Information and Design

By Jean LeMee

"Information" in this context is not related to "meaning" but to the probability that an event will or will not take place.

The amount of information needed for a given task is proportional to the number of decisions one has to make to achieve that task. One needs neither too much nor too little. Just the right amount.

But what is the right amount and how is it measurable?

The probability of P_E an event E is instinctively connected with the amount of information (Surprise) that it brings:

The more probable, the less informative

$$I_E \alpha \frac{1}{P_E}$$

Since, also instinctively, information provided by two combined events should be additive while their probabilities are multiplicative, we can write:

$P_{ij} = P_i P_j$ and $I_{ij} = I_i + I_j$

therefore, the information is taken to be a log function so that

$$I_i = \log(\frac{1}{P_i})$$

Since information is often concerned with binary quantities (yes, no; 0,1; etc...) Base 2 logs are used.

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(1) Principles of Design, Nam Suh; Oxford University Press, NY 1990

$$I_i \triangleq \log_2\left(\frac{1}{P_i}\right)$$

The mean value of information regarding a sequence of events will be:

$$I = \sum_{i=1}^{m} P_i \log_2\left(\frac{1}{P_i}\right) = -\sum_{i=1}^{m} P_i \log_2 P_i$$

This definition of information has been used with great success in communication engineering (C. Shannon).

For purpose of <u>control</u> or <u>design</u>, this definition can be generalized so that "information" becomes a <u>measure of the probability of success,</u> or the system's <u>ability to achieve its goals.</u>

If, for instance, the probability P_0 of achieving a particular goal <u>(fulfilling a particular FR)</u> prior to a particular event (E.G. receipt of a communication) is changed to P_1 after that event takes place, then, the information content brought in by the event is defined as

$$I = \log_2\left(\frac{1}{P_0}\right) - \log_2\left(\frac{1}{P_1}\right) = \log_2\left(\frac{P_1}{P_0}\right)$$

If $P_1 > P_o$	$I > o$ The communication has increased the chance of fulfilling the goal (satisfying the FR's)
If $P_1 < P_o$	$I < o$ The communication decreases the probability of the goal being met (disinformation).
If $P_1 = P_o$	$I = o$ Nothing has changed, no information has been conveyed.

The sensitivity of an information stream may be defined as the ratio of the incremental performance ratio

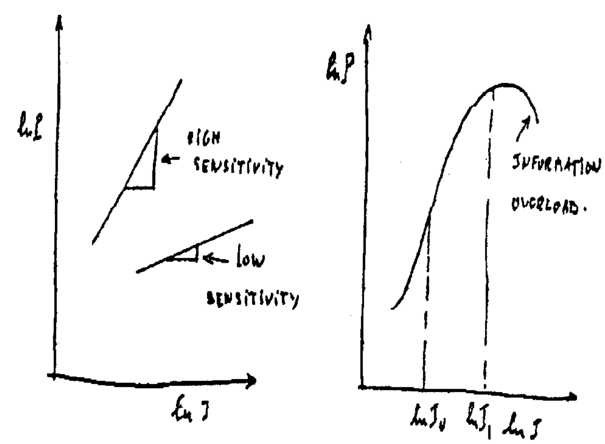
$$\frac{\partial P}{P}$$

to the incremental information

$$\frac{\partial I}{I}$$

Thus

$$S=\frac{\frac{\partial P}{P}}{\frac{\partial I}{I}}=\frac{\partial \ln P}{\partial \ln I}$$



A system should avoid information requirements beyond the minimum needed to attain its goals.

On that basis, one can define the efficiency of information.
E.G. if I_o is the sufficient information for a particular task (i.e. the minimum needed) and I the information supplied, then

$$\frac{I_o}{I} = \eta \triangleq \textit{Information Efficiency}$$

If $I > I_0$ ----> Redundancy

$$R = \frac{I - I_o}{I} = 1 - \eta$$

If $I < I_o$ ---> Deficiency

$$D = \frac{I_o - I}{I_o} = 1 - \frac{1}{\eta}$$

The information content of a design may be defined for a particular FR, FR_i, assuming a uniform distribution of the tolerance throughout the range of the same FR as:

$$I_i = \log_2 \frac{Range\ (of\ Fr_i)}{tolerance\ (on\ Fr_i)}$$

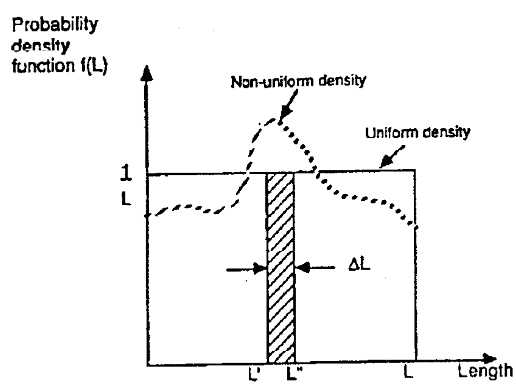


Fig 1 Probability distribution f in making the length measurement of a rod of length L. The probability of the measurement being within the shaded area is $p = \int_{L'}^{L'+\Delta L} f(L)\,dL$. Note that $\int_0^L f(L)\,dL = 1$.

The use of information in design being one of comparing various designs, from the viewpoint of satisfying the FR's the exact form of the information is not important.

When it comes to manufacturing, one has to consider the probability of satisfying the DP's with a particular manufacturing system (Fig. 2).

For each DP, a tolerance is chosen (Design Range). The capability of the manufacturing system is then given in terms of tolerances (System Range).

The common range between required DP tolerances (Design Range) and manufacturing Systems Range is the overlap between these two (Shaded area).

The probability of the manufacturing system being able to produce the specified DP's within the design range can therefore be taken as:

$$P=\frac{common\ range}{system\ range}$$

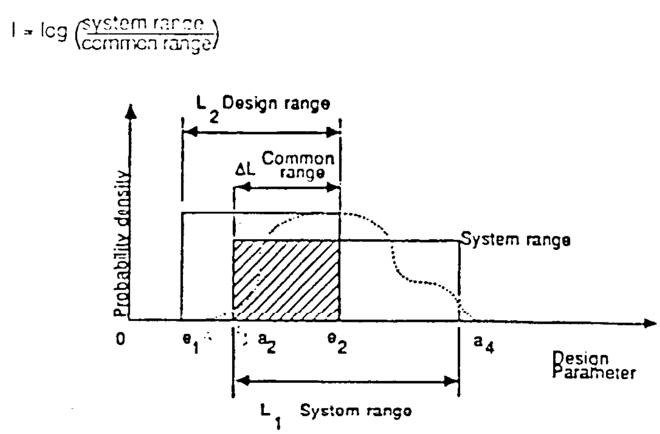


Fig. 2. Probability distribution of a system parameter. The dotted line is for the case of a non-uniform probability distribution. The shaded area is the common range where the designer-specified tolerance and the system tolerance overlap.

Therefore

$$I\text{-}\log_2\frac{(system\ range)}{(common\ range)}$$

If there is no common range, the system cannot satisfy the DP: The Probability of Producing the Part is Zero, (information content would have to be infinity).

If the Design Range (Tolerance on the DP's) covers the System Range (Tolerances the manufacturing system can produce), whatever the system produces is acceptable. The probability of the system meeting the DP is 1 and the corresponding information required is 0.

In a dynamic process (control, design) the timeliness of information is all important and affects system performance. Information loses value over time due to changes in the factors on which the process depends.

Like money, information can be discounted (money is a form of information, e.g. selling futures, etc.).

Once the goal (FR) is achieved, the event is realized, its probability becomes "certainty" therefore $P=1$; $I=o$.

If the goal (FR) is not achieved, the event is not realized, its probability of occurrence is "0" therefore $P=o$, $I=\infty$ i.e. no amount of information can make it happen.

One could define a distribution of the value of information V_I over time to characterize timeliness, with a peak corresponding to the "right time" t_0

$$V_I = \frac{V_{Imax}}{(t_o - t) + 1} \qquad o < t \leq t_o$$

$$= \quad o \qquad t > t_o$$

Many corollaries may be obtained from the two basic axioms. They constitute "Design Rules" that can be used in specific situations.

Cor. 1 (Decoupling of coupled design):
Decouple or separate parts or aspects of solution if FR's are coupled or become interdependent in proposed designs.

Cor. 2 (Minimization of FR's).
Keep number of FR's and constraints to a minimum.

Cor. 3 (Integration of Physical Parts)
...If FR's can be independently satisfied in proposed solution.

Cor. 4 (Use Standardization)
...If consistent with FR's and constraints.

Cor. 5 (Use of Symmetry)
...If consistent with FR's and constraints.

Cor. 6 (Largest Tolerances)
Specify largest allowable tolerance for FR's.

Cor. 7 (Chose Uncoupled Design with Less Information
Satisfying the FR's, than a coupled design.