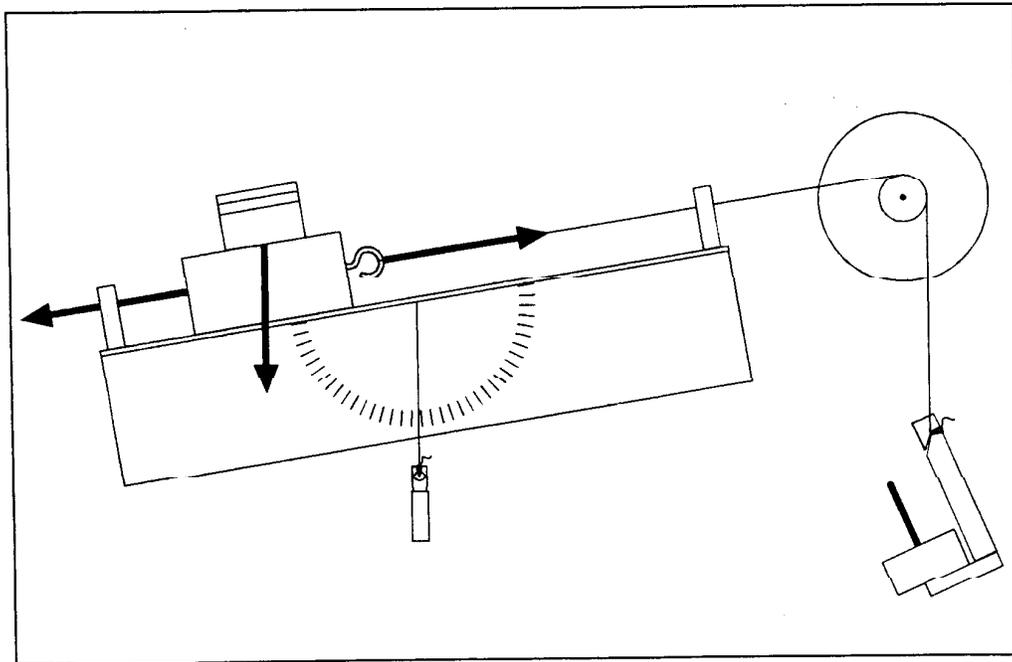
**Introductory Mechanics
System**



**Instruction Manual and
Experiment Guide for
the PASCO scientific
Model ME-9299**

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INTRODUCTORY MECHANICS SYSTEM



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This manual edited by: Eric Ayars

Teacher's guide written by: Eric Ayars

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Introduction

The study of Mechanics begins with Newton's Second Law: force equals mass times acceleration. It is not surprising therefore that much of what is studied in an introductory physics course deals with the many types of forces that exist in nature and the ways in which these forces act on physical bodies. This set of experiments is designed to help you investigate the nature of force by studying a very special case in mechanics—the case in which there is no acceleration, because the vector sum of all the forces acting on the body is equal to zero.

Why study this very special case? There are two reasons. The first is that non-accelerating systems are much easier to observe than accelerating systems. They stay put. By investigating the many ways in which forces can be applied to a body without causing acceleration, a great deal can be learned about the vector nature of forces.

The second reason for studying this special case is that it is not so special after all. Look around. Most solid objects on Earth tend to come to rest, and of course, when they are at rest, the net force acting on them is zero. Throughout the world of our everyday experience, unaccelerated systems are the rule, not the exception.

Preface to the Teacher

The PASCO scientific Model ME-9299 Introductory Mechanics System provides a comprehensive introduction to static mechanics. The experiments in this manual first introduce force as a vector quantity and then build on this concept so that, at the end, the student will understand the equilibrium of a physical body under the application of a variety of forces and torques. Experiments are also included to demonstrate simple harmonic motion with a pendulum and with a spring/mass system.

The experiments are presented in three groups: Basic Experiments, Advanced Experiments, and Simple Machines. Each experiment is designed as a worksheet, to be copied or torn from the manual for student use.

Basic Experiments provide all the essentials for a solid introduction to static mechanics. The concepts of vector forces, torques, and center of mass are explored.

Advanced Experiments allow the student to combine the principles already studied to understand such phenomena as static equilibrium. The phenomena of friction and simple harmonic motion are also investigated.

Simple Machines provide an opportunity for the student to investigate applications of the principles already studied, and also to introduce important new concepts. Levers, inclined planes, and pulley systems are studied using the principles of static equilibrium and also using the principles of work and conservation of energy.

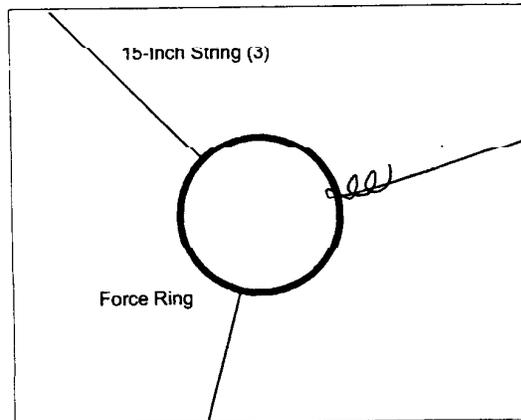
In addition to the equipment provided in this system, a few common items such as pencils, rulers, protractors, paper, and paper clips will be needed for some experiments. (Check the EQUIPMENT NEEDED information at the beginning of each experiment.)

► **NOTE:** Throughout this manual, vectors are designated with boldface letters, such as **F**, **W**, or **F₁**. When the same letters are used in a normal type style, (*F*, *W*, or *F₁*) they refer only to the magnitude of the vector. Since the vector nature of torques is not introduced in these experiments, boldface letters are not used to designate torques.

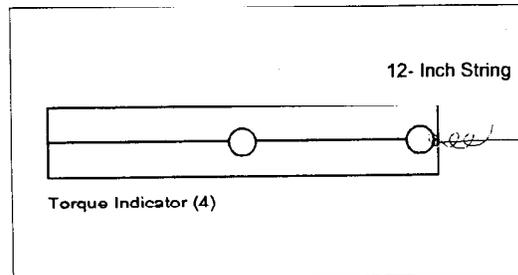
Initial Setup

The setup for several of the experiments in this manual can be simplified by pre-tying the string onto the components. A spool of string is provided with your 9299 Introductory Mechanics System.

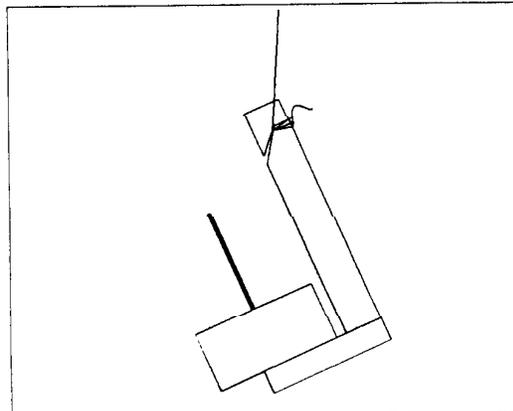
Tie three 0.38m lengths of string to the Force Ring.



Tie one 0.30m length of string to each of the 4 Torque Wheel Indicators.



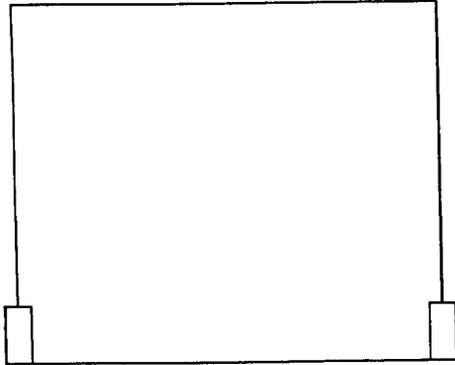
When performing experiments, attach Mass Hangers to the ends of the string by just giving the string a few wraps around the Mass Hanger.



Equipment

Component Identification

- **Experiment Board (1)**

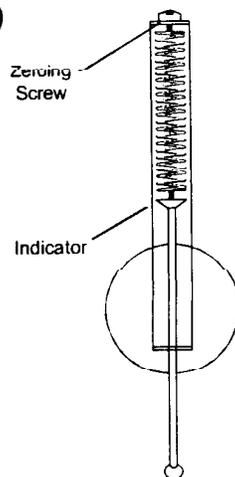


The Spring Balance, Degree Scale, Pulleys, Inclined Plane, and Torque Wheel all attach magnetically to the Experiment Board. Special pens (one brand is called Liquid Chalk), available from any stationary store, can be used to write directly on the board. The writing is easily erased with a damp cloth.

► **IMPORTANT:** When moving or removing any of the magnetically mounted components from the Experiment Board, hold the component by the magnetic mount. This reduces the strain on the component and will increase the life of the system.

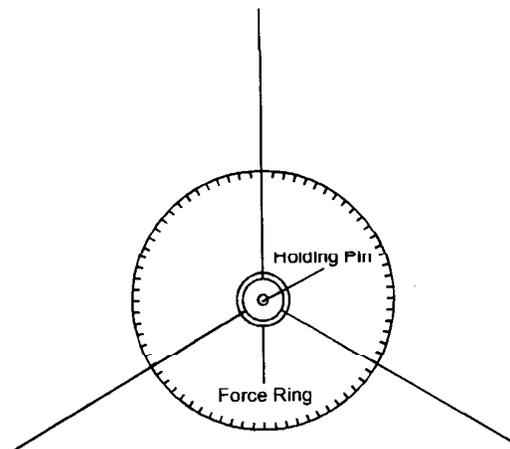
- **Spring Balance (1)**

The Spring Balance has newton, gram, and centimeter scales. The newton and gram scales are accurate to within about 5%. For greater accuracy, use the centimeter scale and the masses provided with the system to calibrate the spring.



► **NOTE:** For accurate measurements, always hang the Spring Balance vertically then, with no force applied, use the zeroing screw to align the indicator with the zero mark on the scales.

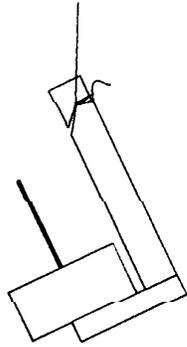
- **Degree Scale with Holding Pin and Force Ring (1)**



The Degree Scale is used to measure the angles of applied forces. Apply the forces to the Force Ring as shown, using the Holding Pin to keep the Force Ring in position. When all the forces are set, adjust the position of the Degree Scale (or adjust the forces) until the Holding Pin is centered in the Force Ring. (Tap the Experiment Board vigorously to ensure that a true equilibrium position has been reached.)

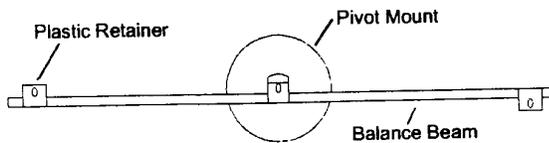
• **Mass Hangers (6)**

When hanging Mass Hangers from a piece of string, you need not tie the Mass Hangers to the string. Just wrap the string two or three times around the Mass Hanger.



• **Balance Beam with three plastic retainers and a Pivot Mount (1)**

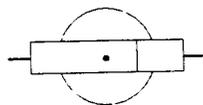
For accurate torque measurements using the Balance



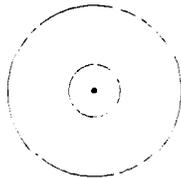
Beam, be sure to take into account the mass of the Plastic Retainers and the center of mass of the beam (see Experiment 6). Of course, by removing the plastic retainers and balancing the beam in the pivot mount before adding torques, you can eliminate the torque produced by gravity acting on the center of mass of the beam.

• **Rolling Mass (1)**

The Rolling Mass has two brackets, allowing two forces to be applied (see Experiment 8). When applying only one force, be sure that the brackets are balanced about the center of the mass. This will prevent a net torque from disturbing the line of application of the force.



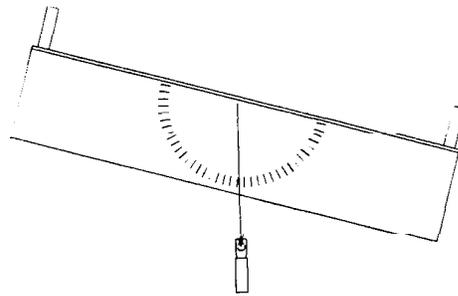
• **Small Pulleys (2); Large Pulley (1)**



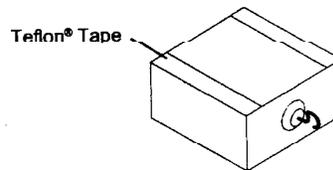
• **Pulley Block (2)**



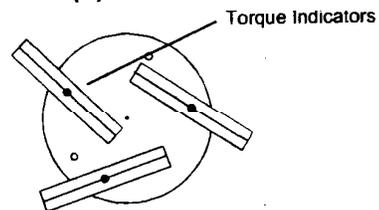
• **Inclined Plane with Plumb Bob (1)**



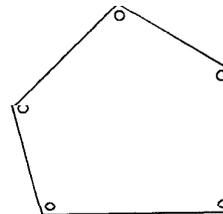
• **Friction Block (1)**



• **Torque Wheel (1)**



• **Planar Mass (1)**



• **Masses:**

100 g (0.1kg) X 2

50 g (0.05kg) X 2

20 g (0.02kg) X 4

10 g (0.01kg) X 2

Mass Hangers are 5g (0.005kg) each.

A spool of string is also included with the system.

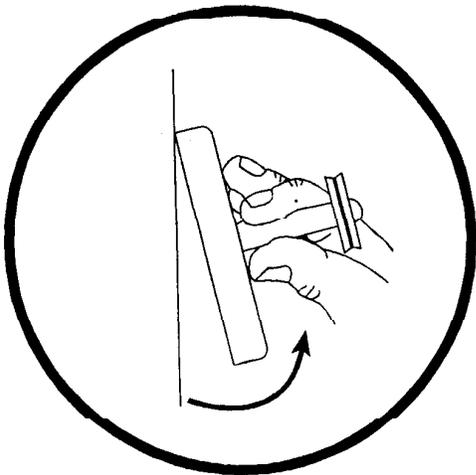
Copy-Ready Experiments

The following experiments are written in worksheet form.
Feel free to photocopy them for use in your lab.

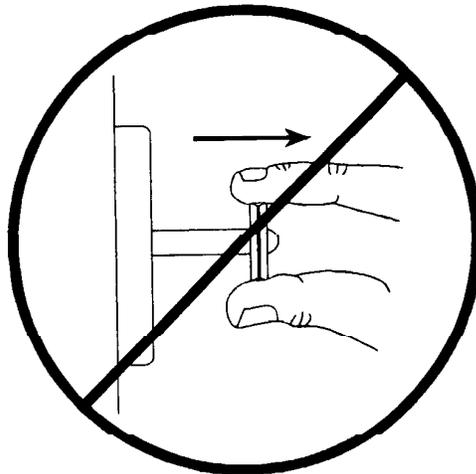
NOTE: The first paragraph in each experiment lists all the equipment needed to perform the experiment. Be sure to read this equipment list first, as the requirements vary with each experiment.

Experiment Setup

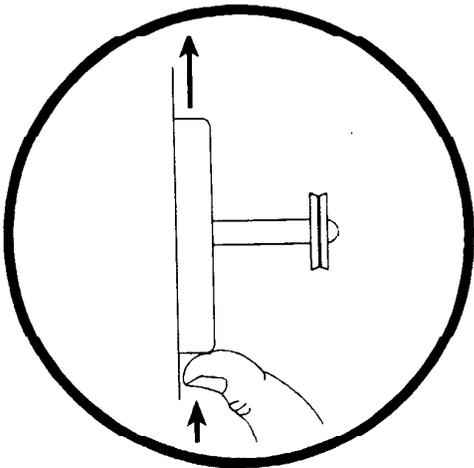
IMPORTANT: When moving or removing any of the magnetically mounted components from the Experiment Board, hold the component by the magnetic mount. This reduces the strain on the component and will increase the life of the system.



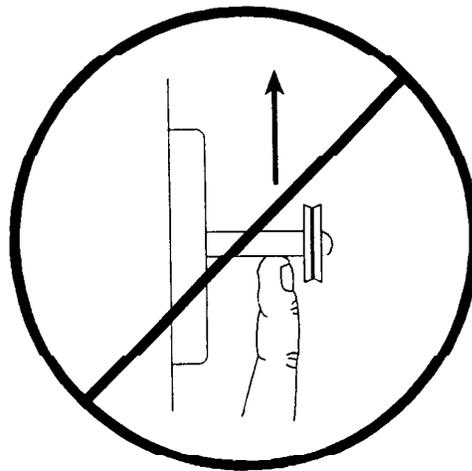
RIGHT



WRONG



RIGHT



WRONG

Experiment 1: Hooke's Law—Measuring Forces

EQUIPMENT NEEDED:

- Experiment Board
- Spring Scale
- Mass Hanger (1)
- Masses

Introduction

The concept of force is defined in Newton's second law as $F = ma$; Force = Mass x Acceleration. Using this law, a force can be determined by measuring the acceleration it produces on a body of known mass. However, this method is rarely practical. A more convenient method is to compare the unknown force with an adjustable force of known magnitude. When both forces are applied to an object, and the object is not accelerated, the unknown force must be exactly opposite—in both magnitude and direction—to the known force.

With this statics system, there are two methods of measuring and applying forces. One method is to hang the calibrated masses. For a mass m , gravity pulls it downward with a magnitude $F = mg$, where g is the acceleration caused by gravity ($g = 9.8 \text{ m/s}^2$ downward, toward the center of the Earth). The Spring Balance provides a second method of applying and measuring forces. In this experiment you will use the known forces provided by the calibrated masses to investigate the properties of the Spring Balance.

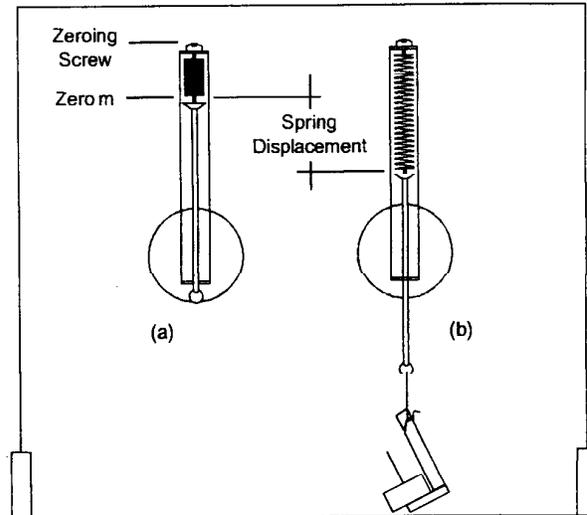


Figure 1.1 Equipment Setup

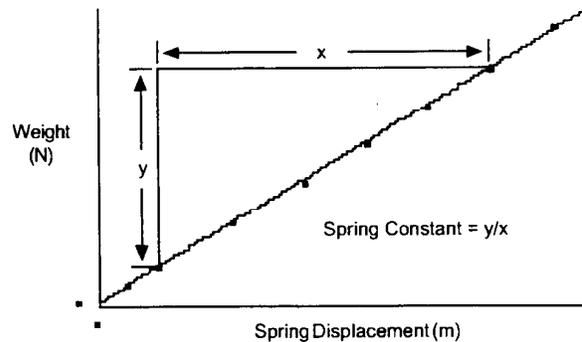


Figure 1.2 Measuring the Spring Constant

Setup

Hang the Spring Balance on the Experiment Board. Be sure the spring hangs vertically in the plastic tube. With no weight on the Spring Balance, adjust the zeroing screw on the top of the Spring Balance until the indicator is aligned with the 0 m mark on the centimeter scale of the Balance as shown in Figure 1.1a

Procedure

- ① Hang a Mass Hanger with a (.02 kg) Mass from the Spring Balance. Measure the spring displacement on the scale as shown in Figure 1.1b. Record this value in the appropriate space in Table 1.1. Be sure to include the mass of the Mass Hanger (.005 kg) in the total mass.

- ② By hanging additional masses from the Mass Hanger, adjust the total mass hanging from the Spring Balance to each of the values shown in the table. For each value, record the spring displacement.
- ③ Using the formula $F = mg$, determine the total weight in newtons for each set of masses that was used. Record your results in the table. (To get the correct force in newtons, you must use the mass values in kilograms.)

► **NOTE:** When using hanging weights to measure force, a unit for mass is often used as if it were a unit of weight. Remember, there is a difference between weight and mass. That is:

Weight = Mass \times (the Acceleration due to gravity).

Weight is a force that depends on mass and gravity. If the gravitational constant changes—on the moon, for example—the weight changes as well, but the mass remains the same.

Calculations

- ① On a separate sheet of paper, construct a graph of Weight versus Spring Displacement with Spring Displacement on the x-axis (see Figure 1.2). Draw the line that best fits your data points. The slope of the graph is the **spring constant** for the spring used in the Spring Balance.
- ② Measure the spring constant from your graph. Be sure to include the units (newtons/meter).
Spring Constant = _____ (newtons/meter)

Questions

- ① The linear relationship between force and displacement in springs is called **Hooke's Law**. If Hooke's Law were not valid, could a spring still be used successfully to measure forces? If so, how?
- ② In what way is Hooke's Law a useful property when calibrating a spring for measuring forces?

Table 1.1

Mass (kg)	Weight (N)	Spring Displacement (m)
.025		
.075		
.125		
.175		
.225		
.275		
.325		

Experiment 2: Adding Forces Resultants and Equilibrants

EQUIPMENT NEEDED:

- Experiment Board
- Degree Scale
- Pulleys (3)
- Masses
- Spring Balance
- Force Ring
- Mass Hangers (3)
- String

Theory

In Figure 2.1, spaceships x and y are pulling on an asteroid with forces indicated by vectors F_x and F_y . Since these forces are acting on the same point of the asteroid, they are called **concurrent forces**. As with any vector quantity, each force is defined both by its direction, the direction of the arrow, and by its magnitude, which is proportional to the length of the arrow. (The magnitude of the force is independent of the length of the tow rope.)

The total force on the asteroid can be determined by adding vectors F_x and F_y . In the illustration, the parallelogram method is used. The diagonal of the parallelogram defined by F_x and F_y is F_r , the vector indicating the magnitude and direction of the total force acting on the asteroid. F_r is called the **resultant** of F_x and F_y .

Another useful vector is F_e , the **equilibrant** of F_x and F_y . F_e is the force needed to exactly offset the combined pull of the two ships. F_e has the same magnitude as F_r , but is in the opposite direction. As you will see in the following experiment, the equilibrant provides a useful experimental method for finding the resultant of two or more forces.

Setup

Set up the equipment as shown in Figure 2.2. The Mass Hanger and mass provide a gravitational force of $F = mg$ downward. However, since the Force Ring is not accelerated, the downward force must be exactly balanced by an equal and opposite, or equilibrant, force. This equilibrant force, F_e , is of course provided by the Spring Balance.

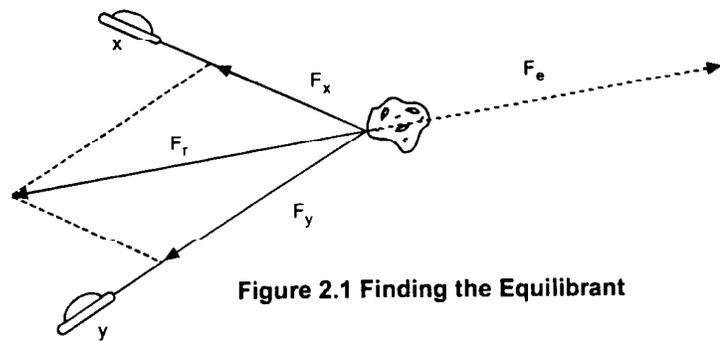


Figure 2.1 Finding the Equilibrant

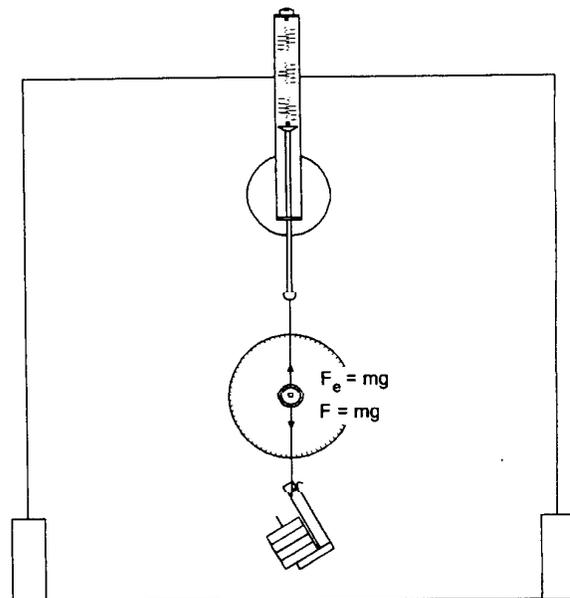


Figure 2.2 Equipment Setup

Procedure

- ① What is the magnitude and direction of F , the gravitational force provided by the mass and Mass Hanger ($F = mg$)?

F : Magnitude = _____ .

Direction = _____ .

- ② Use the Spring Balance and the Degree Plate to determine the magnitude and direction of F_e .

F_e : Magnitude = _____ .

Direction = _____ .

Now use pulleys and hanging masses as shown in Figure 2.3 to set up the equipment so that two known forces, F_1 and F_2 , are pulling on the Force Ring. Use the Holding Pin to prevent the ring from being accelerated. The Holding Pin provides a force, F_e , that is exactly opposite to the resultant of F_1 and F_2 .

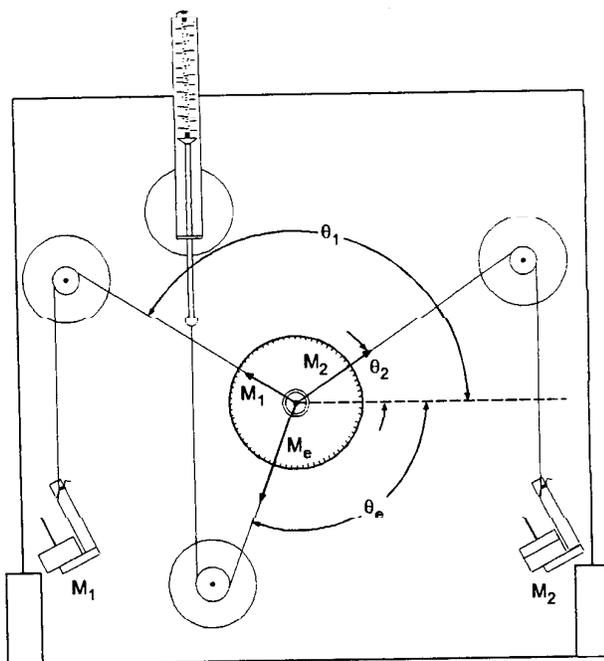


Figure 2.3 Finding the Equilibrant

Adjust the Spring Balance to determine the magnitude of F_e . As shown, keep the Spring Balance vertical and use a pulley to direct the force from the spring in the desired direction. Move the Spring Balance toward or away from the pulley to vary the magnitude of the force. Adjust the pulley and Spring Balance so that the Holding Pin is centered in the Force Ring.

Adjust the Spring Balance to determine the magnitude of F_e . As shown, keep the Spring Balance vertical and use a pulley to direct the force from the spring in the desired direction. Move the Spring Balance toward or away from the pulley to vary the magnitude of the force. Adjust the pulley and Spring Balance so that the Holding Pin is centered in the Force Ring.

NOTE: To minimize the effects of friction in the pulleys, tap as needed on the Experiment Board each time you reposition any component. This will help the Force Ring come to its true equilibrium position.

- ③ Record the magnitude in newtons of F_1 , F_2 , and F_e ; the value of the hanging masses, M_1 , and M_2 (include the mass of the mass hangers); and also θ_1 , θ_2 , and θ_e , the angle each vector makes with respect to the zero-degree line on the degree scale.

F_1 : $M_1 =$ _____ Magnitude = _____ Angle = _____

F_2 : $M_2 =$ _____ Magnitude = _____ Angle = _____

F_e : Magnitude = _____ Angle = _____

- ④ Use the values you recorded above to construct F_1 , F_2 , and F_e on a separate sheet of paper. Choose an appropriate scale (such as 2.0 cm/newton) and make the length of each vector proportional to the magnitude of the force. Label each vector and indicate the magnitude of the force it represents.
- ⑤ On your diagram, use the parallelogram method to draw the resultant of F_1 and F_2 . Label the resultant F_r . Measure the length of F_r to determine the magnitude of the resultant force and record this magnitude on your diagram.
- ⑥ Does the equilibrant force vector, F_e , exactly balance the resultant vector, F_r . If not, can you suggest some possible sources of error in your measurements and constructions?

Vary the magnitudes and directions of F_1 and F_2 and repeat the experiment.

Experiment 3: Resolving Forces—Components

EQUIPMENT NEEDED:

- | | |
|--|---|
| <ul style="list-style-type: none"> – Experiment Board – Force Ring – Mass Hangers (3) – String | <ul style="list-style-type: none"> – Degree Scale – Pulleys (3) – Masses |
|--|---|

Theory

In Experiment 2, you added concurrent forces vectorially to determine the magnitude and direction of the combined force. In this experiment, you will do the opposite; you will find two forces which, when added together, have the same effect as the original force. As you will see, any force vector in the x-y plane can be expressed as the sum of a vector in the x direction and a vector in the y direction.

Setup

Set up the equipment as shown in Figure 3.1.

As shown, determine a force vector, F , by hanging a mass from the Force Ring over a pulley. Use the Holding Pin to hold the Force Ring in place.

Set up the Spring Balance and a pulley so the string from the balance runs horizontally from the bottom of the pulley to the Force Ring. Hang a second Mass Hanger directly from the Force Ring.

Now pull the Spring Balance toward or away from the pulley to adjust the horizontal, or “x-component” of the force. Adjust the mass on the vertical Mass Hanger to adjust the vertical or “y-component” of the force. Adjust the x and y components in this way until the Holding Pin is centered in the Force Ring. (Notice that these x and y components are actually the x and y components of the equilibrant of F , rather than of F itself.)

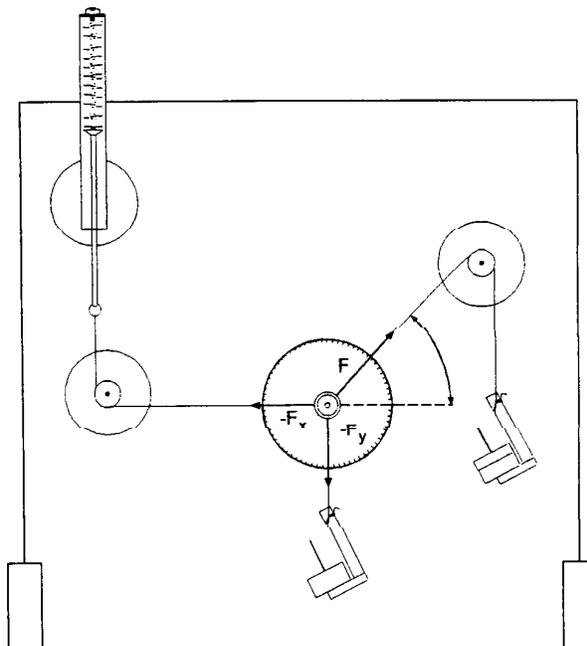


Figure 3.1 Equipment Setup

NOTE: The hanging masses allow the mass to be varied only in 10 gram increments. Using an additional Mass Hanger as a mass allows adjustments in 5 gram increments. Paper clips are convenient for more precise variation. Weigh a known number of clips with the Spring Balance to determine the mass per clip.

Procedure

- ① Record the magnitude and angle of F . Measure the angle as shown in Figure 3.1.

Magnitude = _____ Angle = _____ .

- ② Record the magnitude of the x and y components of the equilibrant of F .

x-Component = _____ y-Component = _____ .

- ③ What are the magnitudes of F_x and F_y , the x and y components of F ?

F_x = _____ F_y = _____ .

Change the magnitude and direction of F and repeat the experiment.

- ④ Record the angle of F , and the magnitudes of F , F_x , and F_y .

F : Magnitude = _____ Angle = _____ .

F_x = _____ F_y = _____

Why use components to specify vectors? One reason is that using components makes it easy to add vectors mathematically. Figure 3.2 shows the x and y components of a vector of length F , at an angle θ with the x-axis. Since the components are at right angles to each other, the parallelogram used to determine their resultant is a rectangle. Using right triangle AOX, the components of F are easily calculated: the x-component equals $F \cos \theta$; the y-component equals $F \sin \theta$.

If you have many vectors to add, simply determine the x and y components for each vector. Add all the x-components together and add all the y-components together. The resulting values are the x and y components for the resultant.

Set up the equipment as in the first part of this experiment, using a pulley and a hanging mass to establish the magnitude and direction of a force vector. Be sure the x-axis of the Degree Plate is horizontal.

- ⑤ Record the magnitude and angle of the force vector, F , that you have constructed.

Magnitude = _____ Angle = _____ .

- ⑥ Calculate F_x and F_y , the magnitudes of the x and y components of F ($F_x = F \cos \theta$; $F_y = F \sin \theta$).

F_x = _____ F_y = _____

Now set up the Spring Balance and a hanging mass, as in the first part of this experiment (Figure 3.1). Using the values you calculated in question 6, position the Spring Balance so it pulls the Force Ring horizontally by an amount F_x . Adjust the hanging mass so it pulls the Force Ring vertically down by an amount F_y .

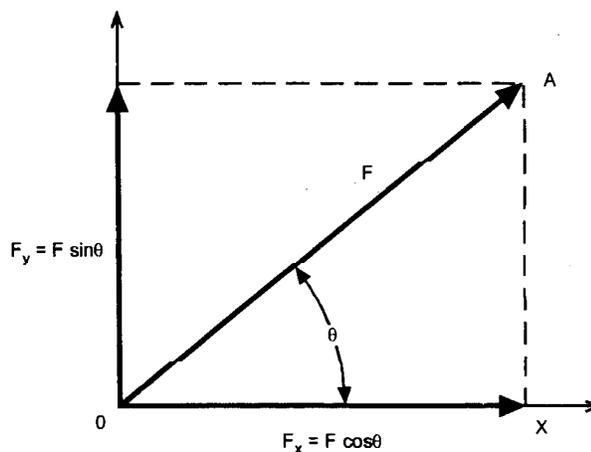


Figure 3.2 Vector Components

Questions

- ① Is the Force Ring at equilibrium in the center of the Degree Plate?

Generally it is most useful to find the components of a vector along two perpendicular axes, as you did above. However, it is not necessary that the x and y axes be perpendicular. If time permits, try setting up the equipment to find the components of a vector along non-perpendicular axes. (Use pulleys to redirect the component forces to non-perpendicular directions.)

- ② What difficulties do you encounter in trying to adjust the x and y components to resolve a vector along non-perpendicular axes?

Experiment 4: Torque—Parallel Forces

EQUIPMENT NEEDED:

- | | |
|---|--|
| <ul style="list-style-type: none"> – Experiment Board – Pivot – Masses | <ul style="list-style-type: none"> – Balance Beam – Mass Hangers (3) – Tape |
|---|--|

Theory

In Experiment 2, you found resultants and equilibrants for concurrent forces—forces that act upon the same point. In the real world, however, forces are often not concurrent. In Figure 4.1, for example, two spaceships are pulling on different points of an asteroid. Two questions might be asked:

- ① Which direction will the asteroid be accelerated in?
- ② Will the asteroid rotate?

If both tow ropes were attached to point A, the resultant would be the force vector shown, F_r . In fact, F_r does point in the direction in which the asteroid will be accelerated (this idea will be investigated further in later experiments). However, what of question 2. Will the asteroid rotate? In this experiment you will begin to investigate the types of forces that cause rotation in physical bodies. In doing so, you will encounter a new concept—torque.

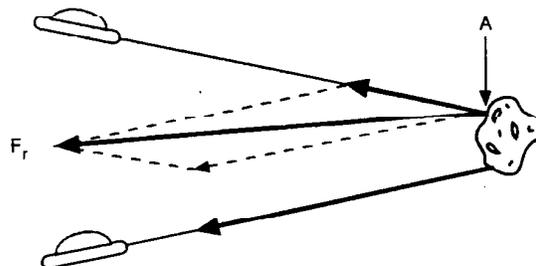


Figure 4.1 Non-Concurrent Forces Equipment Setup

Setup

Using a magic marker pen, draw a horizontal line on the Experiment Board. (The Inclined Plane can be used as a level and a straightedge to ensure a truly horizontal line.) Then set up the equipment as shown in Figure 4.2. Adjust the beam in the pivot retainer until the beam is perfectly balanced on the pivot. Use your horizontal line as a reference.

NOTE: To prevent the pivot point from sliding, place a piece of thin tape against either edge of the pivot retainer. Add additional small pieces of tape, if needed, to rebalance the beam.

Slide a plastic retainer with a hook onto each end of the beam, then hang a Mass Hanger from each hook as shown in Figure 4.3. Position one Mass Hanger approximately half way between the pivot point and the end of the beam. Slide the other Mass Hanger on the beam until the beam is perfectly balanced.

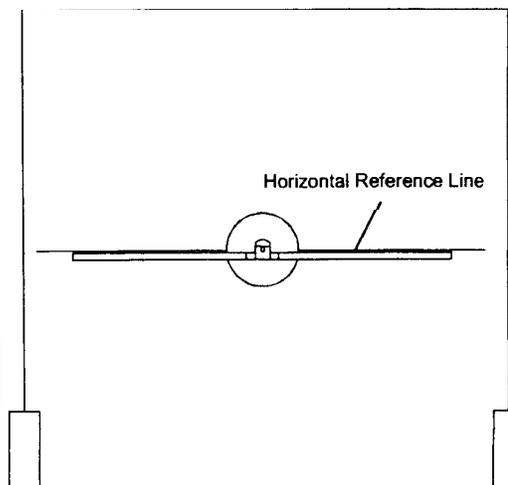


Figure 4.2 Equipment Setup

Procedure

- ① Measure d_1 and d_2 , the distances of each Mass Hanger from the pivot point (see Figure 4.3).

$d_1 =$ _____

$d_2 =$ _____

Add a 50-gram mass to each Mass Hanger.

- ② Is the beam still balanced?

Add an additional 20-gram mass to one Mass Hanger.

- ③ Can you restore the balance of the beam by repositioning the hanger?

Place 75 grams of mass on one Mass Hanger (M_1 in Figure 4.4) and position it approximately half way between the pivot and the end of the beam, as shown. Place various masses on the other Mass Hanger (M_2), and slide it along the beam as needed to rebalance the beam. At each balanced position, measure the mass (M) hanging from each Mass Hanger and the distance (d) between the Mass Hanger and the pivot point, as shown in the illustration. Take measurements for at least 5 different values of M_2 and record your results in Table 4.1. Be sure to include the units of your measurements. Vary M_1 and repeat your measurements.

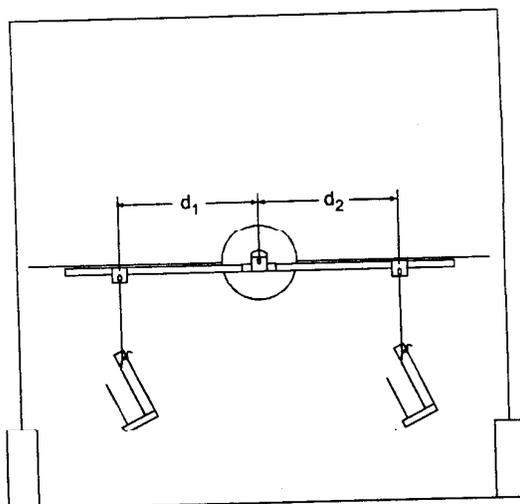


Figure 4.3 Measuring Torques

NOTE: For accurate results, include the mass of the Mass Hangers (5 grams) and of the plastic retainers and hooks (2.2 grams) when determining M_1 and M_2 .

Use the formula $F = Mg$, where g is the acceleration due to gravity, to determine the gravitational force produced by the hanging masses in each case. Then perform the calculations shown in the table to determine τ_1 and τ_2 ; that is, $\tau_1 = F_1 d_1$, and $\tau_2 = F_2 d_2$. Record your calculated values for each balanced position of the beam.

The quantity $\tau_1 = F_1 d_1$ is called the torque of the force F_1 about the pivot point of the balance beam.

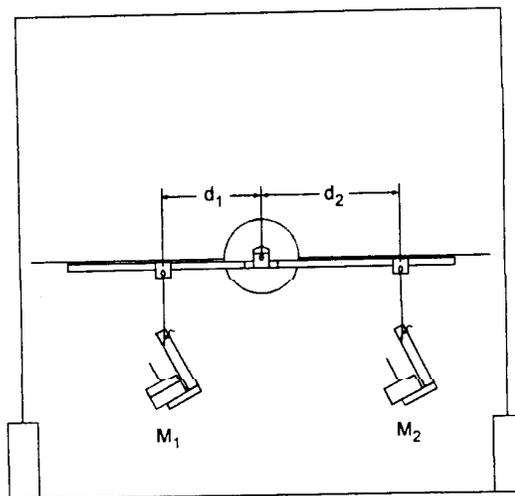


Figure 4.4 More Torques

Experiment 5: Torque: Non-Parallel Forces

EQUIPMENT NEEDED:

- | | |
|---|--|
| <ul style="list-style-type: none"> - Experiment Board - Pivot - Degree Scale - Masses - Tape | <ul style="list-style-type: none"> - Balance Beam - Spring Balance - Mass Hangers (2) - String |
|---|--|

Theory

In Experiment 4, you investigated torques applied to the balance beam, and discovered that when the torques about the point of rotation are balanced, the beam remains balanced. However, all the forces in that experiment were parallel to each other and perpendicular to the balance beam. What happens when one or more of the forces is not perpendicular to the beam.

Fortunately, it turns out that the formula for torque that you determined in Experiment 4 ($\tau = F d$) can be generalized to account for this more general case. The generalized formula is:

$$\tau = F d \sin\theta;$$

where F is the magnitude of the applied force, d is the distance from the pivot point to the point at which the force is applied, and θ is the angle between F and d (see Figure 5.1).

In this experiment, you will investigate the validity of this definition for torque.

Setup

As in Experiment 4, first balance the beam without any applied forces (remove the plastic retainers), and use pieces of tape near the center pivot to ensure that the beam does not slip out of balance during the experiment. Then use a hanging mass and the Spring Balance to apply forces F_1 and F_2 as in Figure 5.2. Attach pieces of tape on each side of the plastic retainers to ensure that they do not slide out of position.

Procedure

- ① Measure d_1 and d_2 and record the values.

$$d_1 = \underline{\hspace{2cm}} \quad d_2 = \underline{\hspace{2cm}}$$

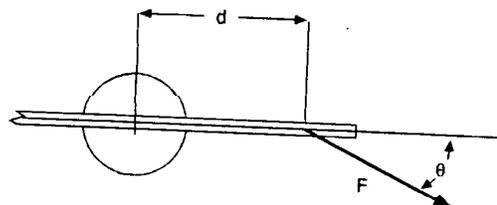


Figure 5.1 Torque

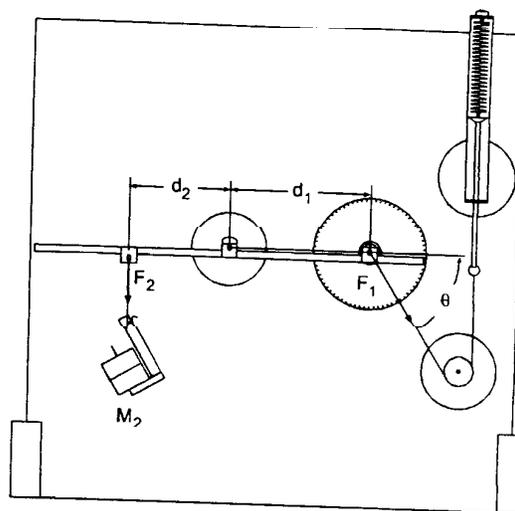


Figure 5.2 Equipment Setup

- Record the mass (M_2) and the magnitude of the force (F_2) of the hanging mass. Use your measured values to calculate the torque of F_2 about the pivot point of the balance beam (use the formula from Experiment 4: $\tau = F d$). Be sure to include the units for each value.

$M_2 =$ _____ $F_2 =$ _____ $\tau_2 =$ _____.

Table 5.1

Angle	F_1 (N)	$\tau_1 = (F_1 d_2) \sin\theta$	$(\tau_1 - \tau_2) / (1/2(\tau_1 + \tau_2))$
30°			
40°			
50°			
60°			
70°			
80°			

By moving the pulley, you can adjust the angle of force F_1 . To accurately measure this angle, the center of the Degree Scale must be aligned beneath the center of the plastic retainer (notice that the retainer has been inverted for this experiment). This ensures that F_1 and a horizontal line from the pivot point intersect at the center of the Degree Scale.

Set the angle of F_1 to each of the values shown in Table 5.1. At each setting, move the Spring Balance toward or away from the pulley, as needed so that the magnitude of F_1 is sufficient to balance the beam, then record the reading on the newton scale of the Spring Balance.

- Perform the calculations shown in the table to determine τ_1 , the torque provided by the Spring Balance about the pivot point of the Balance Beam, and the percentage difference between τ_1 and τ_2 .

To provide a consistent mathematical definition of torque, τ_1 and τ_2 must be determined according to the same formula.

- Apply the generalized definition of torque ($\tau = F d \sin \theta$), to the calculation of τ_1 in step 2 of the experiment. Does this affect your results?

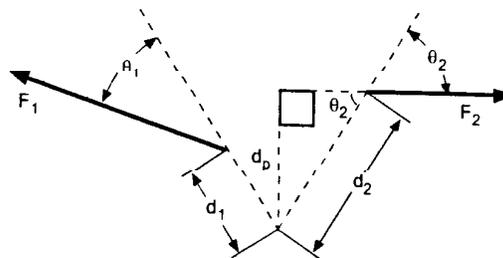


Figure 5.3 Torque Diagram

Figure 5.3 shows a diagram for a calculation of torque. The force F_1 establishes a torque about the point O of magnitude $F_1 d_1 \sin \theta_1$. F_2 establishes a torque of magnitude $F_2 d_2 \sin \theta_2$. However, to simply add the two torques together to determine the total torque would be misleading, because τ_1 and τ_2 tend to cause rotation about point O in opposite directions. When adding two or more torques, add together the magnitudes of all the torques which tend to cause clockwise rotation, then add the magnitudes of all the torques which tend to cause counterclockwise rotation. For the system to be balanced, the sum of the clockwise torques must equal the sum of the counterclockwise torques.

It is helpful to note that the perpendicular distance from the pivot point to the force, d_p , is equal to $d \times \sin \theta$. This can make it easier to calculate the torque.

The Torque Wheel provides an easy method for creating an equilibrium among several non-parallel torques. Figure 5.4 shows a force F applied at an angle θ to the line from the center of the Torque Wheel to the point of application of the force. The torque can be calculated as $\tau = F d \sin \theta$. However, as shown, $d \sin \theta$ is just the perpendicular distance, d_p , between the center of the Torque Wheel and the line of the force, when that line is extended back.

Set up the Torque Wheel as shown in Figure 5.5. Use pulleys and hanging masses to apply 3 torques to the wheel.

Table 5.2

F_1	$d_{1\perp}$	τ_1	F_2	$d_{2\perp}$	τ_2	F_3	$d_{3\perp}$	τ_3	Total Torque

Fill in Table 5.2, using the radial scale on the Torque Wheel to determine the perpendicular distance between the line of each applied force and the center point of the Torque Wheel. Calculate the torque for each force using the formula $\tau = F d_p$ (be sure to indicate whether each torque is clockwise or counterclockwise). Subtract the sum of the clockwise torques from the sum of the counterclockwise torques to determine the total torque.

Repeat the procedure with different forces and angles.

- ⑤ Within the limits of your experimental error, is the total torque equal to zero when the Torque Wheel is in equilibrium?

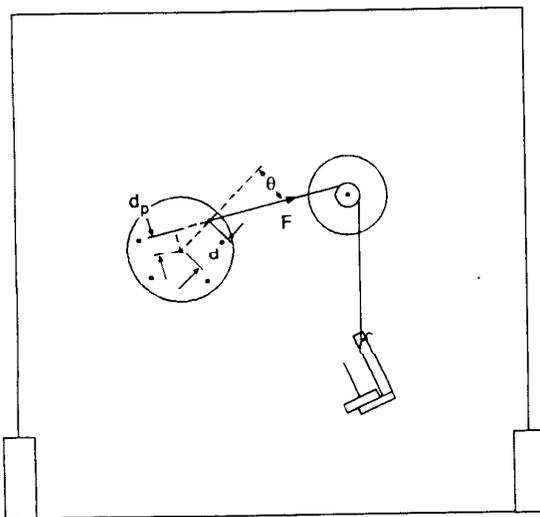


Figure 5.4 Using the Torque Wheel

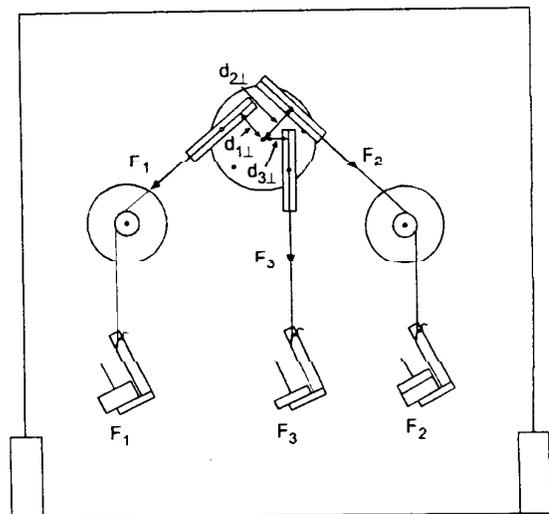


Figure 5.5 Equipment Setup

Experiment 6: Center of Mass

EQUIPMENT NEEDED:

- | | |
|--|---|
| <ul style="list-style-type: none"> - Experiment Board - Planar Mass - Mass Hanger (1) - String | <ul style="list-style-type: none"> - Pivot - Balance Beam - 50-gram Mass |
|--|---|

Theory

Gravity is a universal force; every bit of matter in the universe is attracted to every other bit of matter. So when the balance beam is suspended from a pivot point, every bit of matter in the beam is attracted to every bit of matter in the Earth.

Fortunately for engineers and physics students, the sum of all these gravitational forces produces a single resultant. This resultant acts as if it were pulling between the center of the Earth and the center of mass of the balance beam. The magnitude of the force is the same as if all the matter of the Earth were located at the center of the Earth, and all the matter of the balance beam were located at the center of mass of the balance beam. In this experiment, you will use your understanding of torque to understand and locate the center of mass of an object.

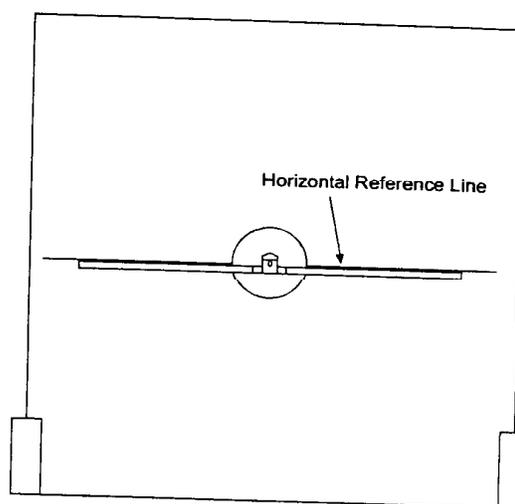


Figure 6.1 Equipment Setup

Setup

Hang the Balance Beam from the pivot as shown in Figure 6.1. As in Experiment 4, use the Inclined Plane as a level and straightedge to draw a horizontal reference line. Adjust the position of the Balance Beam in the pivot so that the beam balances horizontally.

Since the Balance Beam is not accelerated, the force at the pivot point must be the equilibrant of the total gravitational force acting on the beam. Since the beam does not rotate, the gravitational force and its equilibrant must be concurrent forces.

Experiment

- ① Why would the Balance Beam necessarily rotate if the resultant of the gravitational forces and the force acting through the pivot were not concurrent forces?

Think of the Balance Beam as a collection of many small hanging masses. Each hanging mass is pulled down by gravity and therefore provides a torque about the pivot point of the Balance Beam.

- ② What is the relationship between the sum of the clockwise torques about the center of mass and the sum of the counterclockwise torques about the center of mass? Explain.

On the basis of your answer to question 1, use a pencil to mark the center of mass of the balance beam. Then attach a Mass Hanger to each end of the beam. Hang 50 grams from one hanger, and 100 grams from the other, as shown in Figure 6.2. Now slide the beam through the pivot retainer until the beam and masses are balanced and the beam is horizontal. The pivot is now supporting the beam at the center of mass of the combined system (i.e. balance beam plus hanging masses).

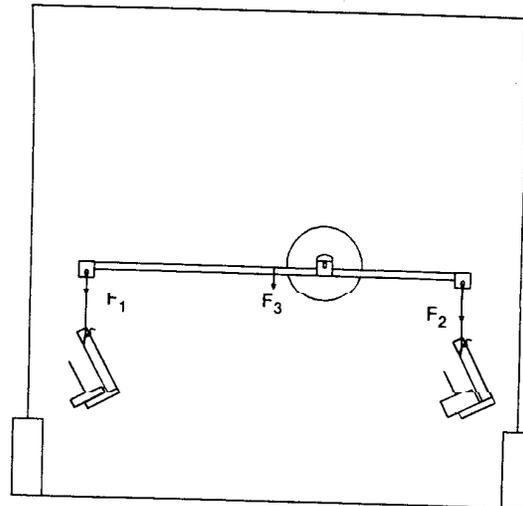


Figure 6.2 Torques and the Center of Mass

- ③ Calculate the torques, τ_1 , τ_2 , and τ_3 provided by the forces F_1 , F_2 , and F_3 acting about the new pivot point, as shown in the illustration. Be sure to indicate whether each torque is clockwise (cw) or counterclockwise (ccw).

$\tau_1 =$ _____ $\tau_2 =$ _____ $\tau_3 =$ _____

- ④ Are the clockwise and counterclockwise torques balanced?

Remove the 50 gram mass and Mass Hanger. Reposition the beam in the pivot to level the beam. Recalculate the torques about the pivot point.

Are the torques balanced?

Hang the Planar Mass from the Holding Pin of the Degree Plate as shown in Figure 6.3. Since the force of the Pin acting on the mass is equilibrant to the sum of the gravitational forces acting on the mass, the line of the force exerted by the Pin must pass through the center of mass of the Planar Mass. Hang a piece of string with a hanging mass from the Holding Pin.

Tape a piece of paper to the Planar Mass as shown. Mark the paper to indicate the line of the string across the Planar Mass. Now hang the Planar Mass from a different point. Again, mark the line of the string. By finding the intersection of the two lines, locate the center of mass of the Planar Mass.

Hang the Planar Mass from a third point.

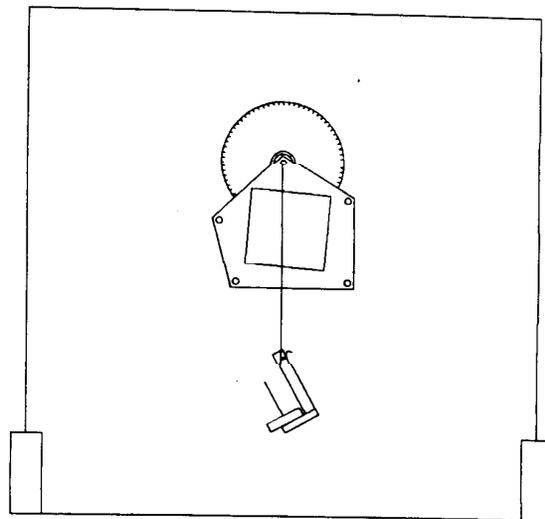


Figure 6.3 Finding the Center of Mass

- ⑥ Does the line of the string pass through the center of mass?
- ⑦ Would this method work for a three dimensional object? Why or why not?

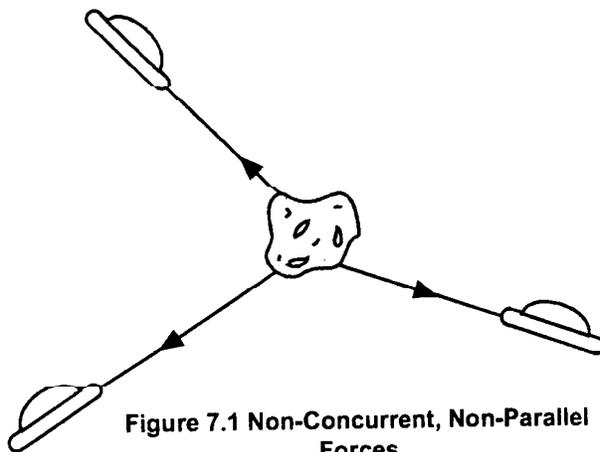
Experiment 7: Equilibrium of Physical Bodies

EQUIPMENT NEEDED:

- | | |
|--|--|
| <ul style="list-style-type: none"> - Experiment Board - Balance Beam - Mass Hangers (3) - String | <ul style="list-style-type: none"> - Spring Balance - Pulley (1) - Masses |
|--|--|

Theory

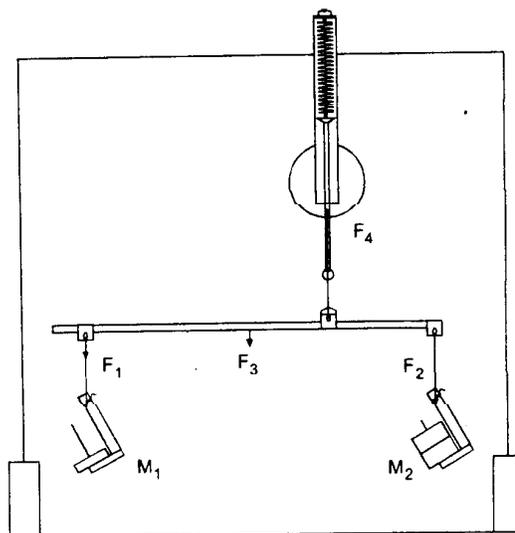
Figure 7.1 shows three spaceships pulling on an asteroid. Which way will the asteroid move? Will it rotate? The answers to these questions depend on the total force and the total torque acting on the asteroid. But any force acting on a body can produce both translational motion (movement of the center of mass of the body in the direction of the force) and rotation. In this experiment you will investigate the interplay between forces and torques by examining all the forces acting on a body in physical equilibrium.



Setup

Using the technique described in Experiment 6, find the center of mass of the balance beam, and mark it with a pencil. Then set up the equipment as shown in Figure 7.2. (The retainer can be pulled from the Pivot Mount and hung from the metal ring, as shown.) By supporting the Balance Beam from the Spring Balance, you can now determine all the forces acting on the beam. As shown in the illustration, these forces include:

- ① F_1 —the weight of Mass M_1 (including the Mass Hanger and plastic retainer).
- ② F_2 —the weight of Mass M_2 (including the Mass Hanger and plastic retainer).
- ③ F_3 —the weight of the Balance Beam, acting through its center of mass.
- ④ F_4 —the upward pull of the Spring Balance (minus the weight of the plastic retainer).



Experiment

Fill in Table 7.1, listing M (the masses in kilograms), F (the magnitude of the forces in newtons), d (the distance in meters from the applied force to the point of suspension), and τ (the torques acting about the point of suspension in newtons x millimeters). Indicate whether each torque is clockwise (cw) or counterclockwise (ccw).

Table 7.1

M_1	F_1	d_1	τ_1	M_2	F_2	d_2	τ_2	M_3	F_3	d_3	τ_3	F_4	d_4	τ_4
-------	-------	-------	----------	-------	-------	-------	----------	-------	-------	-------	----------	-------	-------	----------

- ① Calculate and record the sum of the clockwise and counterclockwise torques. Are the torques balanced?

$\Sigma\tau_{cw} = \underline{\hspace{2cm}}$ $\Sigma\tau_{ccw} = \underline{\hspace{2cm}}$

- ② Calculate the sum of the upward and downward forces. Are these translational forces balanced?

$\Sigma F_{up} = \underline{\hspace{2cm}}$ $\Sigma F_{down} = \underline{\hspace{2cm}}$

- ③ On the basis of your answers to questions 1 and 2, what conditions must be met for a physical body to be in equilibrium (no acceleration)?

In measuring the torques, all distances were measured from the point of suspension of the Balance Beam. This measures the tendency of the beam to rotate about this point of suspension. You can also measure the torques about any other point, on or off the balance beam. Using the same forces as you used in Table 7.1 above, remeasure the distances, measuring from the left end of the balance beam as shown in Figure 7.3. Then recalculate the torques to determine the tendency of the beam to rotate about the left end of the beam. Record your data in Table 7.2. As before, indicate whether each torque is clockwise (cw) or counterclockwise (ccw).

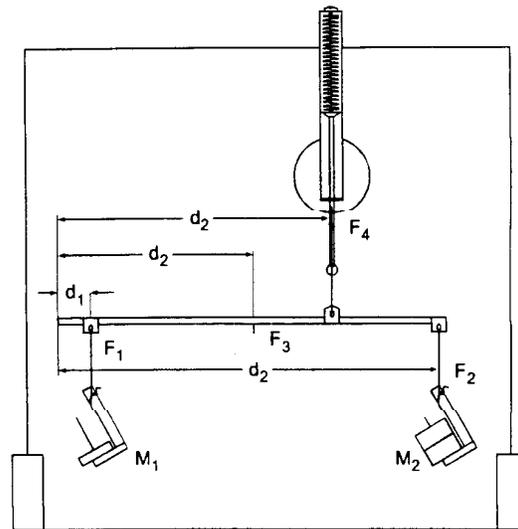


Figure 7.3 Changing the Origin

Table 7.2

M_1	F_1	d_1	τ_1	M_2	F_2	d_2	τ_2	F_3	d_3	τ_3	F_4	d_4	τ_4

- ④ Calculate and record the sums of the clockwise and counterclockwise torques. Are the torques balanced?

$$\Sigma\tau_{cw} = \underline{\hspace{2cm}} \quad \Sigma\tau_{ccw} = \underline{\hspace{2cm}} .$$

Use a pulley and a hanging mass to produce an additional upward force at one end of the beam. (You may need to use tape to secure the string to the beam, to avoid slippage.) Adjust the positions of the remaining hanging masses and the Spring Balance on the beam until the beam is balanced horizontally.

- ⑤ Are all the forces balanced, both for translational and rotational motion?

Diagram your setup and show your calculations on a separate sheet of paper.

Experiment 8: The Inclined Plane

EQUIPMENT NEEDED:

- Experiment Board
- Spring Balance
- Inclined Plane
- Rolling Mass
- Pulley (2)
- Mass Hanger (1)
- Masses
- String

Introduction

Suppose you must design a ramp with a winch to move heavy equipment from one level of an industrial site to another. For a given angle of inclination of the ramp, how much force must the winch deliver to pull the equipment up the ramp? How much weight must the ramp be able to support?

You could solve this problem by building ramps and winches and testing them, or by testing scale models. Alternatively, you could use your knowledge of forces to solve the problem mathematically. In this experiment, you will compare the mathematical solution with data taken directly from a scale model.

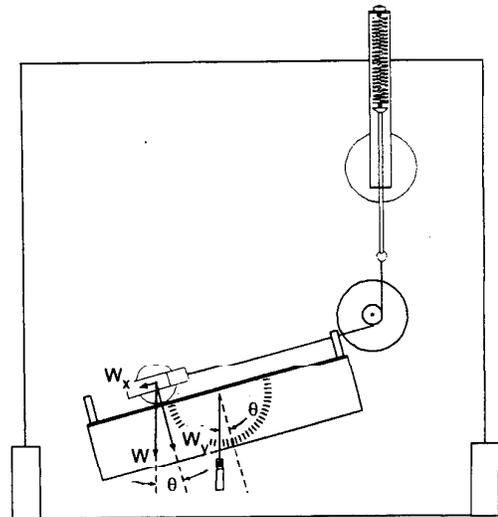


Figure 8.1 Equipment Setup

Experiment

- ① Weigh the Rolling Mass on the Spring Balance and record its mass (M) and weight (W).

M = _____ W = _____

Now set up the equipment as shown in Figure 8.1. To calculate the force that must be exerted by the string and by the ramp, W can be resolved into two components; W_x , a component directed along the surface of the ramp, and W_y , a component perpendicular to the surface of the ramp. The magnitudes of W_x and W_y are easily calculated: $W_x = W \sin\theta$, and $W_y = W \cos\theta$. F_x , the force provided by the string, must be equal and opposite to W_x . F_y , the force provided by the ramp, must be equal and opposite to W_y .

Table 8.1

Angle of Incl.	F_x (Measured)	$W_x = W \sin\theta$ (Calculated)	$\frac{F_x - W_x}{(1/2)(F_x + W_x)} \times 100\%$ (Percent Difference)
15°			
30°			
45°			
60°			
75°			

- ② Adjust the angle of inclination of the ramp to each of the values shown in the table below. At each value, record the experimental value of F_x , as read on the newton scale of the Spring Balance, in Table 8.1. For accurate results, the string must be parallel to the surface of the Inclined Plane.
- ③ For each angle of inclination, calculate W_x , the magnitude of the x-component of W , using the formula shown in the table, then calculate the percent difference between W_x and the measured value of F_x .
- ④ Does the vector model of the forces accurately predict your measured results?

To measure the force of the Rolling Mass on the Inclined Plane, set up the equipment as shown in Figure 8.2. Vary the tilt of the Inclined Plane until the hanging mass and the Rolling Mass are in equilibrium.

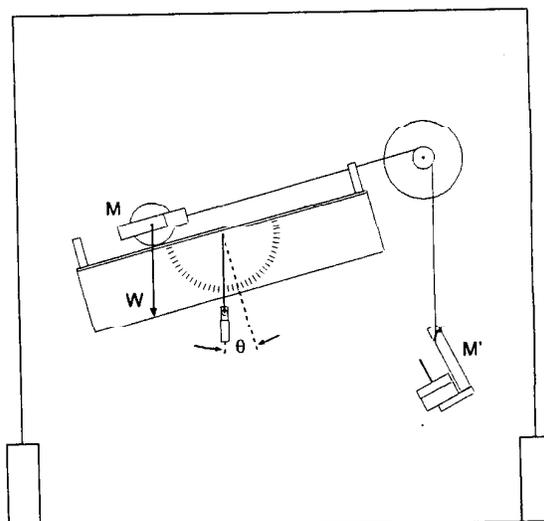


Figure 8.2 Normal Force: Equipment Setup

- ⑤ Record the Mass (M') and Weight (W') of the hanging mass, and the angle of inclination (θ) of the Inclined Plane.

$M =$ _____ $W =$ _____ $\theta =$ _____

- ⑥ Calculate F_x , the component of the weight of the Rolling Mass parallel with the inclined plane ($F_x = W \sin\theta$).
- ⑦ Does $F_x = W$?

Set up the Spring Balance and a pulley as shown in Figure 8.3. Adjust the pulley and the Spring Balance so the string pulls the bracket of the Rolling Mass at a 90° angle to the surface of the Inclined Plane. Pull the Spring Balance up until the force just barely lifts the Rolling Mass off the Inclined Plane.

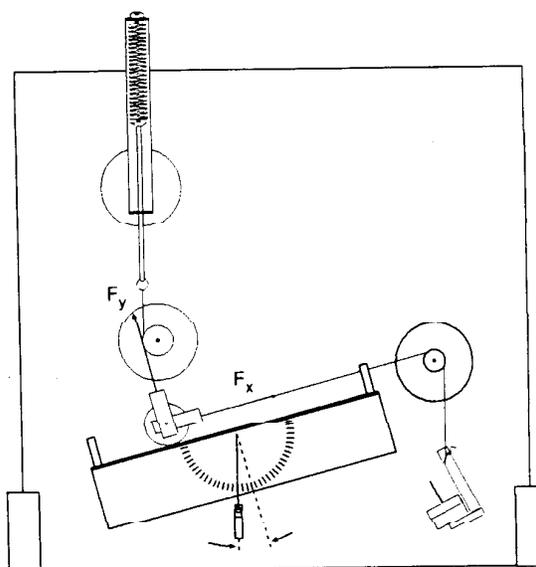


Figure 8.3 Measuring the Normal Force

- ⑧ Read the value of F_y on the newton scale of the Spring Balance.
 - ⑨ Calculate W_y , the magnitude of the y-component of the weight of the Rolling Mass. ($W_y = W \cos\theta$).
- $F_y =$ _____
- $W_y =$ _____

Question

- ① Does the vector model of forces accurately predict the force exerted by the Rolling Mass on the Inclined Plane?

Experiment 9: Sliding Friction

EQUIPMENT NEEDED:

- | | |
|--|--|
| <ul style="list-style-type: none"> - Experiment Board - Friction Block - Pulley (1) - Masses | <ul style="list-style-type: none"> - Inclined Plane - Spring Balance - Mass Hangers (2) - String |
|--|--|

Theory

In most physical systems, the effects of friction are not easily predicted, or even measured. The interactions between objects that cause them to resist sliding against each other seem to be due in part to microscopic irregularities of the surfaces, but also in part to interactions on a molecular level. However, though the phenomena is not fully understood, there are some properties of friction that hold for most materials under many different conditions. In this experiment you will investigate some of the properties of sliding friction—the force that resists the sliding motion of two objects when they are already in motion.

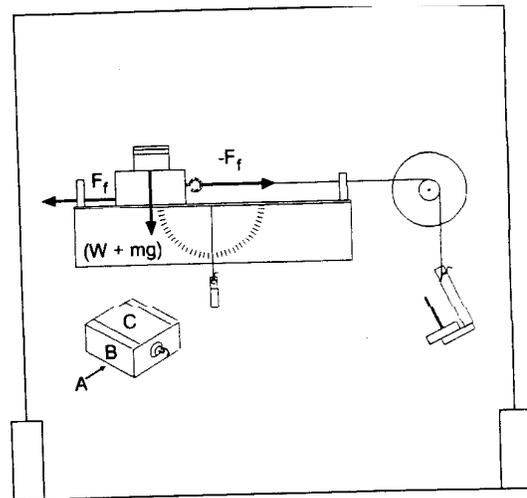


Figure 9.1 Equipment Setup

Procedure

- ① Use the Spring Scale to determine W , the magnitude of the weight of the Friction Block.

$$W = \underline{\hspace{2cm}}$$

Set up the equipment as shown in Figure 9.1. Use the built-in plumb bob to ensure that the Inclined Plane is level. Adjust the position of the pulley so that the string is level with the surface of the Inclined Plane.

Adjust the mass on the Mass Hanger until, when you give the Friction Block a small push to start it moving, it continues to move along the Inclined Plane at a very slow, constant speed. If the block stops, the hanging mass is too light; if it accelerates, the mass is too large. The weight of the hanging mass that is just sufficient to provide a constant slow speed is F_f , the force of the sliding friction of the Friction Block against the Inclined Plane.

Three variables can be varied while measuring F_f . They are:

Normal Force ($W + Mg$)—Place masses of weight W on top of the Friction Block to adjust the normal force between the block and the Inclined Plane.

Contact Material—Using sides A and B of the Friction Block, wood is the material in contact with the Inclined Plane. Using side C, only the two strips of teflon tape contact the Inclined Plane.

Contact Area (A, B, C)—Adjust the area of contact between the Friction Block by having side A, B, or C of the Friction Block in contact with the Inclined Plane. (NOTE: Using side C, the contact area is the surface area of the two strips of teflon tape.)

Adjust the mass on top of the Friction Block to each of the values shown in Table 9.1. At each value of M , adjust the hanging mass to determine the magnitude of F_f . Perform this measurement using side A, B, and C of the Friction Block. For each measurement, calculate the ratio between the magnitude of the sliding friction (F_f) and the magnitude of the normal force ($W + Mg$). This ratio is called the coefficient of friction, μ .

Table 9.1

M (kg)	W + Mg (N)	F_f (N)			μ		
		A	B	C	A	B	C
0							
0.050							
0.100							
0.150							
0.200							
0.250							

NOTE: You will need to adjust the hanging mass in small increments. Paper clips are convenient for this purpose. Weigh a large number of paper clips on the Spring Balance and divide by the number of clips to determine the weight per clip.

Based on your measurements:

Questions

- ① Does the value of sliding friction between two objects depend on the normal force between the two objects? If so, what is the relationship between normal force and sliding friction?
- ② Does the value of sliding friction between two objects depend on the area of contact between the two objects?
- ③ Does the value of sliding friction between two objects depend on the materials that are in contact?
- ④ Combining the results of this experiment and Experiment 8, what would be the value of F_f for the Friction Block (with no additional masses on top) if the inclined plane were at an angle of 45° and side A was in contact with the Inclined Plane?

Experiment 10: Simple Harmonic Motion: Mass on a Spring

EQUIPMENT NEEDED:

- | | |
|--------------------|------------------|
| – Experiment Board | – Spring Balance |
| – Mass Hanger | – Masses |
| – Stopwatch | |

Theory

Figure 10.1 shows a mass hanging from a spring. At rest, the mass hangs in a position such that the spring force just balances the gravitational force on the mass. When the mass is below this point, the spring pulls it back up. When the mass is above this point, gravity pulls it back down. The net force on the mass is therefore a restoring force, because it always acts to accelerate the mass back toward its equilibrium position.

In Experiment 1 you investigated Hook's Law, which states that the force exerted by a spring is proportional to the distance beyond its normal length to which it is stretched (this also holds true for the compression of a spring). This idea is stated more succinctly in the mathematical relationship: $F = -kx$; where F is the force exerted by the spring, x is the displacement of the end of the spring from its equilibrium position, and k is the constant of proportionality, called the spring constant (see Experiment 1).

Whenever an object is acted on by a restoring force that is proportional to the displacement of the object from its equilibrium position, the resulting motion is called **Simple Harmonic Motion**. When the simple harmonic motion of a mass (M) on a spring is analyzed mathematically using Newton's Second Law (the analysis requires calculus, so it will not be shown here), the period of the motion (T) is found to be:

$$T = 2\pi\sqrt{\frac{M}{k}}$$

In this experiment, you will experimentally test the validity of this equation.

Experiment

- ① Measure k , the spring constant for the spring in the Spring Balance (see Experiment 1).

$k =$ _____ (newtons/meter).

Set up the equipment as shown in Figure 10.1, with 120 grams on the Mass Hanger (125 grams total mass, including the hanger). Be sure that the Spring Balance is vertical so that the rod hangs straight down through the hole in the bottom of the balance. This is important to minimize friction against the side as the mass oscillates.

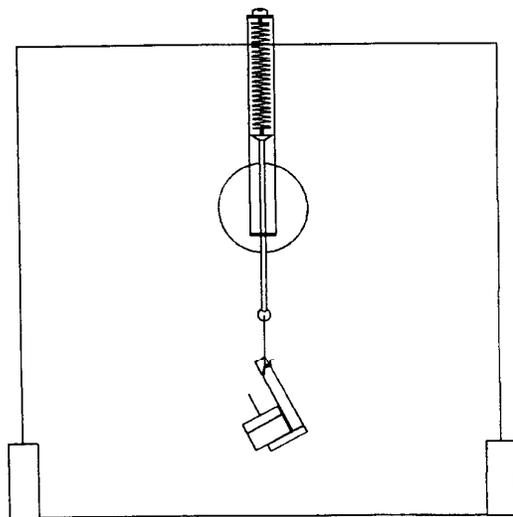


Figure 10.1 Mass on a Spring

Table 10.1

Mass (kg)	# Oscillations	Time (s) (Measured)	Period (s) (Average)	Period (s) (Calculated)
0.125				
0.175				
0.225				

Now pull the rod down a few centimeters. Steady the mass, then let go of the rod. Practice until you can release the rod smoothly, so that the mass and the rod oscillate up and down and there is no rubbing of the rod against the side of the hole.

Set the mass oscillating. Measure the time it takes for at least 10 full oscillations to occur. (Measure the time for as many oscillations as can be conveniently counted before the amplitude of the oscillations becomes too small.) Record the mass, the time, and the number of oscillations counted in Table 10.1. Divide the total time by the number of oscillations observed to determine the period of the oscillations. (The period is the time required for one complete oscillation). Record this value in the table.

Repeat the measurement 5 times. Calculate the period for each measurement. Then add your five period measurements together and divide by 5 to determine the average period over all five measurements.

Use the equation given at the beginning of this Experiment to calculate a theoretical value for the period using each mass value. (Since the spring constant is in units of newtons/m, your mass values used in the equation must be in kg.) Enter this value in the table.

- ② Does your theoretical value for the period accurately predict your experimental value?
Repeat the experiment using masses of 175 and 225 grams (including the mass of the Mass Hanger).
- ③ Does the equation for the period of an oscillating mass provide a good mathematical model for the physical reality?

Experiment 11: Simple Harmonic Motion—the Pendulum

EQUIPMENT NEEDED:

- Experiment Board
- Pivot
- Mass Hanger
- Masses
- String

Theory

Simple harmonic motion is not restricted to masses on springs. In fact, it is one of the most common and important types of motion found in nature. From the vibrations of atoms to the vibrations of airplane wings, simple harmonic motion plays an important role in many physical phenomena.

A swinging pendulum, for example, exhibits behavior very similar to that of a mass on a spring. By making some comparisons between these two phenomena, some predictions can be made about the period of oscillations for a pendulum.

Figure 11.1 shows a pendulum with the string and mass at an angle θ from the vertical position. Two forces act on the mass; the force of the string and the force of gravity. The gravitational force, $F = mg$, can be resolved into two components, F_x and F_y . F_y just balances the force of the string and therefore does not accelerate the mass. F_x is in the direction of motion of the mass, and therefore does accelerate and decelerate the mass.

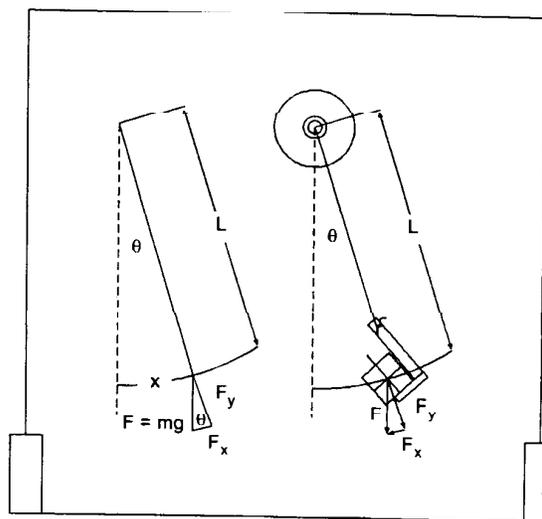


Figure 11.1 Pendulum

Using the two congruent triangles in the diagram, it can be seen that $F_x = mg \sin\theta$, and that the displacement of the mass from its equilibrium position is an arc whose distance, x , is approximately $L \tan\theta$. If the angle θ is reasonably small, then it is very nearly true that $\sin\theta = \tan\theta$. Therefore, for small swings of the pendulum, it is approximately true that $F_x = mg \tan\theta = mgx/L$. (Since F_x is a restoring force, the equation could be stated more accurately as $F_x = -mgx/L$.) Comparing this equation with the equation for a mass on a spring ($F = -kx$), it can be seen that the quantity mg/L plays the same mathematical role as the spring constant. On the basis of this similarity, you might speculate that the period of

motion for a pendulum is just:

where m is the mass, g is the acceleration due to gravity, and L is distance from the pivot point to the center of mass of the hanging mass. In this experiment, you will test the validity of this equation.

Table 11.1

Mass (kg)	L (m)	# Oscillations	Time (s) (Measured)	Period (s) (Average)	Period (s) (Calculated)
-----------	-------	----------------	------------------------	-------------------------	----------------------------

Experiment

- ① Hang a Mass Hanger from the pivot as shown in Figure 11.1. Set the mass swinging, but keep the angle of the swing reasonably small. Measure the time it takes for at least 30 full oscillations to occur. In Table 11.1, record the mass, the distance L , the time, and the number of oscillations counted. Divide the total time by the number of oscillations observed to determine the period of the oscillations. (The period is the time required for one complete oscillation). Record this value in the table.

Repeat the measurement 5 times. Calculate the period for each measurement. Then add your five period measurements together and divide by 5 to determine the average period over all five measurements

Repeat your measurements using a different mass.

Use the equation given at the beginning of this Experiment to calculate a theoretical value for the period in each case ($g = 9.8 \text{ N/m}$; be sure to express L in meters when you plug into the equation). Enter this value in the table.

- ② Does the period of the oscillations depend on the mass of the pendulum?
- ③ Does your theoretical value for the period accurately predict your experimental value?
- Repeat the experiment using a significantly different string length.
- ④ Does the equation for the period of an oscillating mass provide a good mathematical model for the physical reality?

Experiment 12: The Lever

EQUIPMENT NEEDED:

- | | |
|--|--|
| <ul style="list-style-type: none"> - Experiment Board - Pivot - Mass Hanger - Pulley | <ul style="list-style-type: none"> - Balance Beam - Spring Balance - Masses - String |
|--|--|

Theory

The workings of a lever can be understood using the concept of torque. When the torque of the applied force is greater than that of the load, the lever will rotate, raising the load. However, levers can also be understood using the concepts of work and conservation of energy.

In physics, work has a precise mathematical definition. Work is the force applied to an object multiplied by the distance over which that force acts. Whenever work is performed on an isolated system, the energy of the system will increase by exactly the amount of work that was performed. In this experiment, you will apply a measurable amount of work to a lever and observe the change in potential energy of the load.

Setup

Set up the equipment as shown in Figure 12.1. Be sure that the Balance Beam is centered in the pivot and use pieces of tape so the pivot point does not slip during the experiment. Raise or lower the Spring Balance until the Balance Beam is level.

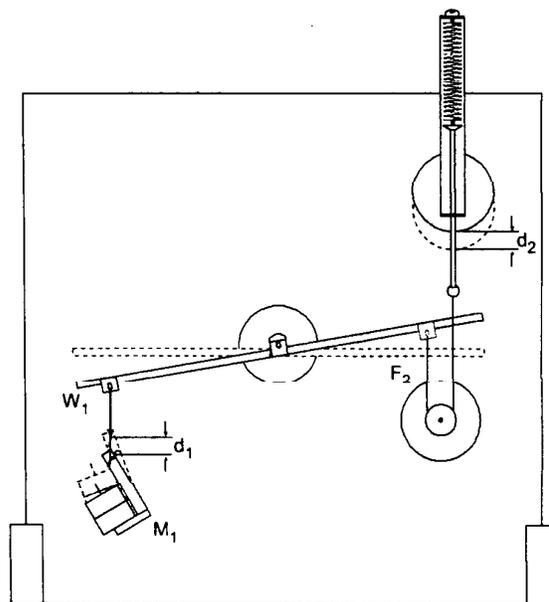


Figure 12.1 Equipment Setup

Procedure

- ① Make and record the necessary measurements to show that the torques provided by the Spring Balance and the Hanging Mass are balanced.

Mark the position of the Spring Balance and the Hanging Mass on the Experiment Board using a Magic Marker or pieces of tape. Slowly push the Spring Balance upward. If you perform this movement slowly enough, the reading on the Spring Balance will not vary appreciably.

- ② Measure distances d_1 and d_2 as shown above. Also record F_2 , the reading on the newton scale of the Spring Balance, and M_1 and W_1 , the mass and weight of the hanging mass.

$$d_1 = \underline{\hspace{2cm}} \quad M_1 = \underline{\hspace{2cm}} \quad W_1 = \underline{\hspace{2cm}}$$

$$d_2 = \underline{\hspace{2cm}} \quad F_2 = \underline{\hspace{2cm}}$$

- ③ Calculate the work performed on the system as you raised the Spring Balance (Work = $F_2 d_2$).

Work = _____ .

- ④ Calculate ΔU , the change in potential energy of the hanging mass as it was raised in the Earth's constant gravitational field ($\Delta U = W_1 d_1$).

$\Delta U =$ _____

- ⑤ Was the work performed on the system equal to the change in energy of the system?

Change the position of the hanging mass and repeat the experiment.

- ⑥ Is the work performed on the system equal to the change in energy of the system for this new configuration?

The lever you just used was a Class I lever—the fulcrum, or pivot point, was between the load (the hanging weight) and the applied force (the Spring Balance). Figure 12.1 and 12.2 show the relationship between the fulcrum, the load, and the applied force for Class II and Class III levers. Repeat the experiment using each of these arrangements. Make whatever measurements are required to determine if the work applied to the system equals the change in energy of the system. On a separate sheet of paper, diagram your results and show your measured values and your calculations. (For accurate results include the mass of the Balance Beam in your calculations—see Experiment 6.)

Questions

- ① Based on your observations, how does a lever make it easier to perform work, such as raising a heavy load? Explain in terms of conservation of energy.

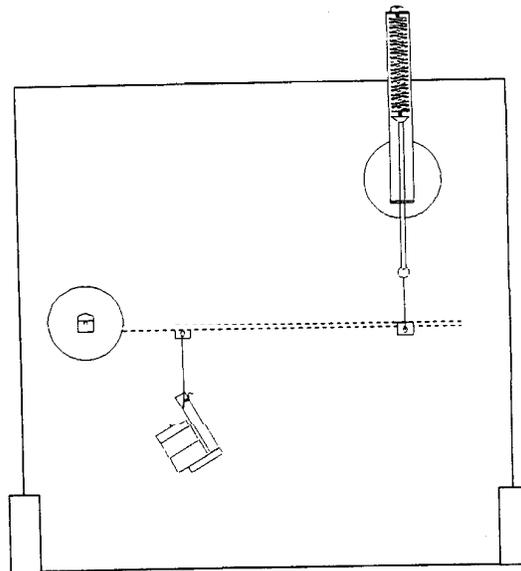


Figure 12.2 Class II Lever

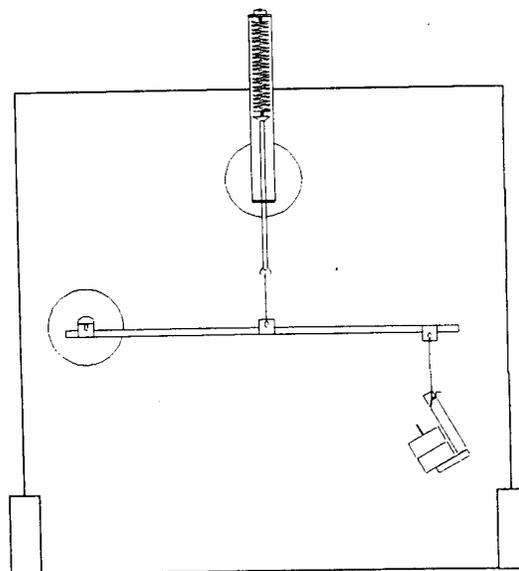


Figure 12.3 Class III Lever

Experiment 13: The Inclined Plane

EQUIPMENT NEEDED:

- | | |
|--|--|
| <ul style="list-style-type: none"> - Experiment Board - Rolling Mass - Pulley | <ul style="list-style-type: none"> - Inclined Plane - Spring Balance - String |
|--|--|

Introduction

The inclined plane, like the lever, is often used to help raise heavy objects. In Experiment 8 you analyzed this use of the inclined plane in terms of the forces that are involved. In this experiment you will take a second look at the inclined plane, using the concepts of work and conservation of energy, as you applied them in Experiment 12. (Review Experiment 8 and 12, if necessary, so that you understand the concepts of work and the resolution of a force into components.)

- ① Use the Spring Balance to determine M , the mass, and W , the magnitude of the weight, of the Rolling Mass.

$$M = \underline{\hspace{2cm}}$$

$$W = \underline{\hspace{2cm}}$$

Set up the equipment as shown in Figure 13.1.

- ② Record F , the magnitude of the force exerted by the Spring Balance on the Rolling Mass.

$$F = \underline{\hspace{2cm}}$$

Use tape or a Liquid Chalk type pen to mark the position of the Spring Balance on the Experiment Board. Now slowly raise the Spring Balance—slow enough so there is no appreciable change in the reading on the Spring Balance.

- ③ Record d_1 , the distance through which the Spring Balance pulled the Rolling Mass.

$$d_1 = \underline{\hspace{2cm}}$$

- ④ Calculate the amount of work performed on the Rolling Mass as you pulled up the Spring Balance.

$$\text{Work} = F d_1 = \underline{\hspace{2cm}}$$

- ⑤ Calculate ΔU , the change in potential energy of the rolling Mass as it was raised in the Earth's gravitational field.

$$\Delta U = Mg \text{ (vertical height raised)}$$

$$= mg \sin \theta = \underline{\hspace{2cm}}$$

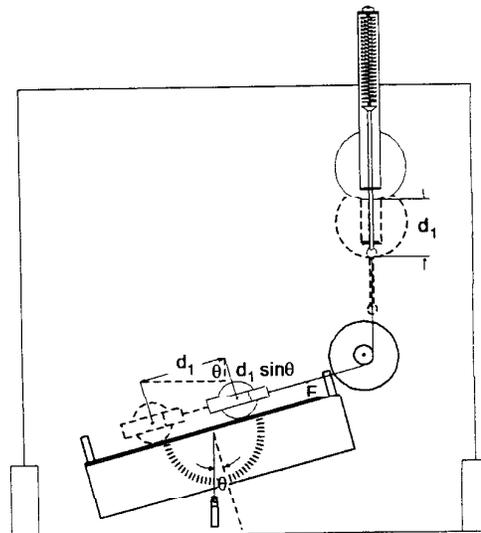


Figure 13.1 Equipment Setup

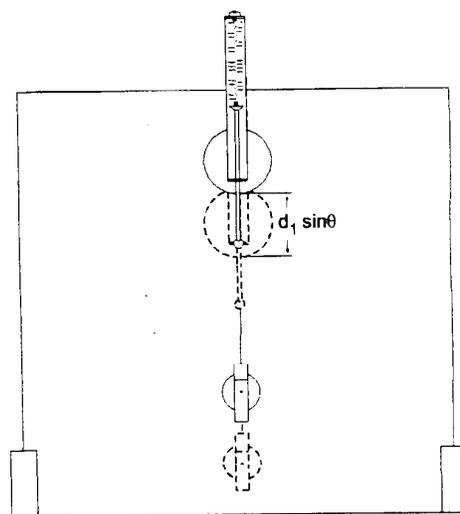


Figure 13.2 Performing Work without the Inclined Plane

- ⑥ Is the change in potential energy equivalent to the work performed on the mass?

Now hang the Rolling Mass directly from the Spring Balance as shown in Figure 13.2. Raise the Spring Balance very slowly a distance $d_1 \sin \theta$. Raise the balance slowly and smoothly enough so that there is no appreciable change in the reading of the Spring Balance.

- ⑦ How much work was performed on the Rolling Mass to lift it straight up by a distance $d_1 \sin \theta$?

- ⑧ What was the change in the potential energy of the Rolling Mass?

$$\Delta U = \underline{\hspace{2cm}} .$$

- ⑨ Does the inclined plane reduce the amount of work necessary to raise the Rolling Mass a given distance? Explain.

- ⑩ What is the advantage of using the inclined plane?

Experiment 14: The Pulley

EQUIPMENT NEEDED:

- Experiment Board
- Large Pulley (1)
- Small Pulley (2)
- Mass Hanger (1)
- Pulley Block
- Spring Balance
- Masses
- String

Theory

As in many of the experiments you have performed, pulleys can be used simply to change the direction of applied forces. However, systems of pulleys can also be arranged to perform much the same function as a lever or an inclined plane, translating small applied forces into much larger forces. In this experiment you will investigate how pulley systems can be used to amplify force and to perform work.

Procedure

In this experiment, the effects of friction are more pronounced than with the lever or the inclined plane. To investigate the effects of friction in the pulleys:

- ① Weigh a Mass Hanger with a 200 gram mass on the Spring Balance. Record your result.
- ② Set up the equipment as in Figure 14.1, using the same Mass Hanger and mass. Record the reading on the Spring Balance.
- ③ Tap on the Experiment Board, just enough so that the string is allowed to move slightly in the pulley. Does the reading on the Spring Balance change? How does pulley friction affect the transfer of force between the Spring Balance and the hanging mass?

In an ideal pulley system there would be no friction in the pulleys. The force from the Spring Balance would be transferred completely to the hanging mass.

As with the lever and the inclined plane, pulley systems can be understood by analyzing either the forces acting in the system or the work performed on and by the system. Set up each of the pulley systems shown in Figure 14.2. For each pulley system, fill in Table 14.1 as follows:

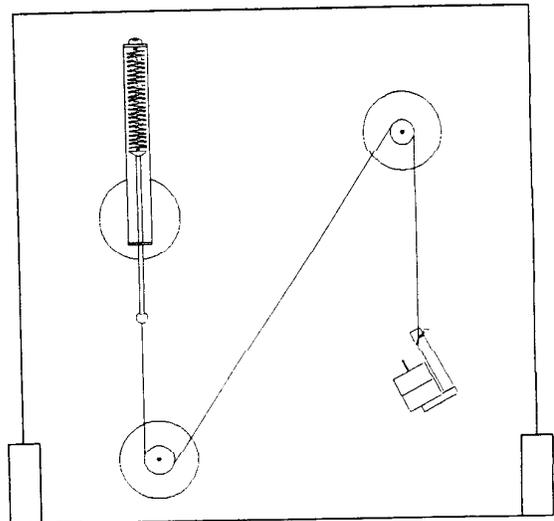


Figure 14.1 Equipment Setup

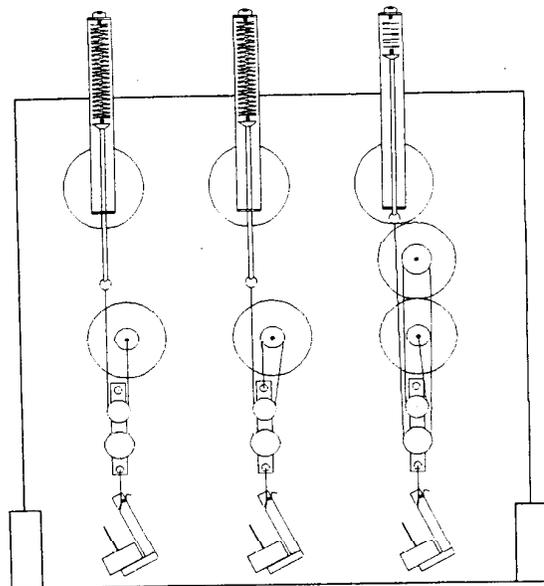
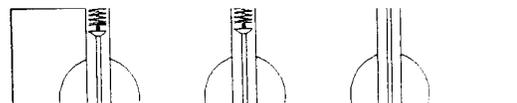


Figure 14.2 Pulley Systems

- ③ Tap on the Experiment Board, just enough so that the string is allowed to move slightly in the pulley. Does the reading on the Spring Balance change? How does



Data and Calculations

Compare forces by measuring:

W—the weight of the hanging mass, weighed directly on the Spring Balance, without pulleys.

Table 14.1

W	F	W/F	d_1	d_2	Work	ΔU
---	---	-----	-------	-------	------	------------

F—the weight of the hanging mass as transferred to the Spring Balance through the pulley system.

Calculate W/F to determine the amplification of the force by the pulley system.

Perform work on the system by slowly raising the Spring Balance. Compare the work performed to the increase in energy of the system by measuring:

d_1 —the distance over which you raised the Spring Balance.

d_2 —the distance over which the load (the hanging mass) was raised.

Calculate:

Work—the work performed in raising the Spring Balance ($F_1 \times d_1$).

ΔU —the increase in potential energy of the hanging mass ($W \times d_2$).

Questions

- ① How does the relationship between W and F parallel the relationship between d_1 and d_2 ? Explain.
- ② Can you explain the value of W/F for each pulley system? (Hint: Assume that, for each system, the tension on the string is constant.)

Experiment 15: Designing a Beam Balance

EQUIPMENT NEEDED:

- | | |
|--|---|
| <ul style="list-style-type: none"> - Experiment Board - Small Pulley (2) - Pulley Block - Masses | <ul style="list-style-type: none"> - Large Pulley (1) - Mass Hanger (1) - Spring Balance - String |
|--|---|

Theory

Figure 15.1 shows a diagram of a simple beam balance. An unknown mass, M_1 , is placed on the Balance Beam, a distance d_1 from the pivot point. A known mass, M_2 , at a distance d_2 from the pivot point is found to just balance the beam. The mass of M_1 can then be determined using the equation:

$$M_1 = \frac{M_2 d_2}{d_1}$$

A practical beam balance using the design shown above would require a ruler to measure d_1 and d_2 and, depending on the range of masses to be measured, a variety of precalibrated masses.

Your task in this experiment is simple in concept, yet it can be quite complicated in practice. Design a beam balance. Feel free to use any or all of the materials included in the Introductory Mechanics System. You might also want to use other readily available items (paper, rulers, and paper clips for example). But keep in mind the qualities that are necessary in a practical design.

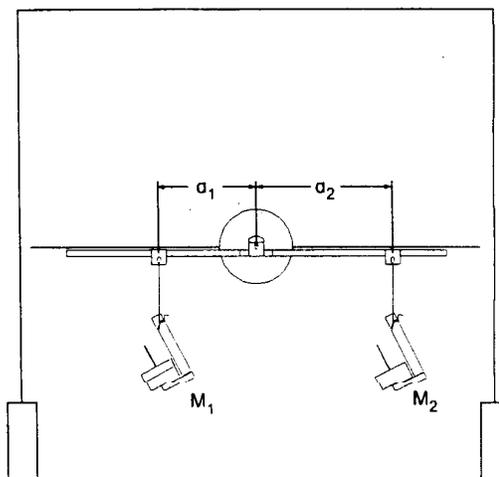


Figure 15.1 Beam Balance

- ① It must perform the task for which it is intended.

You could design the balance to provide measurements as accurately as possible over a small range, or one that will give somewhat less accurate readings but for a much greater range of masses. You could also design it for measuring small masses or for measuring larger masses. Decide what range of masses you want the balance to measure, then optimize the balance for that range.

- ② It should be as simple as possible to use.

Few people would be interested in a beam balance that required them to perform calculations to convert a distance measurement into a mass measurement. A built in scale is, therefore, one way to simplify the use of the balance. Can you think of others?

- ③ It should be as easy as possible to build.

In business terms, it better not cost more to build than people are willing to pay for it. This is not to say that there should be no extra features, just that features should only be added if they sufficiently enhance the overall design.

- ④ It should be appealing. Utility is not the only criteria in an effective design. An attractive design is also important. In general, expense or complexity should be avoided in a design, but if it adds appeal . . . ? You decide.

MAINTENANCE

Aside from periodic cleaning, no regular maintenance is required for the ME-9299 Introductory Mechanics System. All parts may be safely cleaned with soapy water.

NOTE: Clean with a mild detergent. Avoid abrasive cleaners and oil based solvents.

PARTS LIST

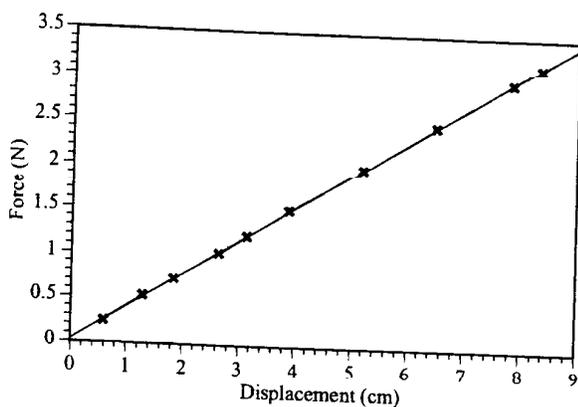
PASCO PART #	DESCRIPTION	QUANTITY
003-02363	Blackboard Assembly	1
648-02366	Left Foot	1
648-02367	Right Foot	1
003-03231	Spring Scale	1
003-02359	Degree Plate	1
616-040	Force Ring	3
648-02344	Holding Pin	1
648-04857	Weight Hanger	6
648-02337	100 gram Mass	2
648-02338	50 gram Mass	2
648-02339	20 gram Mass	4
648-02901	10 gram Mass	2
003-02355	Small Pulley	2
003-02356	Large Pulley	1
003-02360	Pulley Block	1
003-02357	Balance Beam	1
648-02379	Plastic Retainer (Center)	1
648-02023	Plastic Retainer (Side)	3
003-02358	Torque Wheel	1
003-02847	Torque Wheel Indicator	3
003-02361	Inclined Plane	1
003-02362	Rolling Mass	1
003-01331	Friction Block	1
648-02373	Planar Mass	1
699-016	String (Spool)	1
012 02377	Manual	1

Teacher's Guide

Experiment 1: Hooke's Law—Measuring Forces

Notes on Procedure

①

Slope = 37.77 ± 0.074 N/m

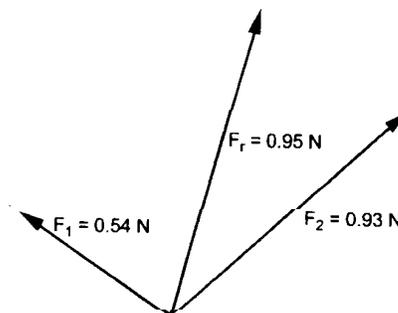
- ② The spring constant is the slope of the graph:
 37.77 ± 0.074 N/m
- ③ If Hooke's law were not valid, then it would still be possible to measure forces with a spring; but the spring scale would have to be non-linear.
- ④ Hooke's law is useful when calibrating a spring for measuring forces because it is not necessary to measure the displacement at every possible mass. Since the spring stretches linearly, it is only necessary to measure the displacement for two different masses and then make a linear scale based on those two measurements.

Experiment 2: Adding Forces—Resultants and Equilibrants

Notes on Procedure

- ① $F = 0.735$ N at 270°
- ② $F_c = 0.73$ N at 90°
- ③ $F_1 = 0.54$ N at 146°
 $F_2 = 0.93$ N at 42°
 $F_e = 1.0$ N at 256°

④, ⑤



- ⑥ The equilibrant force vector does not exactly balance the resultant force vector, but it's close. Sources of error might include static friction and inaccuracy in the spring scale, as well as uncertainty in the masses used.

Experiment 3: Resolving Forces—Components

Notes on Procedure

- ①,⑤ $F = 0.93 \text{ N}$ at 56°
- ②,③ x-component = $-F_x = 0.50 \text{ N}$
 y-component = $-F_y = 0.74 \text{ N}$
- ⑥ $F \cos q = 0.52 \text{ N}$
 $F \sin q = 0.77 \text{ N}$

These values are close to the x and y components from part 2, but not exactly the same. Differences are due to static friction in the pulleys and uncertainties in the masses and spring scale. Generally results should be within 5% or so.

- ⑧ The main difficulty is due to the fact that non-perpendicular components are not entirely independent. Changing the “x” component causes the non-perpendicular “y” component to change also, so balancing things is quite difficult.

Experiment 4: Torque—Parallel Forces

Notes on Procedure

- ① $d_1 = 9.5 \text{ cm} = d_2$
- ② After adding 50g to each weight hanger, the beam is nearly balanced. (It may not be exactly balanced due to variations in the 50g masses.)
- ③ It is possible to restore the balance by moving the hangers so that the heavier mass is nearer to the pivot than the lighter mass.
- ④ $M_1 = 77.2\text{g}$, $d_1 = 7.0 \text{ cm}$, Therefore $T_1 = 5.30$. In each balanced case, T_2 is nearly equal to T_1 .

- ⑤ The torque produced by the upward pull on the pivot point is zero, since the distance from the pivot point to the pivot point is zero.
- ⑥ The clockwise and counterclockwise torques must be equal.

Experiment 5: Torque: Non-Parallel Forces

Notes on Procedure

- ①-⑦ Actual values obtained will vary, but the percent difference between the two opposing torques should not be more than 5-10%.

Table 5.2

	T2 = 6.48		d1 = 14.3
Angle	F1	T1	% diff
30	0.87	6.22	4.08%
40	0.67	6.16	5.08%
50	0.63	6.90	-6.29%
60	0.52	6.44	0.62%
70	0.48	6.45	0.46%
80	0.45	6.34	2.22%

- ④ Applying $T = F d \sin \theta$ does not affect the results, because the angle θ is 90° for F_1 in each case.
- ⑥ In each case, the total torque is small compared to the torques applied to the wheel. It is not always zero, due to frictional torques which we are not able to measure.

Experiment 6: Center of Mass

Notes on Procedure

- ① If the forces were not concurrent there would be a net torque on the balance beam. This torque would cause the bar to rotate.
- ② The sum of the counterclockwise torques is equal to the sum of the clockwise torques. Otherwise there would be a net torque, and the bar would rotate.
- ③-⑤ The torques will not add up to exactly zero; but in general the total of the clockwise and counterclockwise torques should be less than 5% of the largest torque used.

- ⑥ In each case, the line of the string passes through the same point. This point is the center of mass.
- ⑦ In theory, this would also work for a three-dimensional object. The difficulty lies in the actual application: drawing lines through the center of a 3-d object is seldom possible.

Experiment 7: Equilibrium of Physical Bodies

Notes on Procedure

①,② Actual values will vary, but the total of clockwise and counterclockwise torques should be small compared to the magnitude of the torques used.

The first data set is using the spring attachment point as the center of torque measurements, the second is using the left end of the bar.

③ The total force and total torque must both be zero.

④ The total torque should be the same regardless of the point chosen for measurement.

Notes

There is little friction involved with this experiment, and if you measure the weights with an accurate balance and calibrate your spring scale accurately, there should be quite a bit less error than shown here.

Experiment 8: The Inclined Plane

Notes on Procedure

①-③ The vector model predicts the measured results to within 5%.

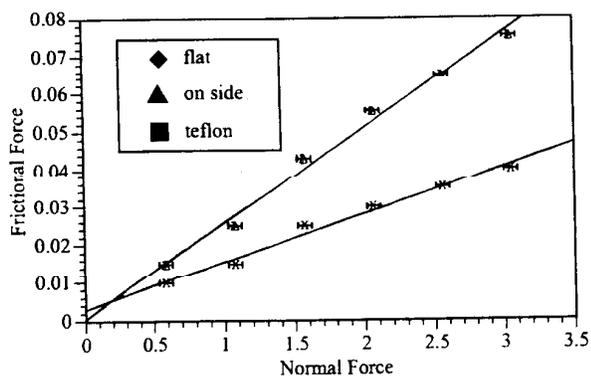
④-⑥ The x-components of the forces are nearly equal.

⑦-⑧ The y-components are also nearly equal.

⑨ The vector model predicts what the measured values will be in both the x and y directions. The predictions should be valid to within 5% or so.

Experiment 9: Sliding Friction

Notes on Procedure



- ② As shown in the graph, the relationship between sliding friction and normal force is roughly linear.
- ③ The first two graph series show that friction is independent of area
- ④ The third graph series shows that there is a dependence on material.
- ⑤ Answers will vary: the formula is $F_f = m g \mu \cos(45^\circ)$, where μ is the coefficient of friction for the wood on metal (.025 in this case)

Experiment 10: Simple Harmonic Motion—Mass on a Spring

Notes on Procedure

- ① The spring constant (from experiment ①) is 37.77 ± 0.074 N/m. (The spring that is supplied with your apparatus may be slightly different.)

- ② There should be good agreement between theoretical and actual values.

Experiment 11: Simple Harmonic Motion—the Pendulum

Notes on Procedure

- ② The length does not affect the period of the pendulum. There will be some variation due to the fact that adding mass to the pendulum changes its effective length.
- ③-④ Overall, there is good correspondence between theoretical and actual.

Notes

Keep the amplitude of the oscillations below 5° or so for best results. Also, remember that the length from the pivot point to the center of mass will change when you stack more mass on the holder.

M (kg)	L (m)	T (s)	
		average	calculated
.0206	0.283	1.061	1.068
.0507	0.283	1.052	1.000
.0507	0.180	0.839	0.852
.1000	1.970	2.774	2.817

Experiment 12: The Lever

Notes

①-⑥ There is some difficulty getting consistent results with this experiment, due primarily to the problem of both keeping the distances small so that our approximations are still valid and then measuring those small distances accurately. (The approximation $\cos(\theta) = 1$ for small θ is made in the lab description, although this is not explicitly stated.) One trick that helps in measuring the distances is to shine a spotlight onto the board from across the room and then measure the displacement of the various shadows. This should reduce parallax error significantly.

③ Using a lever can make it easier to apply a large force to an object. With the lever, you can apply a small force over a long distance, and the result will be a large force over a small distance. The energy put into the long end is equal to the energy gotten out of the short end.

	d1	d2	M1	F2	Work	δU	δ
Class I	2	1.7	105	1.3	2.21	2.058	7.12%
	1.8	2.6	105	0.8	2.08	1.8522	11.58%
	0.7	1.5	205	0.9	1.35	1.4063	-4.08%
Class II	1.5	3.2	205	1.1	3.52	3.0135	15.50%
	0.9	3.4	205	0.7	2.38	1.8081	27.31%
	2.4	2.5	205	1.8	4.5	4.8216	-6.90%
Class III	2.6	1.6	105	1.7	2.72	2.6754	1.65%
	3.4	1.2	105	2.35	2.82	3.4986	-21.47%
	4.5	1.2	105	3.4	4.08	4.6305	-12.63%

Experiment 13: The Inclined Plane

Notes on Procedure

① Roller Weight: 1.62 N

②-⑥ Answers will vary, depending on ramp angle. For a 30° angle, the work and change in potential energy were found to match almost exactly in each

case.

⑦-⑨ The work and change in potential energy were again the same, and the amount of work was the same as for parts 2-6.

⑩ The advantage of using an inclined plane is that the force required to move the object is less.

Experiment 14: The Pulley

Notes on Procedure

- ① 2.1 N
- ② 2.05 N (Answers will vary, depending on the state of your pulleys and the mass used.)
- ③ The friction in the pulleys lessens the amount of force that reaches the spring scale.
- ④ W/F roughly parallels d_1/d_2 , with W/F being slightly lower due to friction.

⑥ The tension in the string is constant throughout the entire length. Therefore, the total force (tension) is just the force of gravity on the object being lifted divided by the number of strings supporting that object. In the cases given for this lab, the number of strings is 2, 3, and 4, respectively; and this corresponds to our data.

W Work	F δU	W/F	d_1	d_2
2.1 2.20	1.10 2.10	1.91	2.0	1.0
2.1 2.25	0.75 2.10	2.80	3.0	1.0
2.1 2.20	0.55 2.10	3.82	4.0	1.0

Technical Support

Feed-Back

If you have any comments about this product or this manual please let us know. If you have any suggestions on alternate experiments or find a problem in the manual please tell us. PASCO appreciates any customer feed-back. Your input helps us evaluate and improve our product.

To Reach PASCO

For Technical Support call us at 1-800-772-8700 (toll-free within the U.S.) or (916) 786-3800.

email: techsupp@PASCO.com

Tech support fax: (916) 786-3292

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Before you call the PASCO Technical Support staff it would be helpful to prepare the following information:

- If your problem is with the PASCO apparatus, note:
Title and Model number (usually listed on the label).

Approximate age of apparatus.

A detailed description of the problem/sequence of events. (In case you can't call PASCO right away, you won't lose valuable data.)

If possible, have the apparatus within reach when calling. This makes descriptions of individual parts much easier.

- If your problem relates to the instruction manual, note:

Part number and Revision (listed by month and year on the front cover).

Have the manual at hand to discuss your questions.